Hudson Zanin

hzanin@unicamp.br
Sala 201

https://www.fee.unicamp.br/dsif/hudson/disciplinas-de-graduacao-e-pos-graduacao

https://www.youtube.com/watch?v=3lTQqEehEhI

MODERN PHYSICS

Foundations of Modern Physics
Part I

Overview

In this lecture, we will learn...

- What is the classical "Doppler Effect";
- The effect of motion of a light source on the characteristics of light other than speed;
- How to compute the special relativistic Doppler effect on light and interpret the effect on observations.

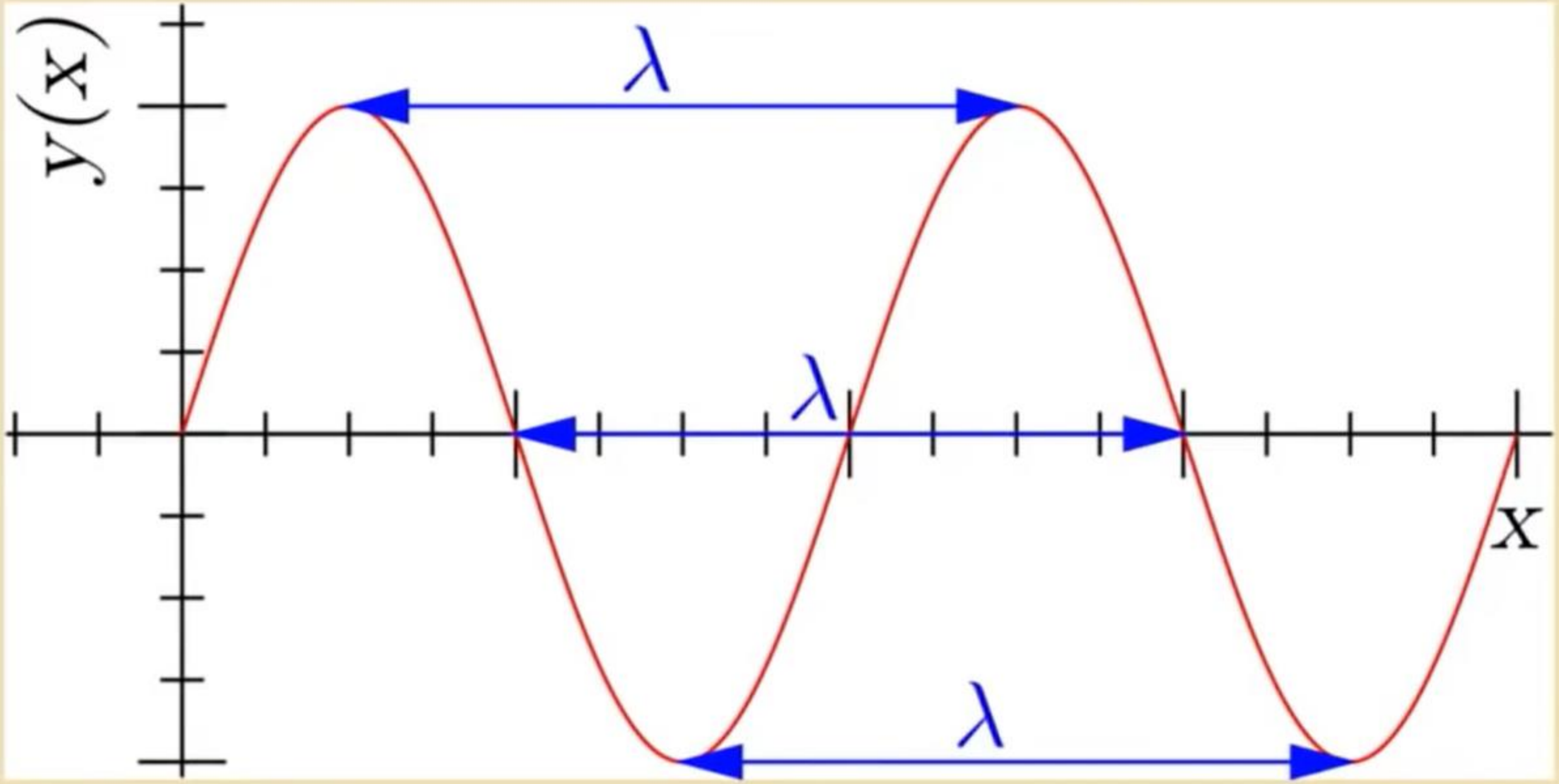
3h31 -

Christian Doppler

(1803—1853)

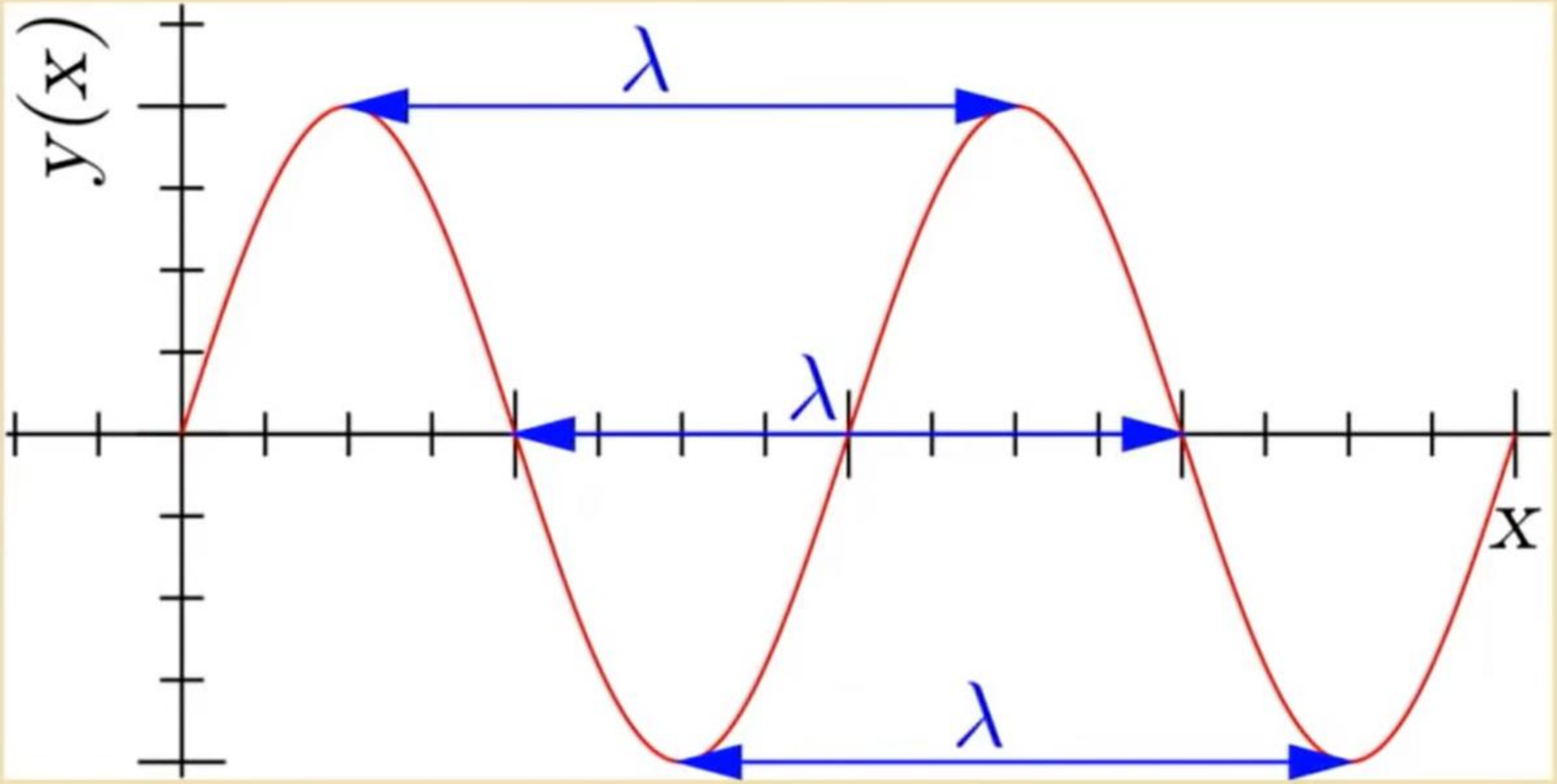
Properties of Waves

Recall *oscillatory phenomena* from introductory physics; specifically, recall simple harmonic motion. This kind of repetitious motion has a time and space structure that allows it to be described using sine or cosine functions of space and time:



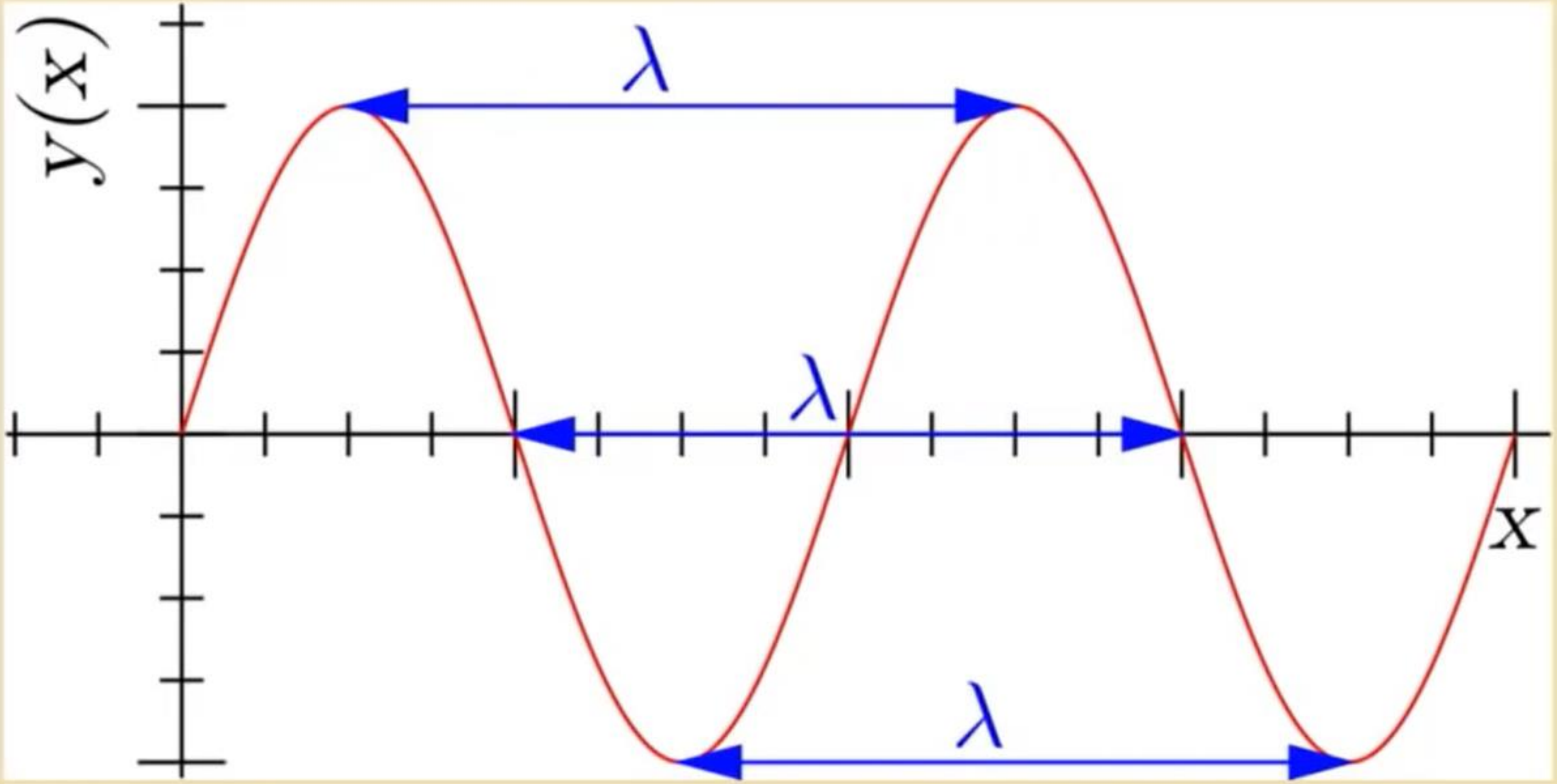
(Image from Wikipedia)

Properties of Waves



The distance between maxima (or minima) of the phenomenon is called the *wavelength*, denoted by λ. The time between maxima (or minima) passing the same spatial point is known as the period, T; the inverse of the period is the rate at which maxima (minima) pass that point and is known as the *frequency*, denoted $f = 1/T$ (or using the Greek letter "nu": $\nu = 1/T$).

Properties of Waves



The speed with which waves move in space during some unit of time is given by the product of frequency and wavelength:

$$v_{wave} = \lambda f \xrightarrow{\text{Light Wave}} c = \lambda f$$

Properties of Waves

We can think of waves of sound or waves of light as being represented by lines or planes; the location of a line (in 2-D) or a plane (in 3-D) indicates a location in space of a maximum of the traveling wave. This is a common way to quickly and simply sketch a wave. The distance between lines/planes is the wavelength. Such a line or plane would be referred to as a "wave front."

The frequency of such a phenomenon can be thought of as how many fronts per second are emitted by the source.

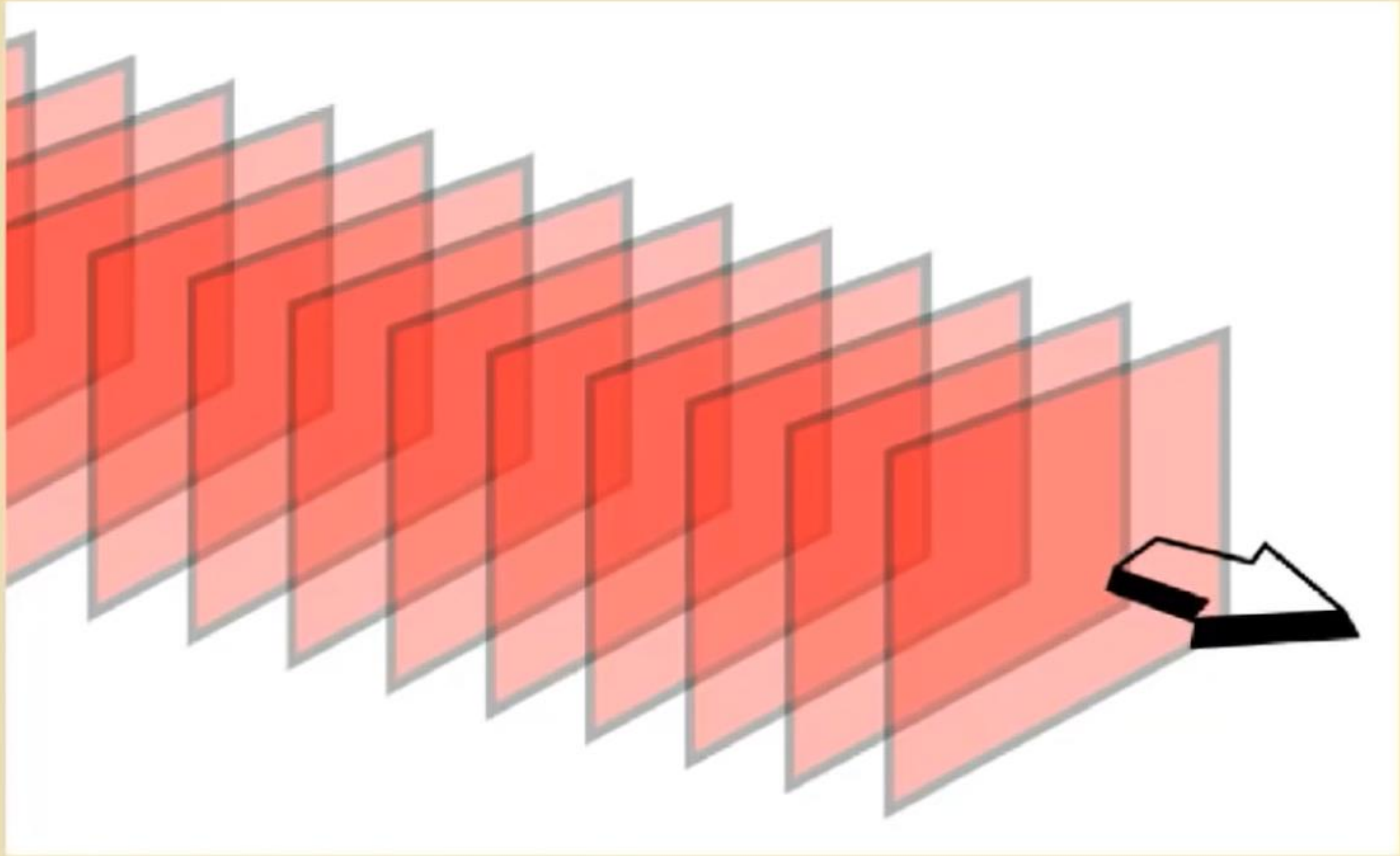


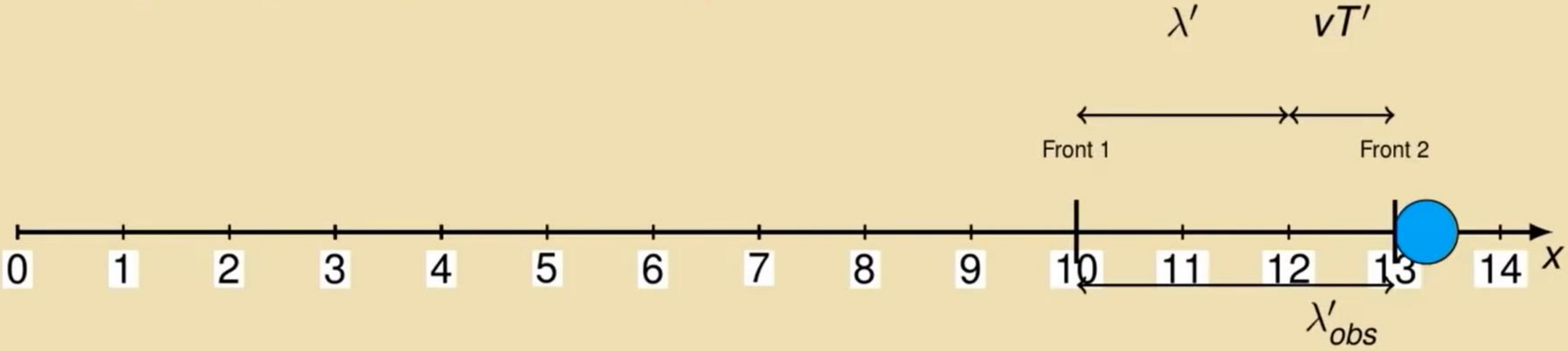
Image from Wikipedia

The Classical Doppler Effect: Sound Waves

The “Doppler Effect” occurs when the source emitting a wave is moving relative to an observer, or when the observer is moving relative to the source. This is illustrated in the image below.

Consider this example using sound waves. In the direction of motion, ahead of the source, the wave fronts are pressed together more densely, shrinking the wavelength and increasing the frequency of the waves. Against the direction of motion, behind the source, the wave fronts are more widely spread apart, increasing the wavelength and decreasing the frequency. To human ears, frequency determines pitch; high-pitched sounds are high-frequency sounds, and vice versa.

Deriving the Special Relativistic Doppler Effect



This is all illustrated above at time t_2', and we can relate all the lengths:

$$\lambda' + vT' \quad = \quad \lambda'_{observed}$$

We can write this in terms of frequencies by remembering that $T = 1/f$ and $c = \lambda f$, so that $\lambda' = c/f'$ and $\lambda'_{obs} = c/f'_{obs}$. We find then that:

$$\frac{(1 + v/c)}{f'} \quad = \quad T'_{obs}$$

Deriving the Special Relativistic Doppler Effect

We are nearly there. Let's begin by defining a convenient symbol, $\beta = v/c$. Then:

$$T'_{obs} \quad = \quad \frac{(1+\beta)}{f'}$$

So far, all of this is in the S' frame - the original frequency of emission from the perspective of the source ($f' = f_{source}$), the relative speed of the source and observer (v), and the period (frequency) with which the person in the source frame would expect the observer to receive the wave fronts, T'_{obs} (f'_{obs}). However, if we now transform into the actual frame of reference of the observer, we know that there is an additional effect we must consider: the relativity of time. Time in the source frame, where all emissions happen at the same location, is proper time; the time dilation relation is given by $T_{obs} = \gamma T'_{obs}$. We then arrive at:

$$T_{obs} \quad = \quad \gamma T'_{obs} = \gamma \frac{(1+\beta)}{f'} = \frac{1+\beta}{\sqrt{1-\beta^2}} \frac{1}{f'} = \sqrt{\frac{1+\beta}{1-\beta}} \frac{1}{f'}$$

$$f_{obs} \quad = \quad \frac{1}{T_{obs}} = \sqrt{\frac{1-\beta}{1+\beta}} f' = \sqrt{\frac{1-\beta}{1+\beta}} f_{source}$$

The Special Relativistic Doppler Effect

$$f_{obs} = \sqrt{\frac{1-\beta}{1+\beta}} f_{source} \text{ (Source moving away from Observer)}$$

The special relativistic Doppler effect, in this case for a source moving away from (receding from) an observer, is a combination of two effects: the classical doppler effect of a moving source that adds extra space between wave fronts and the dilation of time due to relative motion of the source and observer.

For a source that is moving toward an observer (approaching), the sign of the velocity is all that needs to be changed ($\beta \rightarrow -\beta$):

$$f_{obs} = \sqrt{\frac{1+\beta}{1-\beta}} f_{source} \text{ (Source moving toward an Observer)}$$

You can practice this calculation by checking for yourself that this second equation, for an approaching source, is correct.

Tarefa 3h58

https://www.youtube.com/watch?v=3lTQqEehEhI

Demonstre que

$$f_{obs} = \sqrt{\frac{1+\beta}{1-\beta}}\, f_{source} \quad \text{(Source moving toward an Observer)}$$

Some Expectations from the Special Relativistic Doppler Shift

Let's look at some of the consequences we expect as a result of this Doppler shift. For example, if a light source is moving away from us or toward us, what do we expect to happen to the frequency of its light? For a source that is moving away from us at speed β along our line-of-sight, we expect to scale the source frequency by a quantity

$$\sqrt{\frac{1-\beta}{1+\beta}} \leq 1$$

The frequency we observe should always be *lower* than in the source's frame of reference, owing to the "stretching" of its wave fronts combined with the dilation of time. Because $\beta = [0, 1]$, we are taking the ratio of a number less than 1.0 and a number greater than 1.0.

If, instead, the source and observer are moving toward each other, then we scale the source frequency by a quantity:

$$\sqrt{\frac{1+\beta}{1-\beta}} \geq 1$$

This means the observed frequency is always greater than what is observed in the frame of the source, since we are dividing a number greater than (or equal to) 1.0 by a number less than (or equal to) 1.0.

The Special Relativistic Doppler Effect - Frequency and Wavelength

$$f_{obs} = \sqrt{\frac{1-\beta}{1+\beta}} f_{source} \text{ (Source moving away from Observer)}$$

$$f_{obs} = \sqrt{\frac{1+\beta}{1-\beta}} f_{source} \text{ (Source moving toward an Observer)}$$

The above are for frequency. We can derive the equations for wavelength using $c = \lambda f$:

$$\lambda_{obs} = \sqrt{\frac{1+\beta}{1-\beta}} \lambda_{source} \text{ (Source moving away from Observer)}$$

$$\lambda_{obs} = \sqrt{\frac{1-\beta}{1+\beta}} \lambda_{source} \text{ (Source moving toward an Observer)}$$

As expected, when a receding (approaching) source results in a lower (higher) frequency than in the frame of the source, this conversely results in a longer (shorter) wavelength.

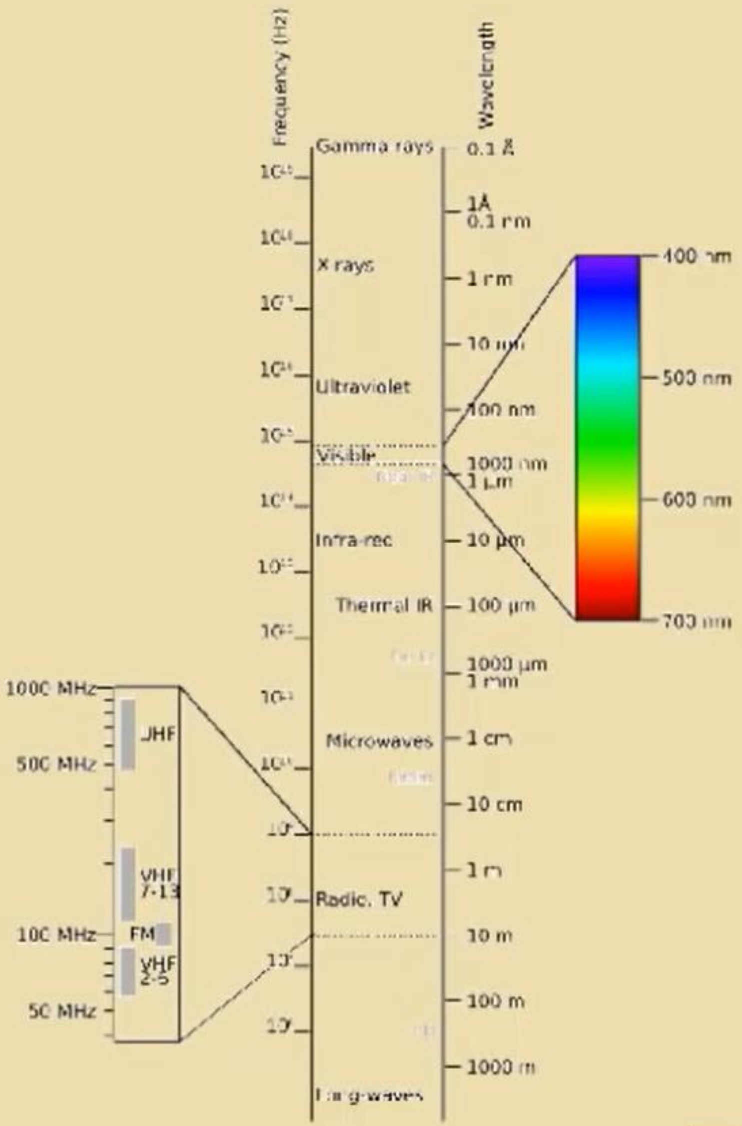
Perceived Light Color Due to the Relativistic Doppler Shift

$$\lambda_{obs} = \sqrt{\frac{1+\beta}{1-\beta}}\lambda_{source} \text{ (Source moving away from Observer)}$$

$$\lambda_{obs} = \sqrt{\frac{1-\beta}{1+\beta}}\lambda_{source} \text{ (Source moving toward an Observer)}$$

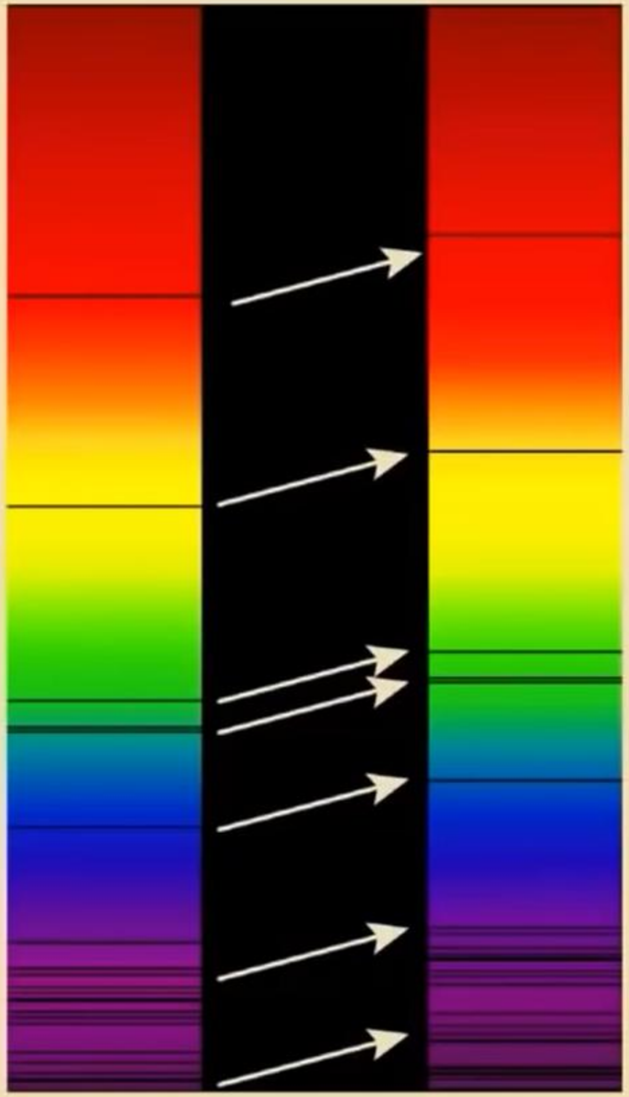
Receding sources of light are said to *Red Shift* compared to when they are at rest, since longer wavelengths are redder than shorter wavelengths of light; conversely, an approaching source is said to be *Blue Shifted* because shorter wavelengths are bluer than longer wavelengths.

This has implications for measuring our place in the cosmos. For instance, without making physical contact with distant stars and galaxies, it's possible to determine whether those objects are receding from Earth or approaching Earth based on the degree of the color shift of their atomic spectra.



The Red (or Blue) Shifting of Astrophysical Objects and Velocity Measurements

For example at the right is a pair of atomic spectra obtained for two objects: the Sun (left), which is not appreciably moving toward or away from us at any one time and the atomic spectrum of a distant supercluster of galaxies named BAS11 (right). Note the difference between the two? Both have the same pattern of missing colors (so-called “absorption lines”), but in the supercluster these are shifted toward the red.



The fact that those missing colors are ***red-shifted*** means the galaxy supercluster is moving away from us. You can use the difference between where the missing wavelengths are present in the Sun and where they are present in the galaxy supercluster to estimate the relative velocity ($\beta = v/c$) with which the supercluster is receding from us.

This kind of measurement is how we know the universe, as a whole, is expanding: all distant objects appear to be receding from Earth, implying the universe as a whole and on the largest distance scale is expanding with time.