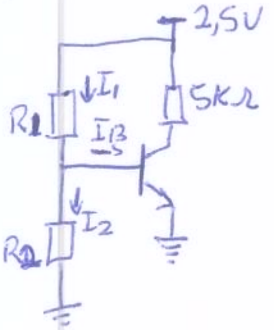
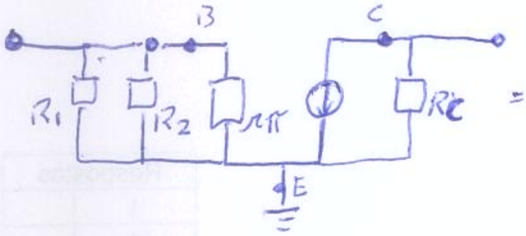
RAZAVI (5.13, 5.17, 5.38d, 5.47b

5.13



$R_{in} \geq 10K\Omega$
$g_m = \frac{1}{260} S$
$\beta = 100$
$I_S = 2.10^{-17} A$
$V_A = \infty$
R_1 e R_{2min}?

$\Rightarrow$



$\Rightarrow R_{in} = R_1 // R_2 // r_\pi$

- $g_m = \frac{I_C}{V_T} \Rightarrow \frac{1}{260} = \frac{I_C}{26.10^{-3}} \Rightarrow \underline{I_C = 1.10^{-4} A}$
- $V_{BE} = V_T \ln \frac{I_C}{I_S} \Rightarrow \underline{V_{BE} = 0,7603V}$
- $I_B = \frac{I_C}{\beta} \Rightarrow \underline{I_B = 1.10^{-6} A}$
- $r_\pi = \frac{\beta}{g_m} \Rightarrow \underline{r_\pi = 26.10^3 K\Omega} \rightsquigarrow$ impedância máxima
- $2,5 - R_1 I_1 - \underbrace{V_B}_{V_{BE}} = 0 \Rightarrow R_1 = \frac{2,5 - V_{BE}}{I_1} \xRightarrow{I_1 = I_2 + I_B} R_1 = \frac{2,5 - V_B}{I_2 + I_B} \Rightarrow \boxed{R_1 = \frac{2,5 - V_{BE}}{\frac{V_{BE}}{R_2} + I_B}}$ (1)
- $r_\pi // \underbrace{R_1 // R_2}_{Req} \geq 10k\Omega \Rightarrow r_\pi // Req \geq 10k\Omega \Rightarrow \cancel{\left(\frac{1}{r_\pi} + \frac{1}{Req}\right)^{-1} \geq \left(\frac{1}{10k\Omega}\right)^{-1}} \Rightarrow \frac{r_\pi Req}{r_\pi + Req} \geq 10.10^3$

$\Rightarrow r_\pi Req \geq 10^4 r_\pi + 10^4 Req \Rightarrow Req.(r_\pi - 10^4) \geq 10^4 r_\pi \Rightarrow \boxed{Req \geq 16.250\Omega}$

- $Req = R_1 // R_2 \Rightarrow Req = \frac{R_1 R_2}{R_1 + R_2} \Rightarrow Req R_1 + Req R_2 = R_1 R_2 \Rightarrow R_1 (R_2 - Req) = Req . R_2$

$R_1 = \frac{Req R_2}{R_2 - Req} \overset{(1)}{\Rightarrow} \frac{2,5 - V_{BE}}{\frac{V_{BE}}{R_2} + I_B} = \frac{Req R_2}{R_2 - Req} \Rightarrow \frac{2,5 - V_{BE}}{\frac{V_{BE} + R_2 I_B}{R_2}} = \frac{Req R_2}{R_2 - Req}$

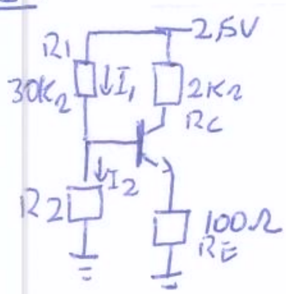
$\Rightarrow Req \cancel{R_2} . \frac{V_{BE} + R_2 I_B}{\cancel{R_2}} = (R_2 - Req)(2,5 - V_{BE}) \Rightarrow Req V_{BE} + R_2 Req I_B = R_2 (2,5 - V_{BE}) - Req (2,5 - V_{BE})$

$\Rightarrow R_2 . (2,5 - V_{BE} - Req I_B) = Req(\cancel{V_{BE}} + 2,5 - \cancel{V_{BE}}) \Rightarrow \boxed{R_2 = 2,3571.10^4 \Omega}$

$\overset{(1)}{\Rightarrow} \boxed{R_1 = 5,2318.10^4 \Omega}$

7

5.17



$\beta = 100$
$I_S = 10^{-17}$
$V_A = \infty$
$V_T = 26mV$

- $V_{CE} \geq V_{BE} \Rightarrow V_{CC} - I_C R_C \geq V_T \ln \frac{I_C}{I_S}$

$\Rightarrow V_T \ln \frac{I_C}{I_S} + I_C R_C - V_{CC} = \emptyset$

$\Downarrow$ Cálculo numérico (fsolve) $\to$ MATLAB

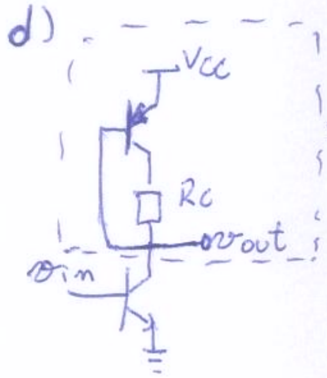
$\Rightarrow \underline{I_C = 8{,}3330 \cdot 10^{-4} A}$ ↳ máximo

- $I_B = \frac{I_C}{\beta} \Rightarrow \underline{I_B = 8{,}3330 \cdot 10^{-6} A}$
- $I_E = \frac{\beta}{\beta+1} \cdot I_C \Rightarrow \underline{I_E = 8{,}2505 \cdot 10^{-4} A}$
- $V_{BE} = V_T \ln \frac{I_C}{I_S} \Rightarrow \underline{V_{BE} = 0{,}8334 V}$
- $V_B = V_{BE} + R_E \cdot I_E \Rightarrow \underline{V_B = 0{,}9159}$
- $I_2 = I_1 - I_B \Rightarrow \frac{V_B}{R_2} = \frac{V_{CC} - V_B}{R_1} - I_B \Rightarrow \boxed{R_2 = 2{,}059 \cdot 10^4 \Omega}$ ↳ máx

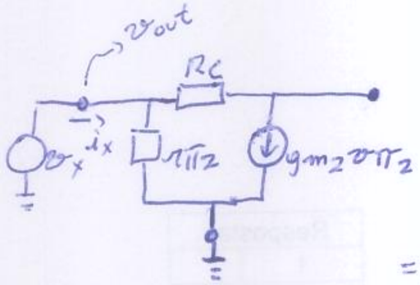
8

5.38

d)



$\Rightarrow$

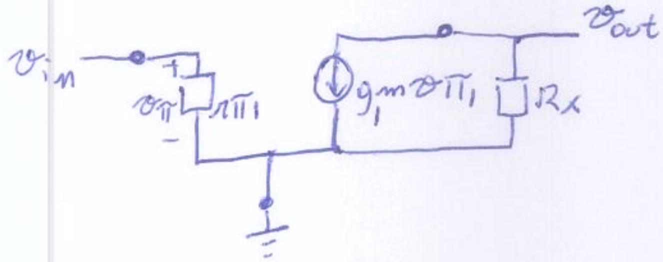


$$i_x = \frac{\overbrace{v_{\pi 2}}^{v_x}}{r_{\pi 2}} + g_{m2} v_{\pi 2}$$

$$i_x = \frac{v_x}{r_{\pi 2}} + g_{m2} v_{\cancel{x}}$$

$$\Rightarrow \frac{v_x}{i_x} = \frac{1}{\frac{1}{r_{\pi 2}} + g_{m2}} \Rightarrow R_x = r_{\pi 2} \parallel \frac{1}{g_{m2}}$$

$\Downarrow$



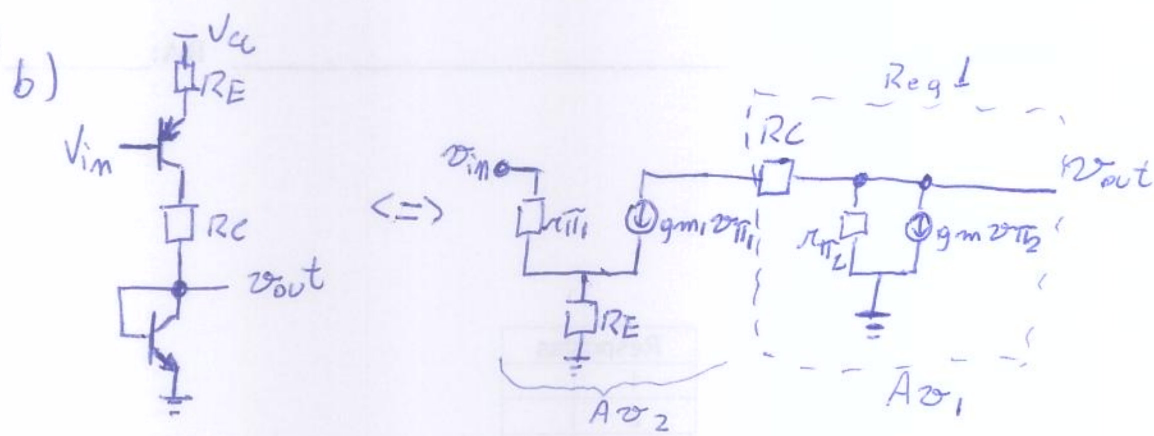
$\Rightarrow$

$$\boxed{R_{in} = r_{\pi 1}}$$

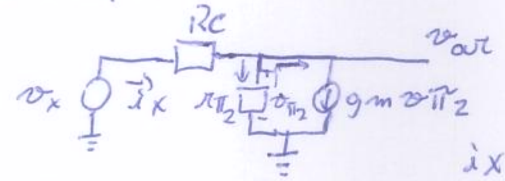
$$v_{out} = -v_{in} \cdot g_{m1} \cdot R_x \Rightarrow \boxed{A_v = -g_{m1}\left(r_{\pi 2} \parallel \frac{1}{g_{m2}}\right)}$$

$$R_{out} = R_x \Rightarrow \boxed{R_{out} = r_{\pi 2} \parallel \frac{1}{g_{m2}}}$$

(5,47) b)



- Req 1



$$i_x = i_{r\pi_2} + g_m v_{\pi_2}$$

$$\boxed{i_x = \frac{v_{\pi_2}}{r_{\pi_2}} + g_m v_{\pi_2}} \Rightarrow (1)$$

$$i_x = \frac{v_x - v_{\pi_2}}{R_C} \Rightarrow \boxed{v_{\pi_2} = v_x - i_x R_C} \; (2)$$

$$\overset{(1)}{\Longrightarrow} i_x = \frac{v_x - i_x R_C}{r_{\pi_2}} + g_m(v_x - i_x R_C) \Rightarrow i_x = \frac{v_x}{r_{\pi_2}} - \frac{i_x R_C}{r_{\pi_2}} + g_{m_2} v_x - g_{m_2} i_x R_C$$

$$i_x\left(1 + \frac{R_C}{r_{\pi_2}} + g_{m_2} R_C\right) = v_x \cdot \left(\frac{1}{r_{\pi_2}} + g_{m_2}\right) \Rightarrow Req_1 = \frac{v_x}{i_x} = \frac{1 + R_C\left(\frac{1}{r_{\pi_2}} + g_{m_2}\right)}{\left(\frac{1}{r_{\pi_2}} + g_{m_2}\right)}$$

$$Req_1 = \left(1 + R_C\left(\frac{1}{r_{\pi_2}} + g_{m_2}\right)\right) \cdot \left(\frac{1}{r_{\pi_2}} + g_{m_2}\right)^{-1} \Rightarrow Req_1 = \left(\frac{1}{r_{\pi_2}} + g_{m_2}\right)^{-1} + R_C$$

$$\boxed{\underset{\hookrightarrow \frac{v_x}{i_x}}{Req_1} = R_C + r_{\pi_2} // \frac{1}{g_{m_2}}}$$

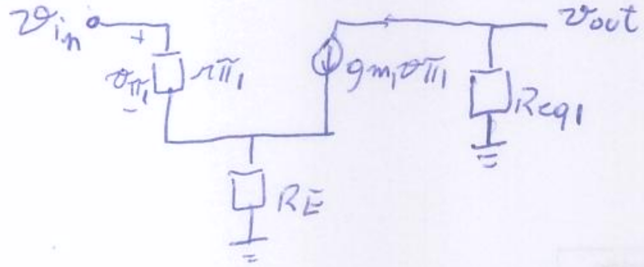
- $Av_1 =$

$$v_{out}' = v_{\pi_2} \overset{(2)}{\Rightarrow} v_{out}' = v_x - i_x \cdot R_C \Rightarrow v_{out}' = v_x - \frac{v_x R_C}{Req_1} \Rightarrow \underbrace{\frac{v_{out}'}{v_x}}_{Av_1} = 1 - \frac{R_C}{R_C + r_{\pi_2} // \frac{1}{g_{m_2}}}$$

$$Av_1 = \frac{R_C + r_{\pi_2} // 1/g_{m_2} - R_C}{R_C + r_{\pi_2} // \frac{1}{g_{m_2}}} \Rightarrow \boxed{Av_1 = \frac{r_{\pi_2} // 1/g_{m_2}}{R_C + r_{\pi_2} // 1/g_{m_2}}}$$

10

• A_{v_2} -



$v_{\pi_1} = v_{in} - v_{RE}$

$v_{\pi_1} = v_{in} - R_E\left(\frac{v_{\pi_1}}{r_{\pi_1}} + g_{m1} v_{\pi_1}\right)$

$$v_{\pi_1} = v_{in} - R_E v_{\pi_1}\left(\frac{1}{r_{\pi_1}} + g_{m1}\right) \Rightarrow v_{\pi_1}\left(1 + R_E\cdot\left(r_{\pi_1} // \frac{1}{g_{m1}}\right)^{-1}\right) = v_{in}$$

$$\boxed{v_{\pi_1} = \frac{v_{in}}{1 + R_E\left(r_{\pi_1} // \frac{1}{g_{m1}}\right)^{-1}}}$$ (A)

$$v_{out} = -g_{m1} v_{\pi_1}\cdot R_{eq1} = -g_{m1}\frac{v_{in}}{1 + R_E\left(r_{\pi_1} // \frac{1}{g_{m1}}\right)^{-1}}\cdot R_{eq1}$$

$$\underbrace{\frac{v_{out}}{v_{in}}}_{A_{v_2}} = \frac{-R_{eq1}}{\frac{1}{g_{m1}}\cdot\left[1 + R_E\cdot\left(\frac{1}{r_{\pi_1}} + g_{m1}\right)\right]} = \frac{-R_{eq1}}{\frac{1}{g_{m1}} + \frac{R_E}{g_{m1} r_{\pi_1}} + R_E\frac{g_{m1}}{g_{m1}}}$$

$$\frac{v_{out}}{v_{in}} = \frac{-R_{eq1}}{\frac{1}{g_{m1}} + R_E + \frac{R_E}{\beta}} \Rightarrow A_{v_2} = \frac{-R_{eq1}}{\frac{1}{g_{m1}} + R_E\cdot\left(1 + \frac{1}{\beta}\right)}$$

$$\boxed{A_{v_2} = \frac{R_C + r_{\pi 2} // \frac{1}{g_{m2}}}{\frac{1}{g_{m1}} + R_E\left(1 + \frac{1}{\beta}\right)}}$$

• $Z_{in} = \frac{v_{in}}{i_{in}}$

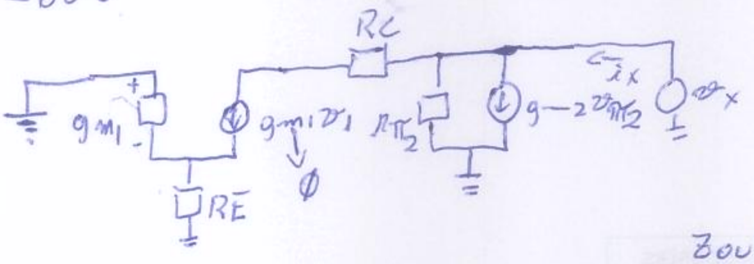
(A) $\Rightarrow v_{in} = v_{\pi_1}\left(1 + R_E\cdot\left(r_{\pi_1} // \frac{1}{g_{m1}}\right)^{-1}\right)$

$i_{in} = \frac{v_{\pi}}{r_{\pi_1}}$

$$Z_{in} = r_{\pi_1}\cdot\left[1 + R_E\cdot\left(\frac{1}{r_{\pi_1}} + g_{m1}\right)\right] = r_{\pi_1} + R_E r_{\pi_1}\left(\frac{1}{r_{\pi_1}} + \frac{\beta}{r_{\pi_1}}\right)$$

$$\boxed{Z_{in} = r_{\pi_1} + R_E\cdot(1+\beta)}$$

• Zout



$$i_x = g_{m2} v_{\pi 2} + \frac{v_{\pi 2}}{r_{\pi 2}}$$

$$v_{\pi 2} = v_x$$

$$i_x = v_x \cdot \left(g_{m2} + \frac{1}{r_{\pi 2}}\right)$$

$$Z_{out} = \frac{v_x}{i_x} = \frac{1}{g_{m2} + \frac{1}{r_{\pi 2}}} \Rightarrow Z_{out} = \frac{1}{g_{m2}} // r_{\pi 2}$$

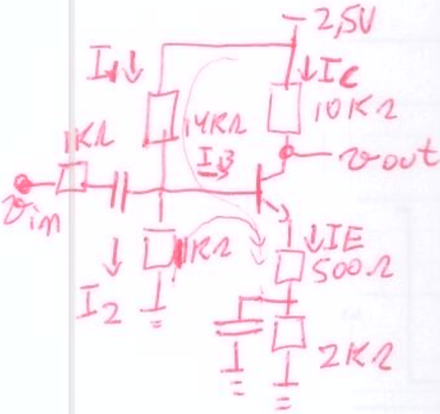
• $Av = Av_1 \cdot Av_2 \Rightarrow Av = \dfrac{r_{\pi 2} // 1/g_{m2}}{\cancel{(R_C + r_{\pi 2} // 1/g_{m2})}} \cdot \dfrac{\cancel{(R_C + r_{\pi 2} // \frac{1}{g_{m2}})}}{\frac{1}{g_{m1}} + R_E\left(1 + \frac{1}{\beta}\right)}$

$$\boxed{Av = \frac{r_{\pi 2} // \frac{1}{g_{m2}}}{\frac{1}{g_{m1}} + R_E \cdot \underbrace{\left(1 + \frac{1}{\beta}\right)}_{\sim 1}}}$$

AM

12

5.52 c) A_v? $I_S = 8.10^{-16} A$; $\beta = 100$; $V_A = \infty$



$$\bullet\ I_B = I_1 - I_2$$

$$\bullet\ I_1 = \frac{V_{cc} - (500+2000).I_E - V_{BE}}{14.000}$$

$$\bullet\ I_2 = \frac{V_{BE} + (500+2000).I_E}{11.000}$$

$$\bullet\ I_B = \frac{V_{cc} - 2500 I_E - V_{BE}}{14.000} - \frac{V_{BE} + 2500 I_E}{11000}$$

$$\Rightarrow I_B = \frac{I_C}{\underbrace{\beta}_{100}} \ e\ I_E = \frac{\beta+1}{\underbrace{\beta}_{100}} . I_C \Rightarrow \boxed{I_B = I_C.10^{-2}\ e\ I_E = 0,9901\, I_C}$$

$$I_C.10^{-2} = \frac{2,5 - 2500.0,9901 I_C - V_{BE}}{14000} - \frac{V_{BE} + 2500.0,9901 I_C}{11.000}$$

$$I_C = \frac{110(2,5 - 2,4752.10^3 I_C - V_{BE}) - 140.(V_{BE} + 2,4752.10^3 I_C)}{140 . 110}$$

$$I_C = \frac{275 - 2,7278.10^5 I_C - 110 V_{BE} - 140 V_{BE} - 3,4653.10^5 I_C}{15.400}$$

$$15.400 I_C = -6.1881.10^5 I_C - 250 V_{BE} + 275$$

$$\Rightarrow 6.3421.10^5 I_C + 250 V_{BE} - 275 = 0$$

$$V_{BE} = \underbrace{V_T}_{0,026} \ln \frac{I_C}{I_S}$$

$$\Rightarrow 6,3421.10^5 I_C + 250.0,026 \ln \frac{I_C}{8.10^{-16}} - 275 = 0$$

$$6,3421.10^5 I_C + 6,5 . \ln \frac{I_C}{8.10^{-16}} - 275 = 0$$

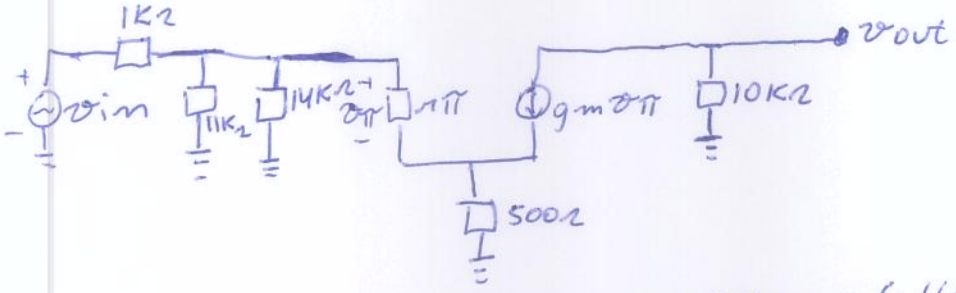
$$\Rightarrow \boxed{I_C = 1,6651.10^{-4} A}$$

$$\bullet\ g_m = \frac{I_C}{\underbrace{V_T}_{0,026}} \Rightarrow \boxed{g_m = 6,4041.10^{-3} S}$$

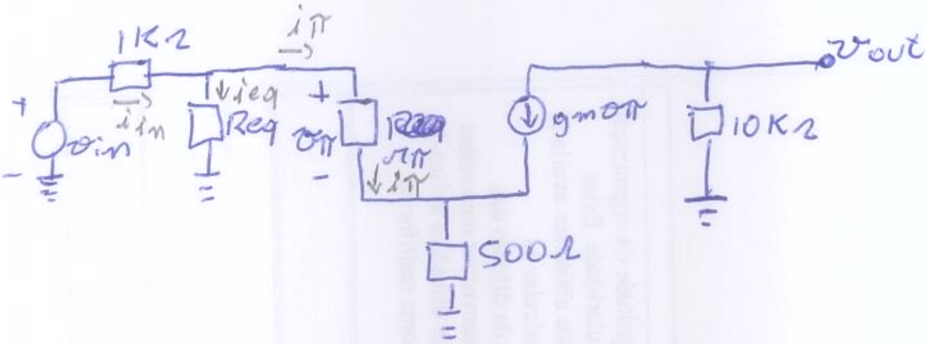
$$\boxed{r_o = \infty}$$

$$\bullet\ r_\pi = \frac{\beta}{g_m} \Rightarrow \boxed{r_\pi = 1,5615.10^4 \Omega}$$

- Modelo de Pequenos Sinais



$$\Downarrow \quad R_{eq} = 11K\Omega // 14K\Omega = 6.160\Omega$$



$$v_{in} = 1000\, i_{in} + v_\pi + 500 \cdot (i_\pi + g_m v_\pi)$$

$$v_{in} = 1000\, i_{in} + v_\pi + 500 \left(\frac{v_\pi}{r_\pi} + g_m v_\pi \right)$$

$$v_{in} = 1000 \cdot i_{in} + v_\pi \cdot \left[1 + 500 \left(\frac{1}{r_\pi} + g_m \right) \right]$$

$$v_{in} = 1000 \cdot (\underbrace{i_{eq}}_{\frac{v_\pi + 500 \cdot (i_\pi + g_m v_\pi)}{R_{eq}}} + \underbrace{i_\pi}_{\frac{v_\pi}{r_\pi}}) + v_\pi \left[1 + 500 \left(\frac{1}{r_\pi} + g_m \right) \right]$$

$$v_{in} = 1000 \cdot \left(\frac{v_\pi + 500(\overset{v_\pi / r_\pi}{i_\pi} + g_m v_\pi)}{R_{eq}} + \frac{v_\pi}{r_\pi} \right) + v_\pi \left[1 + 500 \left(\frac{1}{r_\pi} + g_m \right) \right]$$

$$v_{in} = \frac{1000}{R_{eq}} \cdot \left(v_\pi + 500 \left(\frac{v_\pi}{r_\pi} + g_m v_\pi \right) + \frac{v_\pi}{r_\pi} \right) + v_\pi \left[1 + 500 \left(\frac{1}{r_\pi} + g_m \right) \right]$$

$$v_{in} = \frac{1000\, v_\pi}{R_{eq}} \left(1 + 500 \left(\frac{1}{r_\pi} + g_m \right) + \frac{1}{r_\pi} \right) + v_\pi \left[1 + 500 \left(\frac{1}{r_\pi} + g_m \right) \right]$$

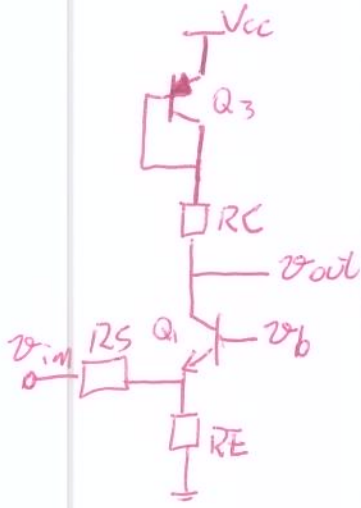
$$v_{in} = v_\pi \cdot \left\{ \frac{1000}{R_{eq}} \cdot \left[1 + 500 \left(\frac{1}{r_\pi} + g_m \right) + \frac{1}{r_\pi} \right] + 1 + 500 \left(\frac{1}{r_\pi} + g_m \right) \right\}$$

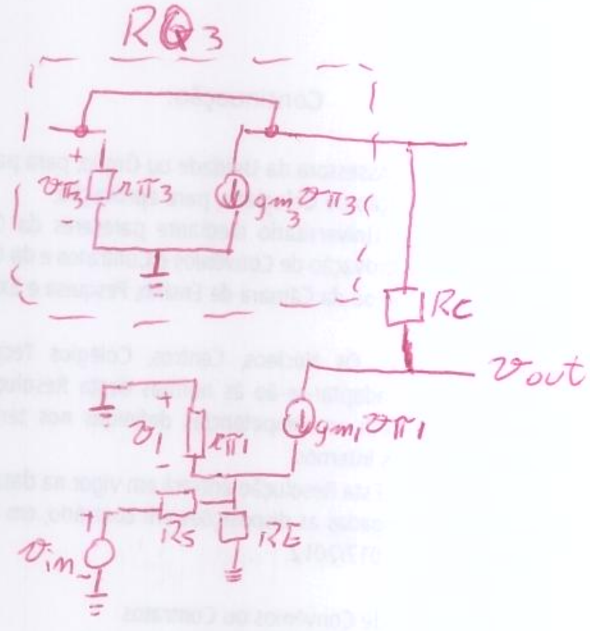
$$v_{out} = -10000 \cdot g_m v_\pi$$

$$A_v = \frac{v_{out}}{v_{in}} = -\left(\frac{\left\{ \frac{1000}{R_{eq}} \cdot \left[1 + 500 \left(\frac{1}{r_\pi} + g_m \right) + \frac{1}{r_\pi} \right] + 1 + 500 \left(\frac{1}{r_\pi} + g_m \right) \right\}}{10.000\, g_m} \right)^{-1}$$

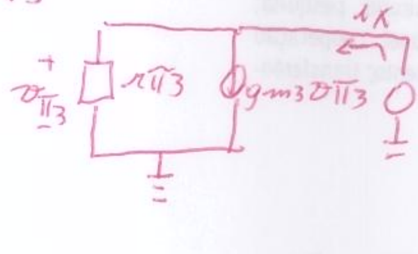
$$\boxed{A_v = 13{,}0127}$$

5.55 d)





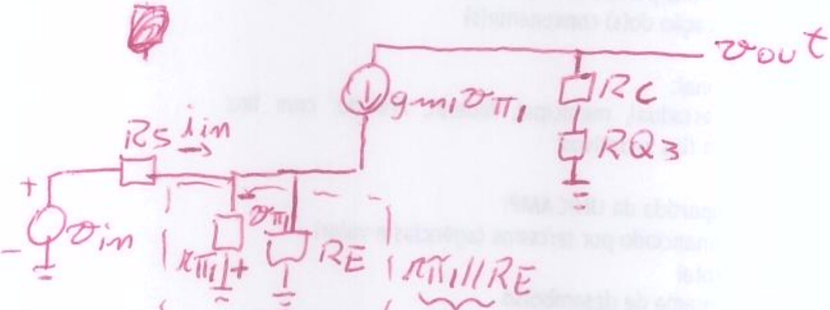
• R_{Q3}



$$\Rightarrow i_x = \frac{v_{\pi 3}}{r_{\pi 3}} + g_{m3} v_{\pi 3}$$

$$i_x = v_x \left(\frac{1}{r_{\pi 3}} + g_{m3} \right) \Rightarrow \boxed{R_{Q3} = \frac{v_x}{i_x} = r_{\pi 3} // 1/g_{m3}}$$

• Modelo de pequenos sinais equivalente



• $v_{Req} = -v_{\pi 1}$

• $v_{Req} = (g_m v_{\pi 1} + i_{in}) \cdot R_{eq} = \left[g_m v_{\pi 1} + \frac{v_{Req}}{R_{eq}} - g_{m1} v_{\pi 1} \right] R_{eq}$

• $-v_{\pi 1} = v_{\pi 1} \left[g_{m1} - \frac{1}{R_{eq}} + \right.$

• ~~$v_{Req} = R_{eq} \cdot i$~~

• $v_{in} = v_{RS} + v_{Req} = R_S \cdot i_{in} - v_{\pi 1} = R_S \cdot (i_{Req} - g_{m1} v_{\pi 1}) - v_{\pi 1}$

• $v_{in} = R_S \cdot \left[\frac{v_{Req}}{R_{eq}} - g_{m1} v_{\pi 1} \right] - v_{\pi 1}$

- $v_{in} = R_S \, v_{\pi 1}\left[\dfrac{-1}{R_{eq}} - g_{m1}\right] - v_1$

- $v_{in} = -v_{\pi 1} \cdot \left[R_S\left(\dfrac{1}{R_{eq}} + g_{m1}\right) + 1\right]$

- $v_{out} = -g_{m1}\, v_{\pi 1} \cdot (R_C + R_{Q3})$

- $$A_v = \frac{v_{out}}{v_{in}} = \frac{+g_{m1}(R_C + R_{Q3})}{R_S \cdot \left(\dfrac{1}{R_{eq}} + g_{m1}\right) + 1} \begin{array}{l} \div \left(\dfrac{1}{R_{eq}} + g_{m1}\right) \\ \div \left(\dfrac{1}{R_{eq}} + g_{m1}\right) \end{array}$$

$$\left(R_{eq} /\!/ \frac{1}{g_{m1}}\right)$$

$$A_v = \frac{-g_{m1}(R_C + R_{Q3}) \cdot \left(\dfrac{1}{R_{eq}} + g_{m1}\right)^{-1}}{R_S + 1 \cdot \left(\dfrac{1}{R_{eq}} + g_{m1}\right)^{-1}}$$

$$\boxed{A_v = \frac{+g_{m1}\left(R_C + r_{\pi 3} /\!/ \dfrac{1}{g_{m3}}\right) \cdot \left(r_{\pi 1} /\!/ R_E /\!/ \dfrac{1}{g_{m1}}\right)}{R_S + r_{\pi 1} /\!/ R_E /\!/ 1/g_{m1}}}$$

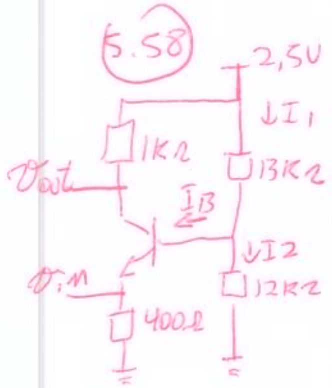
dm

obs: $r_{\pi 1} = \dfrac{\beta}{g_{m1}} \Rightarrow \boxed{r_{\pi 1} /\!/ \dfrac{1}{g_{m1}} \cong \dfrac{1}{g_{m1}}}$

$$\boxed{\Rightarrow A_v \cong \frac{g_{m1}\left(R_C + r_{\pi 3} /\!/ \dfrac{1}{g_{m3}}\right)}{R_S + R_E /\!/ 1/g_{m1}} \cdot R_E /\!/ 1/g_{m1}}$$

dm

5.58



$\beta = 100$; $I_S = 8.10^{-16} A$; $V_A = \infty$

a) $2,5 = 13000 I_1 + V_{BE} + 400 I_E$

$2,5 = 13000(I_B + I_2) + V_{BE} + 400 \frac{1}{\alpha} I_C$

$2,5 = 13000 \cdot \left[\frac{I_C}{\beta} + \frac{V_{BE} + 400 I_E}{12.000} \right] + V_{BE} + (400/\alpha) I_C$

$2,5 = 13000 \left[\frac{I_C}{\beta} + \frac{V_{BE} + (400/\alpha) I_C}{12000} \right] + V_{BE} + (400/\alpha) I_C$

$$2,5 = \frac{13000}{\beta} I_C + \frac{13000}{12000} \cdot \left(V_{BE} + \frac{400}{\alpha} I_C \right) + V_{BE} + \frac{400}{\alpha} I_C$$

$$2,5 = \frac{13000}{\beta} I_C + \frac{13000}{12000} \cdot V_{BE} + \frac{13000}{12000} \frac{400}{\alpha} I_C + \frac{400}{\alpha} I_C + V_{BE}$$

$$I_C \cdot \left(\frac{13000}{\beta} + \frac{13000}{12000} \frac{400}{\alpha} + \frac{400}{\alpha} \right) + V_{BE} \left(\frac{13000}{12000} + 1 \right) - 2,5 = 0$$

$$\Rightarrow 971,6667 I_C + 2,0833 V_{BE} - 2,5 = 0$$

obs

$\boxed{\alpha = \frac{\beta+1}{\beta} = 0,9901}$

$$\Rightarrow V_{BE} = \underset{0,026}{V_T} \ln \frac{I_C}{I_S \Rightarrow 8.10^{-16}}$$

$$\Rightarrow 971,6667 I_C + 2,0833 \cdot 0,026 \ln \frac{I_C}{8.10^{-16}} - 2,5 = 0$$

$$971,6667 I_C + 0,0542 \ln \frac{I_C}{8.10^{-16}} - 2,5 = 0$$

$$\boxed{I_C = 1,0191.10^{-3} A}$$

$$V_{BE} = \underset{0,026}{V_T} \ln \frac{I_C}{I_S \Rightarrow 8.10^{-16}} \Rightarrow \boxed{V_{BE} = 0,7247V}$$

$V_E = 400.I_E = 400.\alpha I_C \Rightarrow \boxed{V_E = 0,4036V}$

$V_B = V_E + V_{BE} \Rightarrow \boxed{V_B = 1,112V}$

$\cancel{V_C = 1000.I_C \Rightarrow V_C = 1,0191V}$

$V_C = 2,5 - 1000 I_C \Rightarrow \boxed{V_C = 1,4809V}$

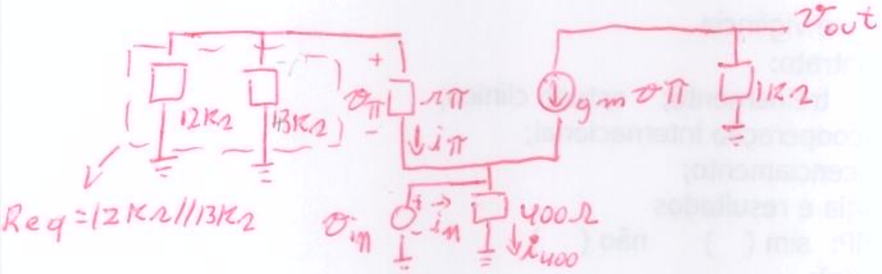
$V_B > V_E$

$V_B > V_C$

$V_B > V_E$
$V_C > V_B$ $\Rightarrow$ Região Ativa OK

b) $g_m = \dfrac{I_C}{V_T} = \dfrac{1{,}0491 \cdot 10^{-3}}{0{,}26} \Rightarrow \boxed{g_m = 0{,}039205}$

$r_\pi = \dfrac{\beta}{g_m} = \dfrac{100}{0{,}0392} \Rightarrow \boxed{r_\pi = 2{,}5513 \cdot 10^{3}\,\Omega}$



- $v_{in} + v_\pi + v_{Req} = 0$

$v_{in} = -v_\pi - Req \cdot i_\pi = -v_\pi - Req \cdot \dfrac{v_\pi}{r_\pi}$

$v_{in} = -v_\pi \cdot \left(1 + \dfrac{Req}{r_\pi}\right)$

- $v_{out} = -g_m v_\pi \cdot 1000$

- $A_v = \dfrac{v_{out}}{v_{in}} = \dfrac{1000 \overbrace{g_m}^{\sim 0{,}0392}}{1 + \dfrac{12000 // 13000}{2{,}5513 \cdot 10^{3}}} \Rightarrow \boxed{A_v = 11{,}3749}$

- R_{in}

$i_{in} + \underbrace{i_\pi}_{\sim \frac{v_\pi}{r_\pi}} + g_m v_\pi - \underbrace{i_{400}}_{\sim v_{in}/400} = 0$

$i_{in} = -\dfrac{v_\pi}{r_\pi} - g_m v_\pi + \dfrac{v_{in}}{400} \sim -v_\pi\left(1 + \dfrac{Req}{r_\pi}\right)$

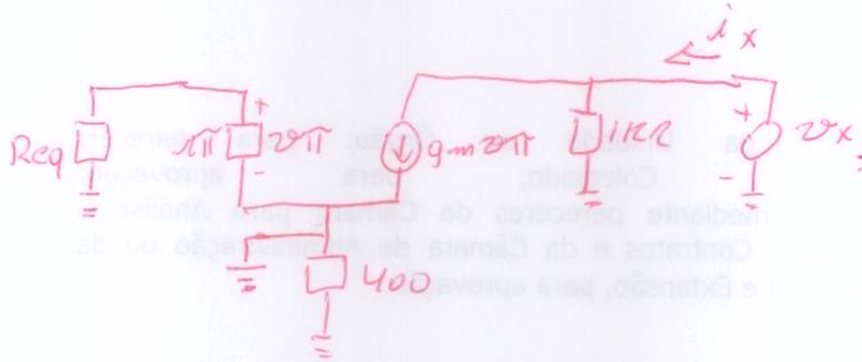
$i_{in} = -v_\pi \left(\dfrac{1}{r_\pi} + g_m + \dfrac{1}{400} \cdot \left(1 + \dfrac{Req}{r_\pi}\right)\right)$

$v_{in} = -v_\pi \left(1 + \dfrac{Req}{r_\pi}\right)$

$R_{in} = \dfrac{v_{in}}{i_{in}} = \dfrac{1 + \dfrac{Req}{r_\pi}}{\dfrac{1}{r_\pi} + g_m + \dfrac{1}{400} \cdot \left(1 + \dfrac{Req}{r_\pi}\right)} \Rightarrow \cancel{R_{in} = \ldots\,\Omega}$

$\boxed{R_{in} = 71{,}4867\,\Omega}$

• Rout

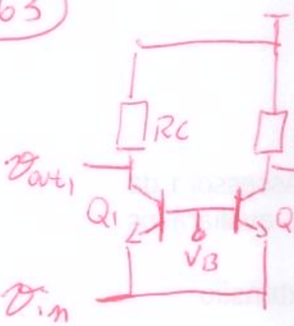


$\Rightarrow v_\pi = \emptyset \Rightarrow g_m v_\pi = \emptyset$

$\Rightarrow \boxed{R_{out} = 1k\Omega}$

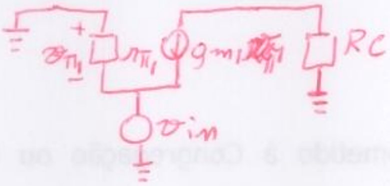
$$\boxed{I_C = I_S \exp\left(\frac{V_{BE}}{V_T}\right)} \qquad \boxed{g_m = \frac{I_C}{V_T}}$$

(5.63)



$$\left.\begin{aligned} I_{S1} &= 2\, I_{S2} \\ V_{BE1} &= V_{BE2} \end{aligned}\right\} \; I_{C1} = 2 \cdot I_{C2}$$

$$\Rightarrow g_{m1} = 2 g_{m2}$$



$v_{in} = -v_{\pi}$

$v_{out} = -g_{m_1} v_{\pi_1} R_C$

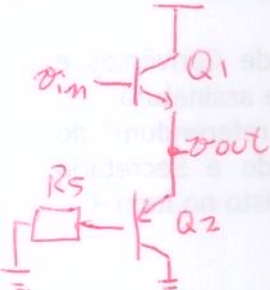
$$\boxed{A_v = g_{m_1} R_C}$$

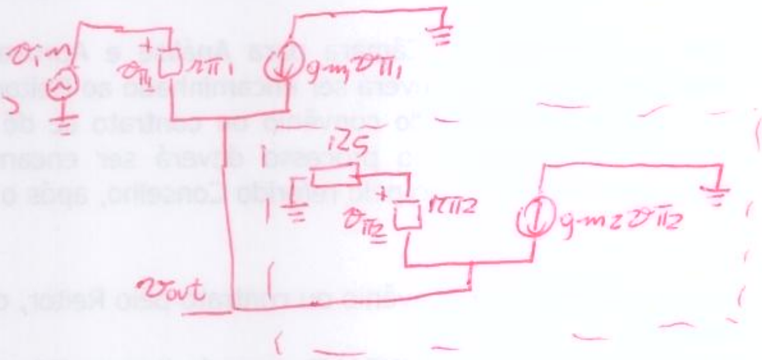
$$\frac{v_{out1}}{v_{in}} = g_{m1} R_C = 2 \cdot g_{m2} R_C$$

$$\frac{v_{out2}}{v_{in}} = g_{m2} R_C$$

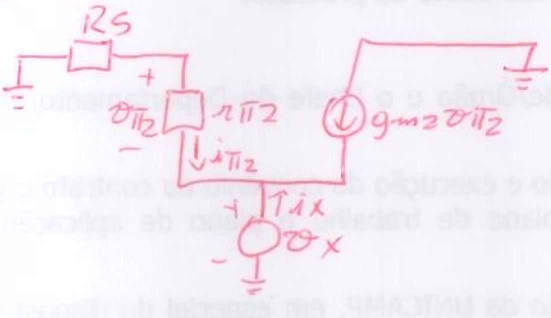
$$\Big\} \; \boxed{A_{v_1} = 2 A_{v_2}} \text{ ou } \boxed{v_{out_1} = 2\, v_{out2}}$$

(5.68) c)





• R_{Q_2}



• $v_x = -v_{\pi_2} - v_{RS}$

$$v_x = -v_{\pi_2} - \frac{v_{\pi_2}}{r_{\pi_2}} \cdot R_S$$

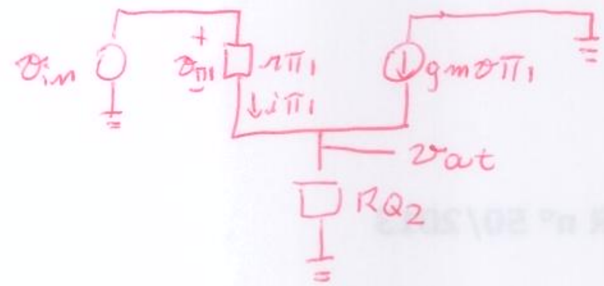
$$v_x = -v_{\pi_2} \cdot \left(1 + \frac{R_S}{r_{\pi_2}}\right)$$

• $i_x = -g_m v_{\pi_2} - i_{r\pi_2}$

$$i_x = -g_m v_{\pi_2} - \frac{v_{\pi_2}}{r_{\pi_2}}$$

$$i_x = -v_{\pi_2} \cdot \left(g_m + \frac{1}{r_{\pi_2}}\right)$$

$$\Rightarrow R_{Q_2} = \frac{v_x}{i_x} = \frac{1 + \frac{R_S}{r_{\pi_2}}}{g_{m_2} + \frac{1}{r_{\pi_2}}} \Rightarrow \boxed{R_{Q_2} = \left(1 + \frac{R_S}{r_{\pi_2}}\right) \cdot \left(\frac{1}{g_{m_2}} \,\|\, r_{\pi_2}\right)}$$



$$v_{in} = v_{\pi 1} + v_{RQ2} = v_{\pi 1} + R_{Q2}\cdot(i_{\pi 1} + g_m v_{\pi 1}) = v_{\pi 1} + R_{Q2}\left(\frac{v_{\pi 1}}{r_{\pi 1}} + g_{m1} v_{\pi 1}\right)$$

$$v_{in} = v_{\pi 1}\cdot\left(1 + R_{Q2}\cdot\left(\frac{1}{r_{\pi 1}} + g_{m1}\right)\right)$$

$$v_{out} = v_{RQ2} = R_{Q2}\left(\frac{v_{\pi 1}}{r_{\pi 1}} + g_{m1} v_{\pi 1}\right)$$

$$v_{out} = v_{\pi 1} R_{Q2}\left(\frac{1}{r_{\pi 1}} + g_{m1}\right)$$

$$A_v = \frac{v_{out}}{v_{in}} = \frac{R_{Q2}\left(\frac{1}{r_{\pi 1}} + g_{m1}\right)}{1 + R_{Q2}\left(\frac{1}{r_{\pi 1}} + g_{m1}\right)} \cdot \frac{\left(\frac{1}{r_{\pi 1}} + g_{m1}\right)^{-1}}{\left(\frac{1}{r_{\pi 1}} + g_{m1}\right)^{-1}}$$

$$A_v = \frac{R_{Q2}}{\left(\frac{1}{r_{\pi 1}} + g_{m1}\right)^{-1} + R_{Q2}} \Rightarrow \boxed{A_v = \frac{R_{Q2}}{r_{\pi 1} // \frac{1}{g_{m1}} + R_{Q2}}}$$

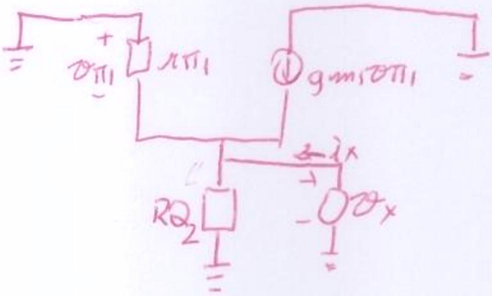
R_{in}

$$\left.\begin{array}{l} v_{in} = v_{\pi 1}\left(1 + R_{Q2}\left(\frac{1}{r_{\pi 1}} + g_{m1}\right)\right) \\ i_{in} = \frac{v_{\pi 1}}{r_{\pi 1}} \end{array}\right\} R_{in} = r_{\pi 1}\left(1 + R_{Q2}\left(\frac{1}{r_{\pi 1}} + g_{m1}\right)\right)$$

$$R_{in} = r_{\pi 1} + R_{Q2}\left(1 + g_{m1} r_{\pi 1}\right)$$

$$\boxed{R_{in} = r_{\pi 1} + R_{Q2}(1+\beta)}$$

• Rout



$$v_x = -v_{\pi_1}$$

$$i_x + g_{m_1} v_{\pi_1} - \frac{v_x}{R_{eq}} = 0$$

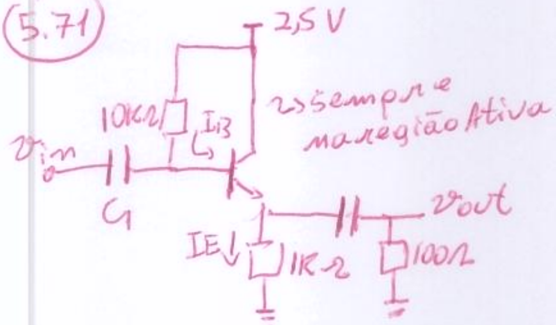
$\hookrightarrow r_{\pi_1} // R_{Q_2}$

$$i_x - g_{m_1} v_x - \frac{v_x}{R_{eq}} = 0$$

$$\frac{v_x}{i_x} = \left(g_{m_1} + \frac{1}{R_{eq}}\right)^{-1}$$

$$R_{out} = \frac{1}{g_{m_1}} // R_{eq} \Rightarrow R_{out} = \frac{1}{g_{m_1}} // r_{\pi_1} // R_{Q_2}$$

5.71



A_v? ; $I_S = 7.10^{-16}A$; $\beta = 100$; $V_A = 5V$

$$2,5 - 10.000 I_B - V_{BE} - 1000 I_E = 0$$

$$2,5 = 10.000 \frac{I_C}{\beta} + V_{BE} + 1000 \frac{I_E}{\alpha} = \emptyset$$

$$I_C \cdot \left(\underbrace{\frac{10.000}{\beta}}_{\beta \to 100} + \underbrace{\frac{1000}{\alpha}}_{\alpha \to 0,9901} \right) + V_{BE} = 2,5$$

$$\Rightarrow \begin{cases} I_C \cdot 1110 + V_{BE} = 2,5 \Rightarrow \boxed{V_{BE} = 2,5 - 1110 I_C} \\ I_C = \underbrace{I_S}_{7.10^{-16}} \exp \frac{V_{BE}}{\underbrace{V_T}_{\to 0,026}} \cdot \left(1 + \frac{V_{CE}}{\underbrace{V_A}_{\to 5V}}\right) \\ \bullet\ V_{CE} = 2,5 - 1000 I_E \Rightarrow V_{CE} = 2,5 - 1000 \frac{I_C}{\alpha} \Rightarrow \boxed{V_{CE} = 2,5 - 1010 I_C} \end{cases}$$

$$\cancel{\left[7.10^{-16} \cdot \exp \frac{V_{BE}}{0,026} \cdot \left(1 + \frac{2,5 - 1010 I_C}{5}\right) \right] \cdot 1110}$$

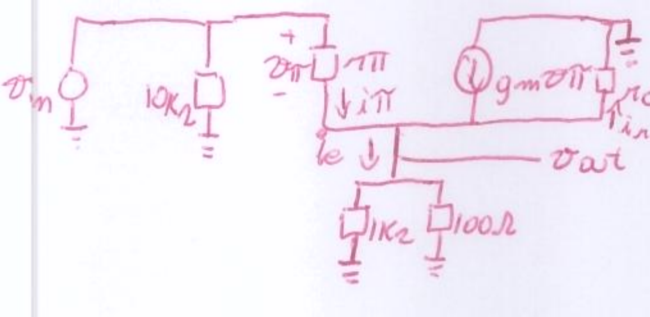
$$I_C = I_S \exp\left(\frac{2,5 - 1110 I_C}{0,026}\right) \cdot \left(1 + \frac{2,5 - 1010 I_C}{5}\right)$$

$$I_C - 7.10^{-16} \exp\left(\frac{2,5 - 1110 I_C}{0,026}\right) \cdot \left(1 + \frac{2,5 - 1010 I_C}{5}\right) = 0$$

$$\boxed{I_C = 1,5897.10^{-3} A}$$

$$V_{BE} = 2,5 - 1110 \cdot I_C \Rightarrow \boxed{V_{BE} \cong 0,7355V}$$

- $g_m = \frac{I_C}{V_T} \Rightarrow \boxed{g_m = 0,0611 S}$
- $r_o = \frac{V_A}{I_C} \Rightarrow \boxed{r_o = 3,1453.10^3}$
- $r_\pi = \frac{\beta}{g_m} \Rightarrow \boxed{r_\pi = 1,6355.10^3 \Omega}$



- $v_{out} = (1000 // 100) \cdot \left(\frac{v_\pi}{r_\pi} + g_m v_\pi\right)$
- $\cancel{v_{out} = (1000 // 100) \cdot v_\pi \cdot \left(\frac{1}{r_\pi} + g_m\right)}$
- $\cancel{v_{in} = v_\pi + v_{out}}$

• $v_{out} = 100 // 1000 \cdot i_e = 90,9091 \cdot (i_\pi + g_m v_\pi - i_{r_o})$

$$v_{out} = 90,9091 \left(\frac{v_\pi}{r_\pi} + g_m v_\pi - \frac{v_{out}}{r_o} \right)$$

$$v_{out} = 90.9091 \left[v_\pi \underbrace{\left(\frac{1}{r_\pi} + g_m \right)}_{0,0618} - \frac{v_{out}}{r_o} \right]$$

$g_m \to 0,0611$ $\quad r_\pi \to 1,6355 \cdot 10^3$ $\quad r_o \to 3,1453 \cdot 10^3$

$$v_{out} = 5,6139 \, v_\pi - 0,0289 \, v_{out}$$

$$\boxed{v_{out} = 5,4562 \, v_\pi}$$

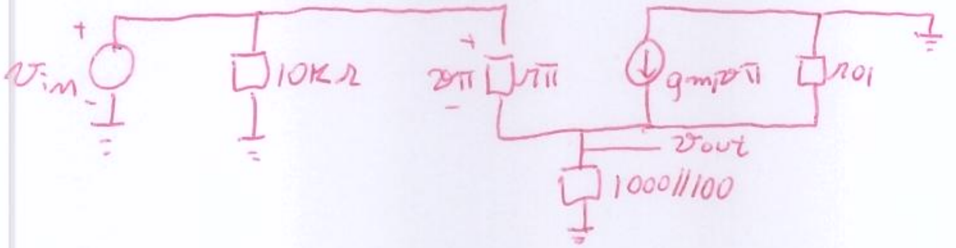
• $v_{in} = v_\pi + v_{out} \Rightarrow v_{out} = v_{in} - v_\pi \Rightarrow \boxed{v_\pi = v_{in} - v_{out}}$

$$\Rightarrow v_{out} = 5,4562 \, (v_{in} - v_{out})$$

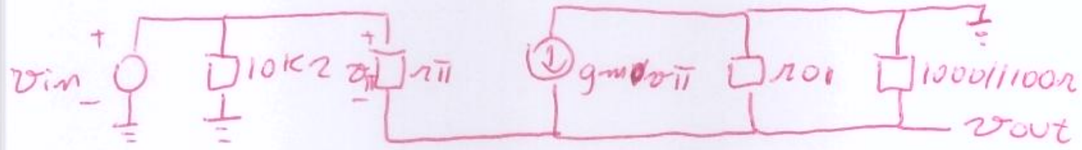
$$v_{out} \cdot 6,4562 = 5,4562 \, v_{in}$$

$$\boxed{A_v = \frac{v_{out}}{v_{in}} = 0,8451}$$

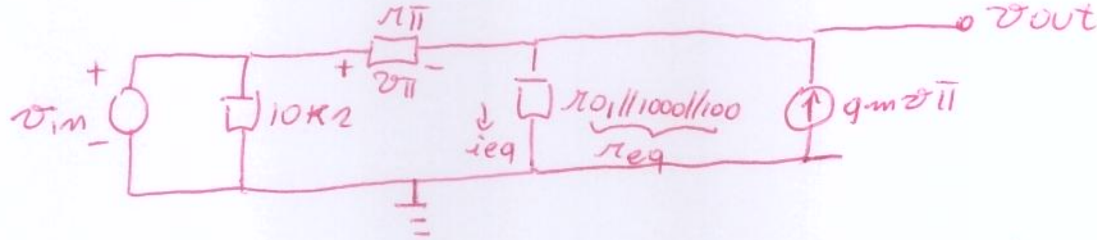
outro modo de resolver



$\Downarrow$



$\Downarrow$



- $v_{in} = v_\pi + r_{eq} \cdot i_{eq} = v_\pi + r_{eq} \cdot \left(\frac{v_\pi}{r_\pi} + g_m v_\pi \right) = v_\pi + r_{eq} v_\pi \left(\frac{1}{r_\pi} + g_m \right)$

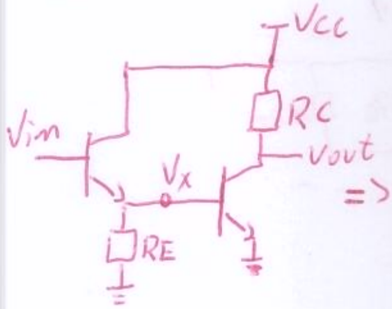
$$v_{in} = v_\pi \left[1 + r_{eq} \left(\frac{1}{r_\pi} + g_m \right) \right]$$

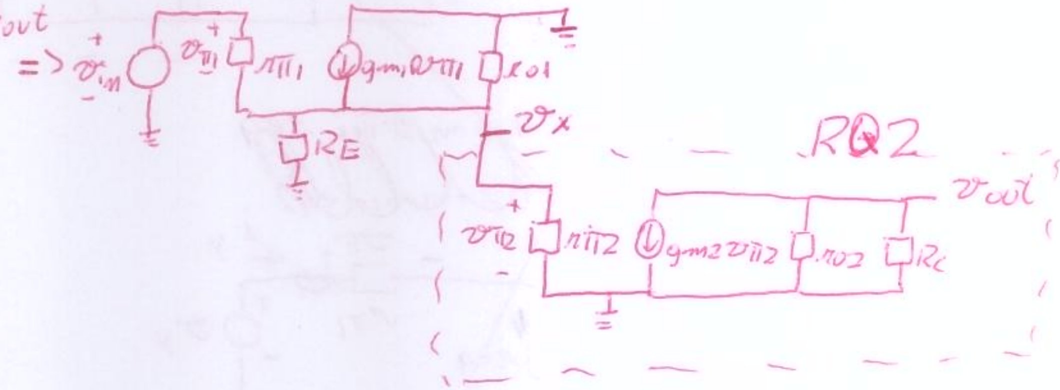
- $v_{out} = r_{eq} \cdot i_{eq} \Rightarrow v_{out} = v_\pi r_{eq} \left(\frac{1}{r_\pi} + g_m \right)$

- $A_v = \frac{v_{out}}{v_{in}} = \dfrac{r_{eq} \cdot \left(\frac{1}{r_\pi} + g_m \right)}{1 + r_{eq} \cdot \left(\frac{1}{r_\pi} + g_m \right)} \begin{matrix} \cdot \left(\frac{1}{r_\pi} + g_m \right)^{-1} \\ \cdot \left(\frac{1}{r_\pi} + g_m \right)^{-1} \end{matrix} = \dfrac{r_{eq}}{\left(\frac{1}{r_\pi} + g_m \right)^{-1} + r_{eq}}$

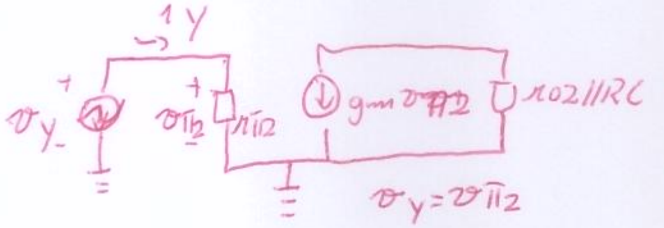
$$A_v = \frac{r_{o1}//1000//100}{\left(\frac{1}{r_\pi} + g_m \right)^{-1} + r_{o1}//1000//100} \Rightarrow \boxed{A_v = 0,845}$$

(5.72)



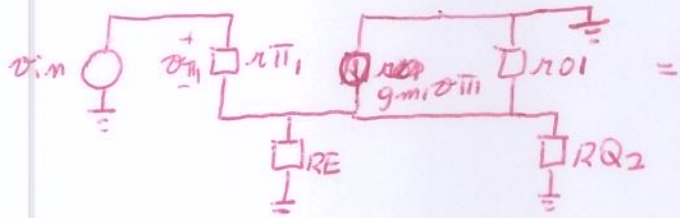


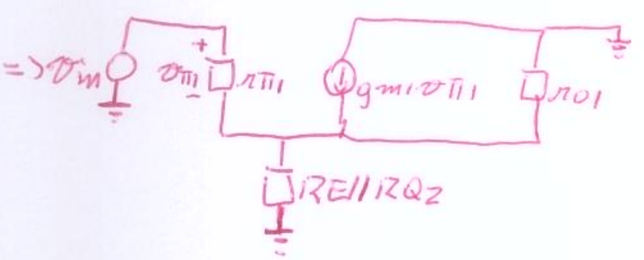
- RQ_2



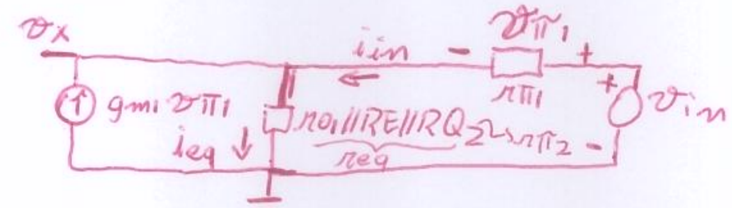
$$\Rightarrow \boxed{RQ_2 = r_{\pi 2}}$$

$$\Rightarrow \boxed{Av_2 = -g_{m2}\; r_{o2} \,//\, R_C}$$





$$\Rightarrow$$



- $v_{in} = v_\pi + r_{eq} \cdot i_{eq} = v_{\pi 1} + r_{eq} \cdot (i_{in} + g_{m1} v_{\pi 1}) = v_\pi + r_{eq}\left(\frac{v_{\pi 1}}{r_{\pi 1}} + g_{m1} v_{\pi 1}\right)$
- $v_{in} = v_{\pi 1}\left[1 + r_{eq}\left(\frac{1}{r_{\pi 1}} + g_{m1}\right)\right]$
- $i_{in} = \frac{v_{\pi 1}}{r_{\pi 1}} \Rightarrow \underbrace{R_{in}}_{\frac{v_{in}}{i_{in}}} = r_{\pi 1} \cdot \left[1 + r_{eq}\left(\frac{1}{r_{\pi 1}} + g_{m1}\right)\right]$

$$R_{in} = r_{\pi 1} + r_{\pi 1} R_{eq}\left(\frac{1}{r_{\pi 1}} + g_{m1}\right) = r_{\pi 1} + R_{eq}\left(1 + \underbrace{r_{\pi 1} g_{m1}}_{\beta}\right)$$

$$\boxed{R_{in} = r_{\pi 1} + r_{o1} \| R_E \| r_{\pi 2} \cdot (1+\beta)}$$

- $v_{\cancel{x}} = v_{req} = r_{eq} \cdot \left(\frac{v_{\pi 1}}{r_{\pi 1}} + g_{m1} v_{\pi 1}\right) = r_{eq} \cdot v_{\pi 1}\left(\frac{1}{r_{\pi 1}} + g_{m1} v_{\pi 1}\right)$

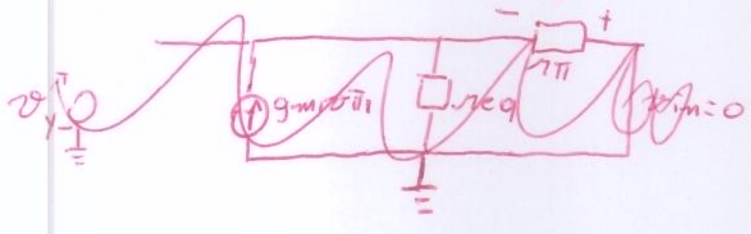
$$A_{v1} = \frac{v_x}{v_{in}} = \frac{r_{eq} \cdot \left(\frac{1}{r_{\pi 1}} + g_{m1}\right)}{1 + r_{eq} \cdot \left(\frac{1}{r_{\pi 1}} + g_{m1}\right)} \cdot \frac{\left(\frac{1}{r_{\pi 1}} + g_{m1}\right)^{-1}}{\left(\frac{1}{r_{\pi 1}} + g_{m1}\right)^{-1}}$$

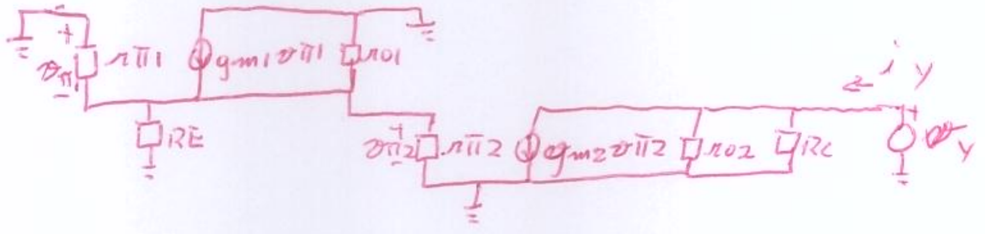
$$A_{v1} = \frac{r_{eq}}{r_{eq} + \underbrace{\left(\frac{1}{r_{\pi 1}} + g_{m1}\right)^{-1}}_{\rightarrow \frac{g_{m1}}{\beta_1} + g_{m1} \cong g_m}} \Rightarrow A_{v1} \cong \frac{r_{eq}}{r_{eq} + \frac{1}{g_{m1}}}$$

$$\boxed{A_{v1} \cong \frac{r_{o1} \| R_E \| R_{Q2}^{\sim r_{\pi 2}}}{\frac{1}{g_{m1}} + r_{o1} \| R_E \| R_{Q2}}} \Rightarrow A_v = A_{v1} \cdot A_{v2} \Rightarrow$$

$$\boxed{A_v = -g_{m2}(r_{o2} \| R_C) \cdot \frac{r_{o1} \| R_E \| r_{\pi 2}}{\frac{1}{g_{m1}} + r_{o1} \| R_E \| r_{\pi 2}}}$$

- Rout





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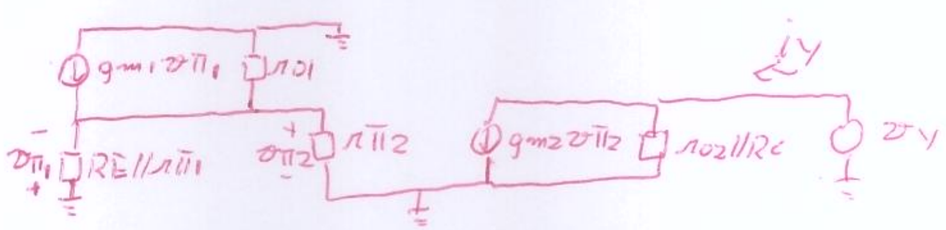

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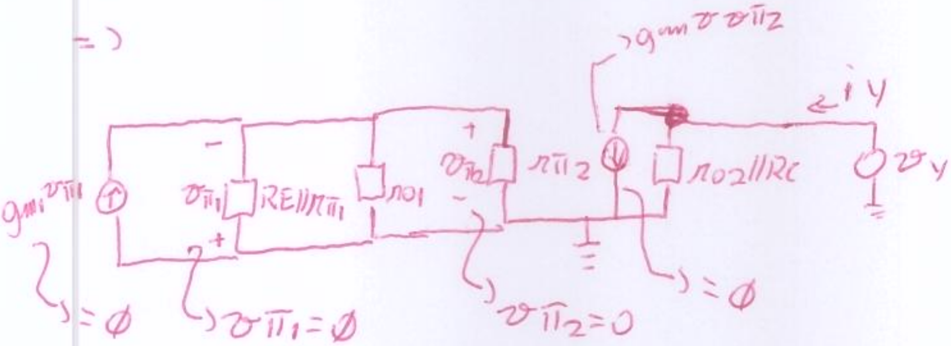
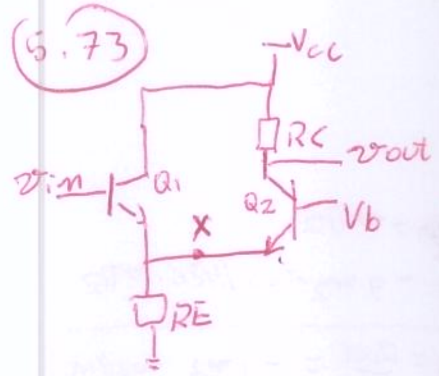
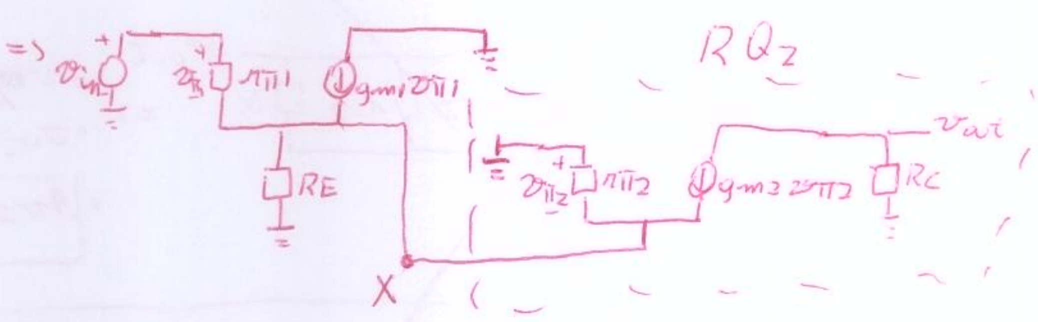


$\Rightarrow \boxed{R_{out} = r_{o2} \| R_C}$

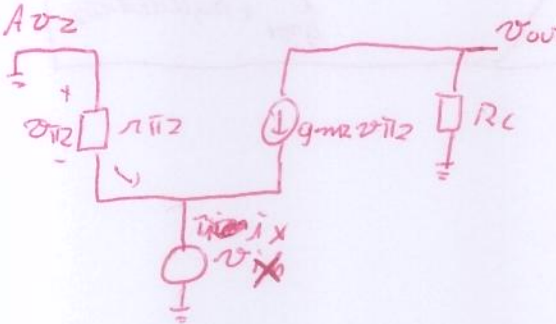
(5.73)



$\Rightarrow$



• RQ2 e Av2



$v_x = -v_{\pi 2}$

$v_{out} = -g_{m2} v_{\pi} R_C$

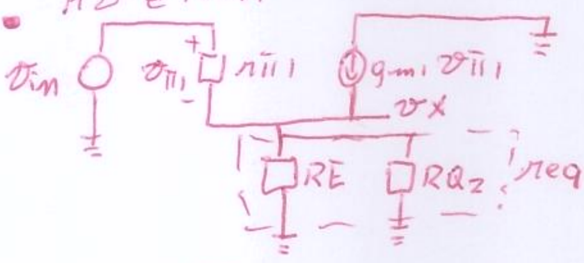
$v_{out} = g_{m2} R_C v_x$

$$\boxed{\frac{v_{out}}{v_x} = g_{m2} R_C}$$

$$i_x = -g_{m2} v_{\pi 2} - \frac{v_{\pi 2}}{r_{\pi 2}} \Rightarrow i_x = -v_{\pi 2}\left(g_{m2} + \frac{1}{r_{\pi 2}}\right) \Rightarrow i_x = v_x\left(g_{m2} + \frac{1}{r_{\pi 2}}\right)$$

$$R_{Q2} = \frac{v_x}{i_x} = \left(g_{m2} + \frac{1}{r_{\pi 2}}\right)^{-1} \Rightarrow \boxed{R_{Q2} = \frac{1}{g_{m2}} \parallel r_{\pi 2}}$$

• Av e Rin



$\Rightarrow$

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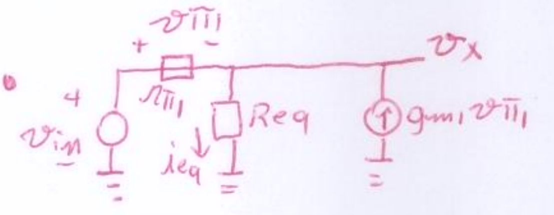

$$\Rightarrow v_x = R_{eq} \cdot i_{eq} = R_{eq} \cdot \left(\frac{v_{\pi 1}}{r_{\pi 1}} + g_{m1} v_{\pi 1}\right)$$

$$v_x = v_{\pi 1} R_{eq}\left(\frac{1}{r_{\pi 1}} + g_{m1} v_{\pi 1}\right)$$

$$v_{in} = v_{\pi 1} + \underbrace{R_{eq} \cdot i_{eq}}_{v_x} = v_{\pi 1} + v_{\pi 1} R_{eq}\left(\frac{1}{r_{\pi 1}} + g_{m1} v_{\pi 1}\right)$$

$$v_{in} = v_{\pi 1} \cdot \left[1 + R_{eq}\left(\frac{1}{r_{\pi 1}} + g_{m1}\right)\right] \cdot \cancel{R_{eq}}$$

$$\frac{v_x}{v_{in}} = \frac{R_{eq}\left(\frac{1}{r_{\pi 1}} + g_{m1}\right)}{1 + R_{eq}\left(\frac{1}{r_{\pi 1}} + g_{m1}\right)} \Rightarrow \frac{v_x}{v_{in}} = \frac{R_{eq}}{\left(\frac{1}{r_{\pi 1}} + g_{m1}\right)^{-1} + R_{eq}} \Rightarrow \frac{v_x}{v_{in}} = \frac{R_{eq}}{r_{\pi 1} \parallel g_{m1} + R_{eq}}$$

$$\boxed{\frac{v_x}{v_{in}} = \frac{R_E \| \frac{1}{g_{m2}} \| r_{\pi 2}}{\underbrace{(r_{\pi 1} \| g_{m1})}_{\sim \frac{1}{g_{m1}}} + R_E \| \frac{1}{g_{m2}} \| r_{\pi 2}}}$$

$$A_v = \frac{v_{out}}{v_x} \cdot \frac{v_x}{v_{in}} \Rightarrow \boxed{A_v \simeq g_{m2} R_C \cdot \frac{R_E \| 1/g_{m2} \| r_{\pi 2}}{\frac{1}{g_{m1}} + R_E \| \frac{1}{g_{m2}} \| r_{\pi 2}}}_{u_1}$$

- $i_{in} = \frac{v_\pi}{r_\pi}$
- $R_{in} = \frac{v_{in}}{i_{in}} = r_{\pi 1}\left[1 + R_{eq}\left(\frac{1}{r_{\pi 1}} + g_{m1}\right)\right] = r_{\pi 1} + R_{eq}(1 + r_{\pi 1} g_{m1})$

$$\boxed{R_{in} = r_{\pi 1} + R_{eq}(\beta_1 + 1)} \Rightarrow \boxed{R_{in} = r_{\pi 1} + (\beta + 1) \cdot \left(R_E \| \frac{1}{g_{m2}} \| r_{\pi 2}\right)}_{u_1}$$

- R_{out}

$v_{in} = 0 \Rightarrow v_{\pi_1} = 0 \Rightarrow g_{m1} v_{\pi_1} = 0 \Rightarrow v_{\pi_2} = 0 \Rightarrow g_{m2} v_{\pi_2} = 0 \Rightarrow \boxed{R_{out} = R_C}_{u_1}$