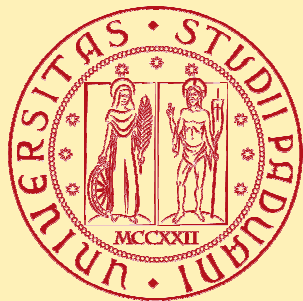


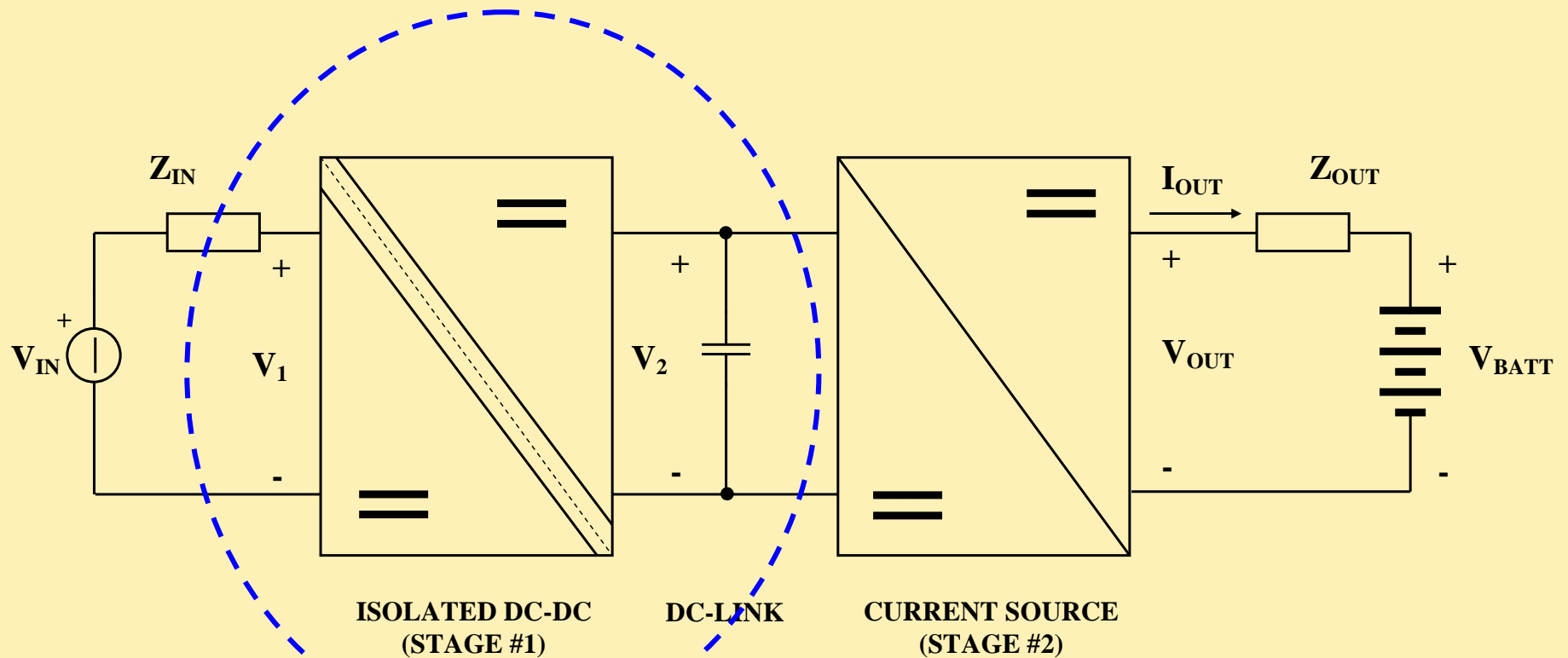
Unified Analysis of Isolated Bidirectional Converters for Battery Charging

Simone Buso, Giorgio Spiazzi
University of Padova - DEI



- **Introduction**
- **Review of Considered Converter Topologies**
 - **Single-Phase Dual Active Bridge (DAB)**
 - **Single-Phase Series Resonant DAB (SR-DAB)**
 - **Three-Phase Dual Active Bridge (DAB)**
 - **Three-Phase Series Resonant DAB (SR-DAB)**
 - **Interleaved Boost with Coupled Inductor (IBCI)**
- **Unified Analysis**
- **Soft-switching conditions**
- **Transferred Power Calculation**

Introduction



Focus on the isolated stage
(V_1 = high voltage port, V_2 = low voltage port)

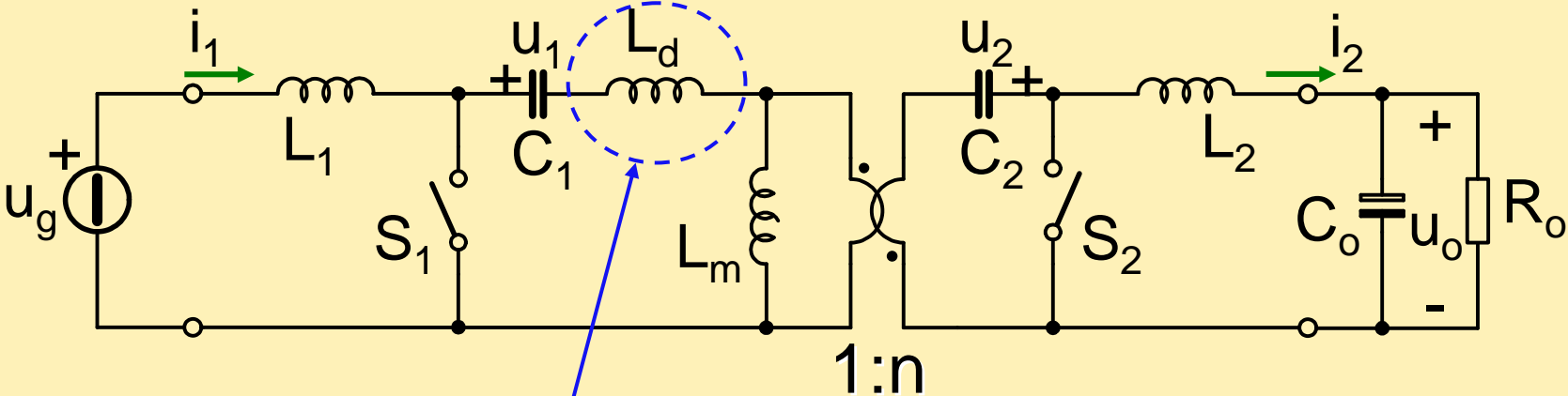
Isolated Bidirectional Topologies

- **topologies with reduced switch count**
 - **Flyback**
 - **Cuk**
 - **Sepic**
 -
- **topologies with dual bridge (or half-bridge or push-pull) configuration**
 - **Dual Active Bridge (DAB)**
 - **Interleaved Boost with Coupled Inductors (IBCI)**
 -
- **topologies with dual bridge configuration and high frequency resonant networks**
 - **Series Resonant DAB (SR-DAB)**
 -

Reduced Switch Count Topologies

- High voltage and/or current stress on active components
- Limited soft-switching operation
- Transformer leakage inductance requires suitable snubber circuits

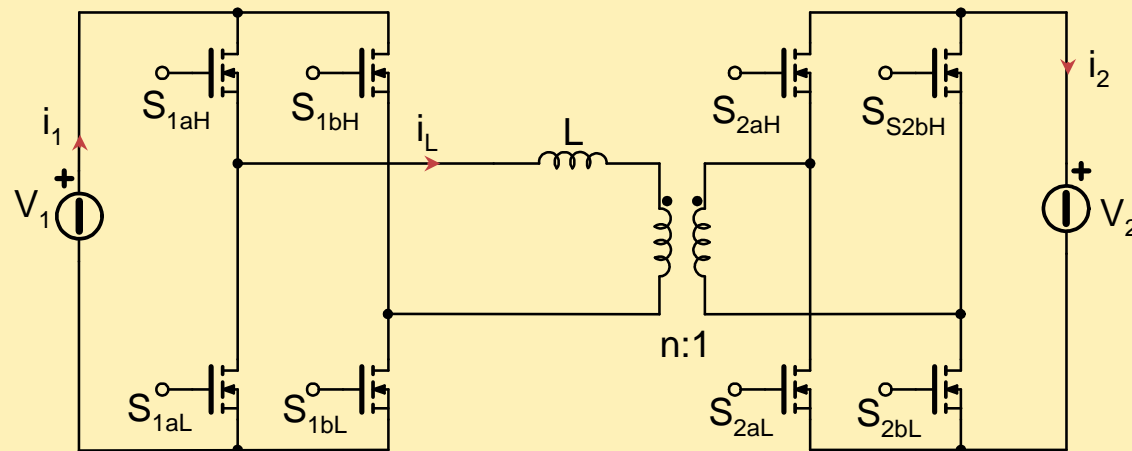
Example: bidirectional Cuk converter



Switch overvoltage!

Dual Active Bridge (DAB)

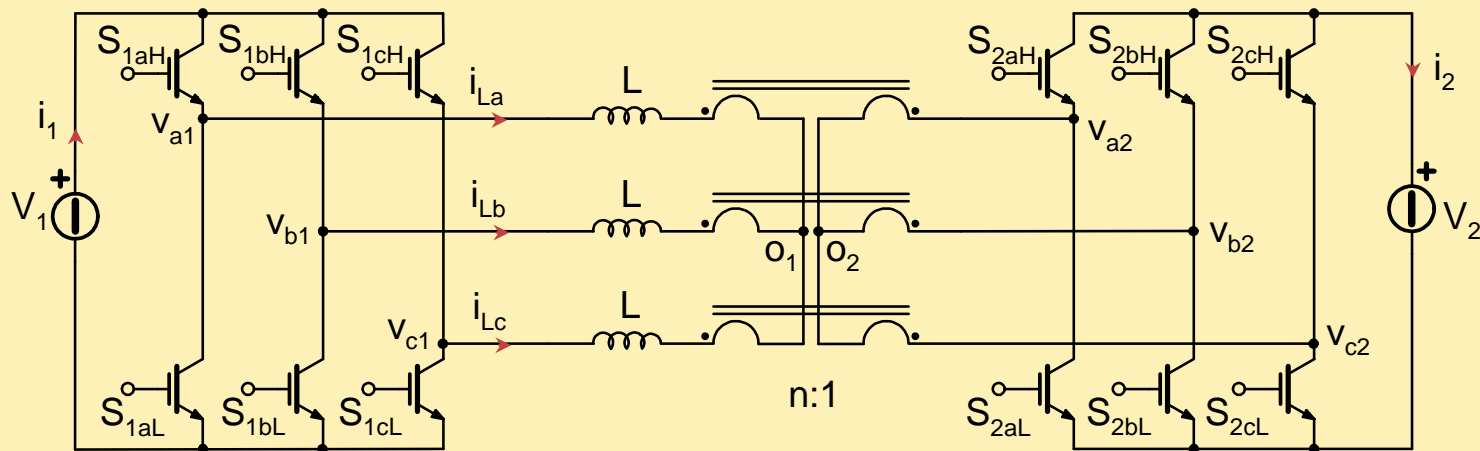
Single-phase:



- **Simple phase-shift modulation**
- **Extended soft-switching operation**
- **Exploitation of transformer leakage inductance**
- **Optimum design for constant port voltages V_1 and V_2**

Dual Active Bridge (DAB)

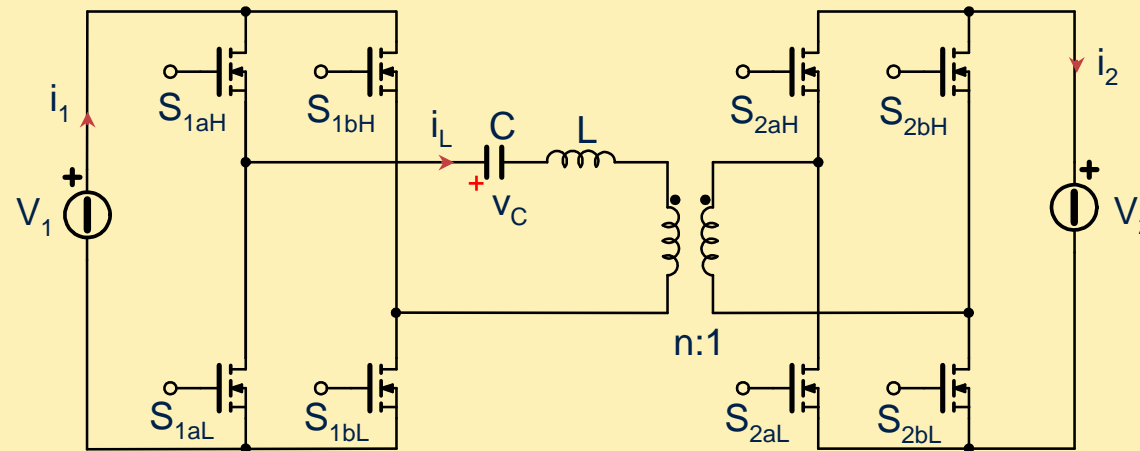
Three-phase:



- Simple phase-shift modulation
- Extended soft-switching operation
- Exploitation of transformer leakage inductance
- Optimum design for constant port voltages V_1 and V_2
- Reduced input and output current ripples

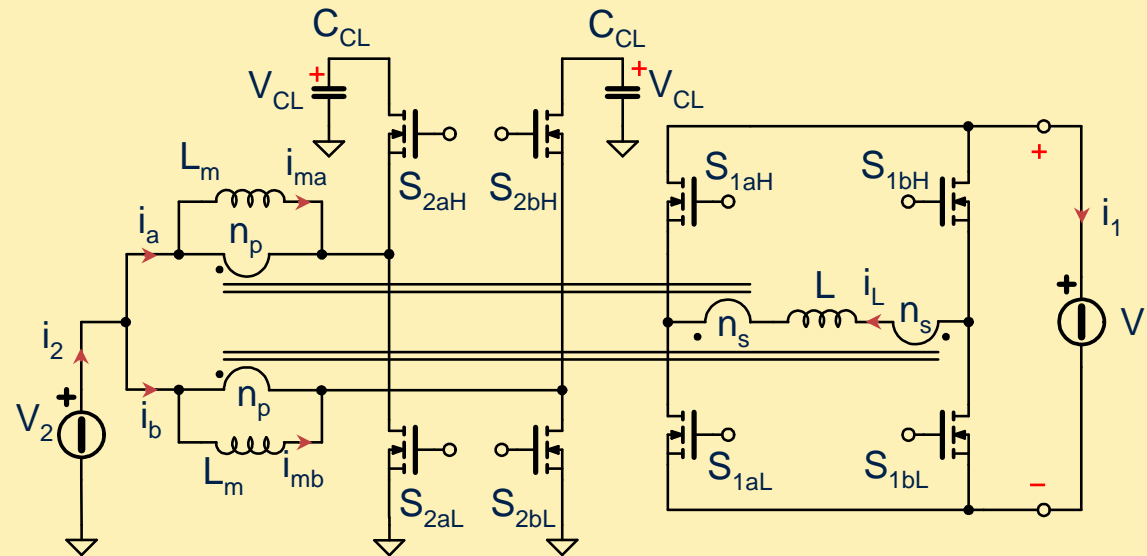
Series Resonant Dual Active Bridge (SR-DAB)

Single-phase:



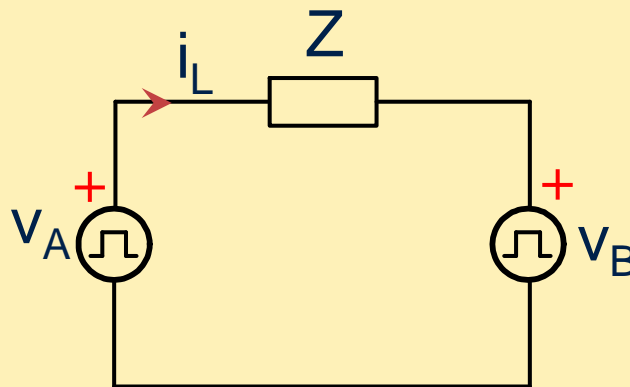
- Same characteristics as single-phase DAB
- Higher degree of freedom (two parameters: L and C)
- Inherent protection against transformer saturation (with C split between primary and secondary)

Interleaved Boost with Coupled Inductors (IBCI)



- Simple phase-shift modulation
- Extended soft-switching operation
- Exploitation of mutual inductor leakage inductance
- Duty-cycle control of port 2 switches for variable port voltages V_1 and V_2
- Reduced port 2 current ripple (low-voltage high-current port)

- All the aforementioned topologies control the power transfer between the two ports by modulating the voltage applied to a current shaping impedance

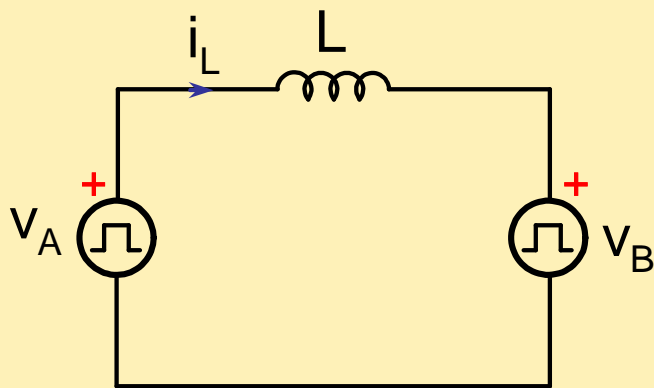


- For DAB and IBCI topologies, Z is a simple inductor while for SR-DAB topologies Z is a series resonant L-C tank

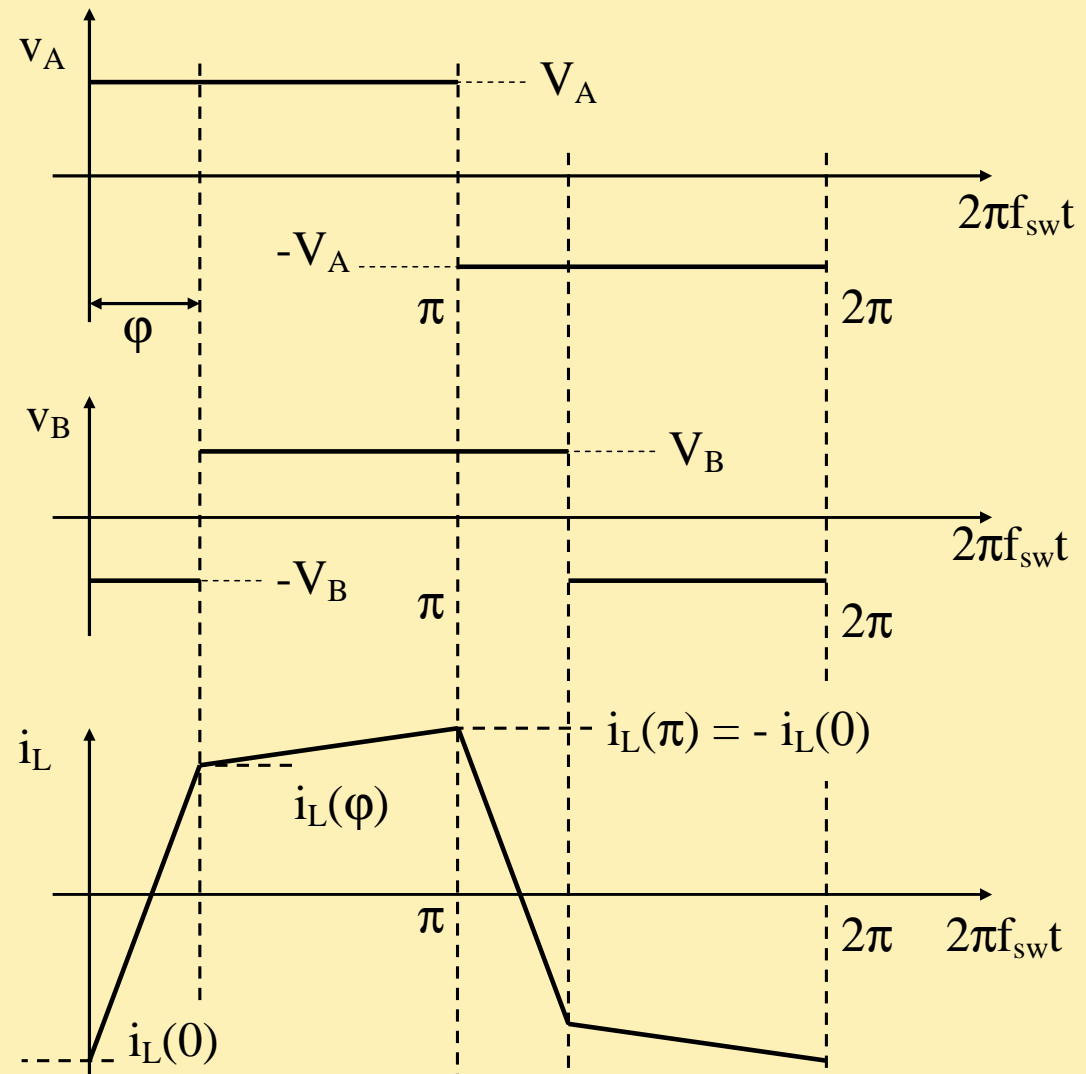
Phase-Shift Modulation

Example: power transfer from v_A to v_B

Single-phase DAB



$$V_A > V_B$$



Phase-Shift Modulation

- v_A and v_B are square wave voltages of amplitudes V_A and V_B , respectively

$$V_A = V_1 \qquad V_B = nV_2$$

- The power transfer between ports 1 and 2 is controlled through the phase-shift angle φ

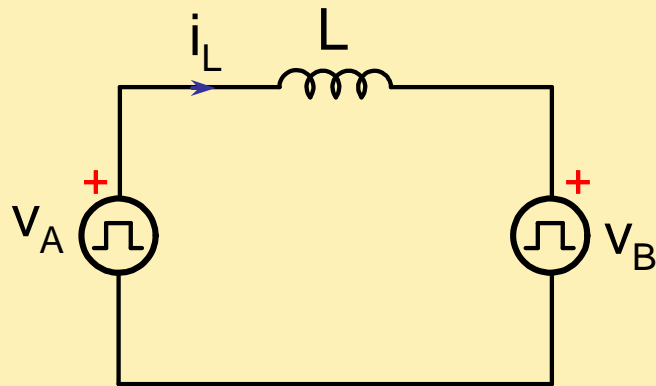
$$0 \leq \varphi \leq \pi/2 \qquad v_A \xrightarrow{P} v_B$$

$$-\pi/2 \leq \varphi \leq 0 \qquad v_A \xleftarrow{P} v_B$$

- Inductor current has a piecewise linear behavior (DAB)

Plus Phase-Shift Modulation

IBC1 converter

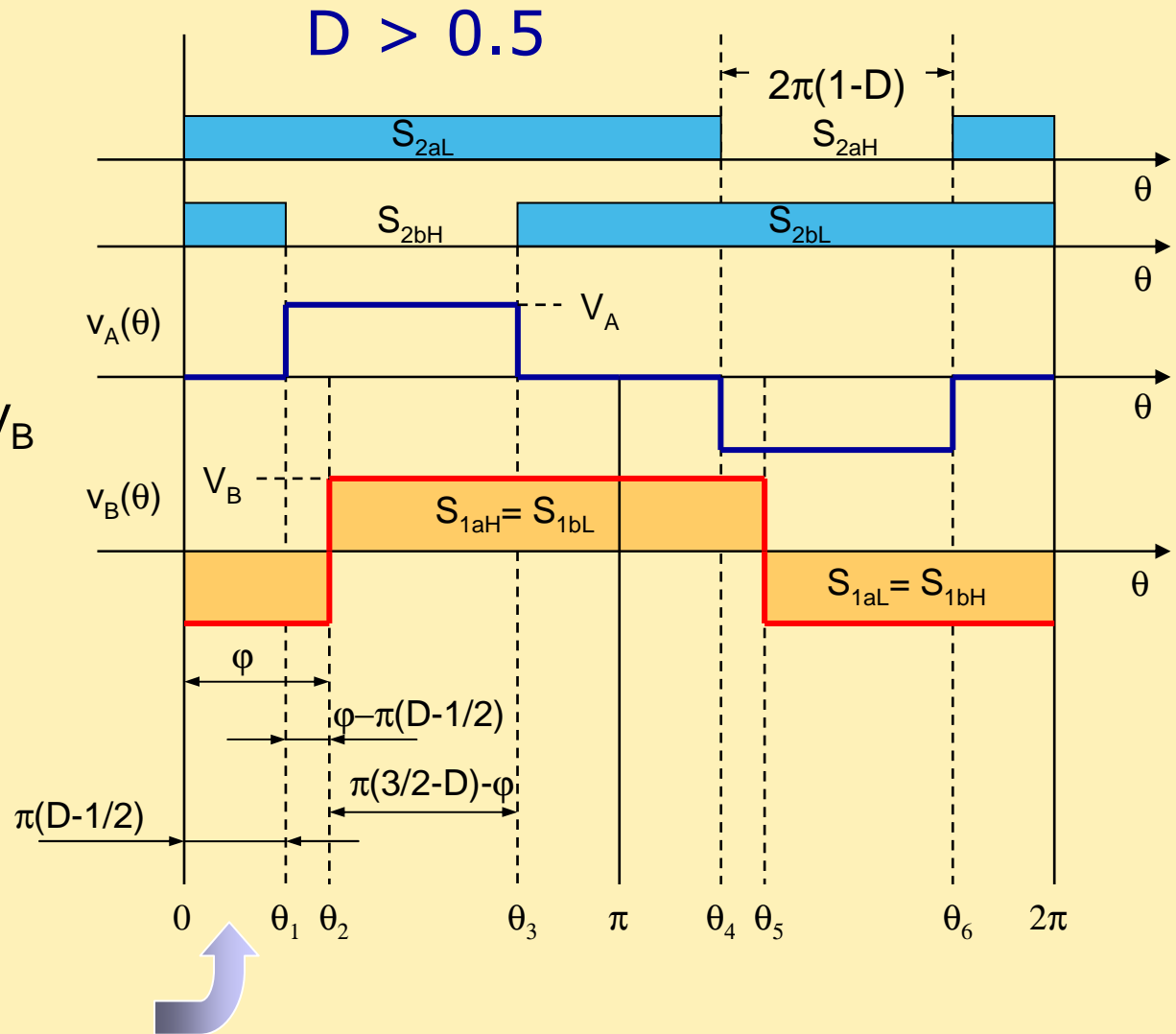


Case A:

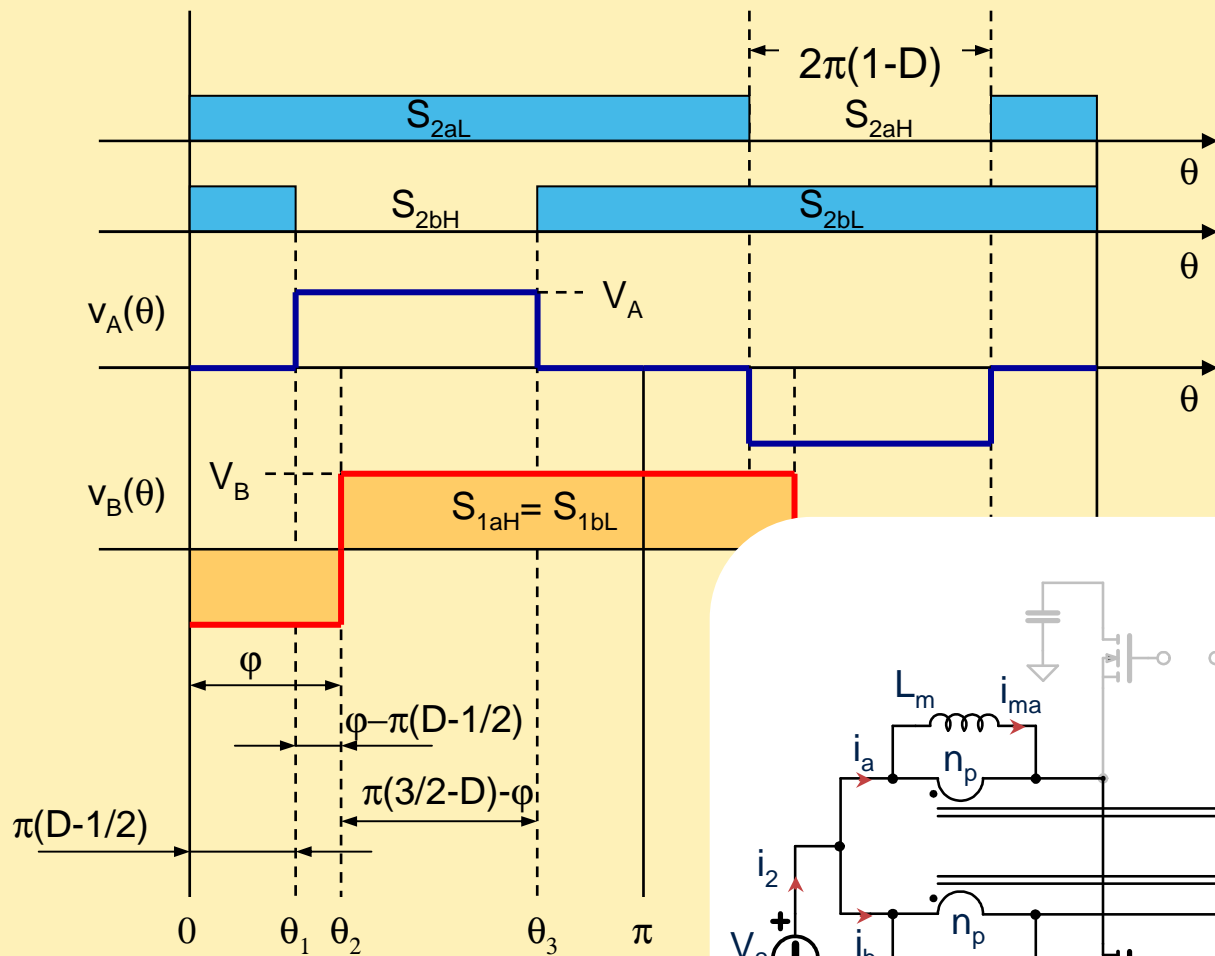
$$0 < \varphi < \pi(D-1/2)$$

Case B:

$$\pi(D-1/2) < \varphi < \pi/2$$



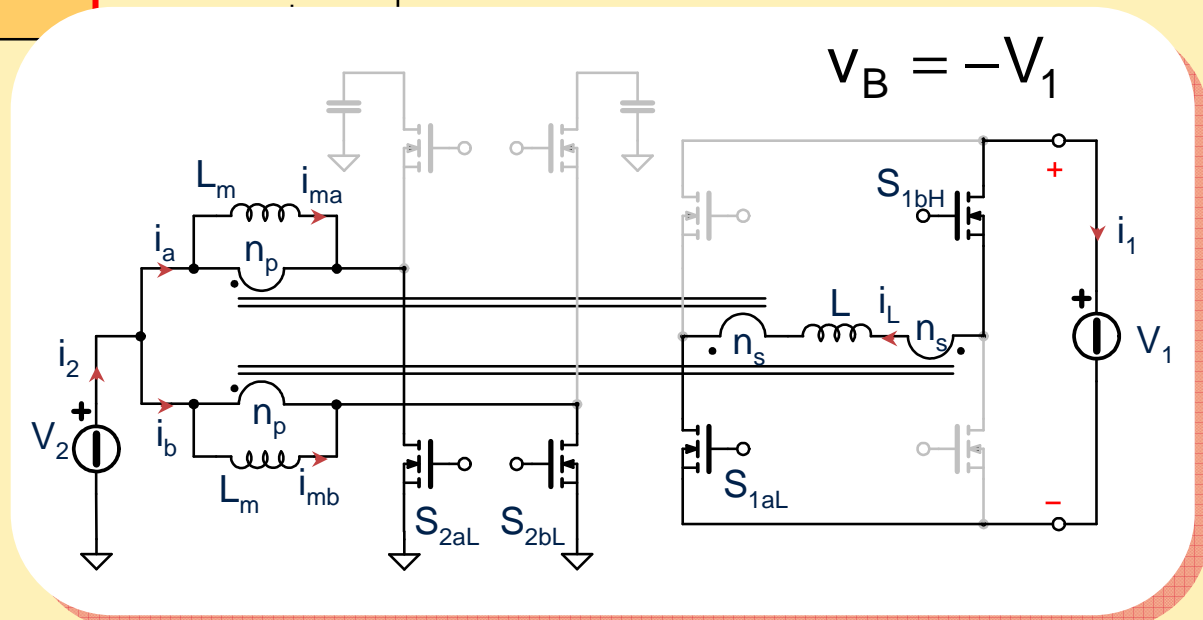
Plus Phase-Shift Modulation



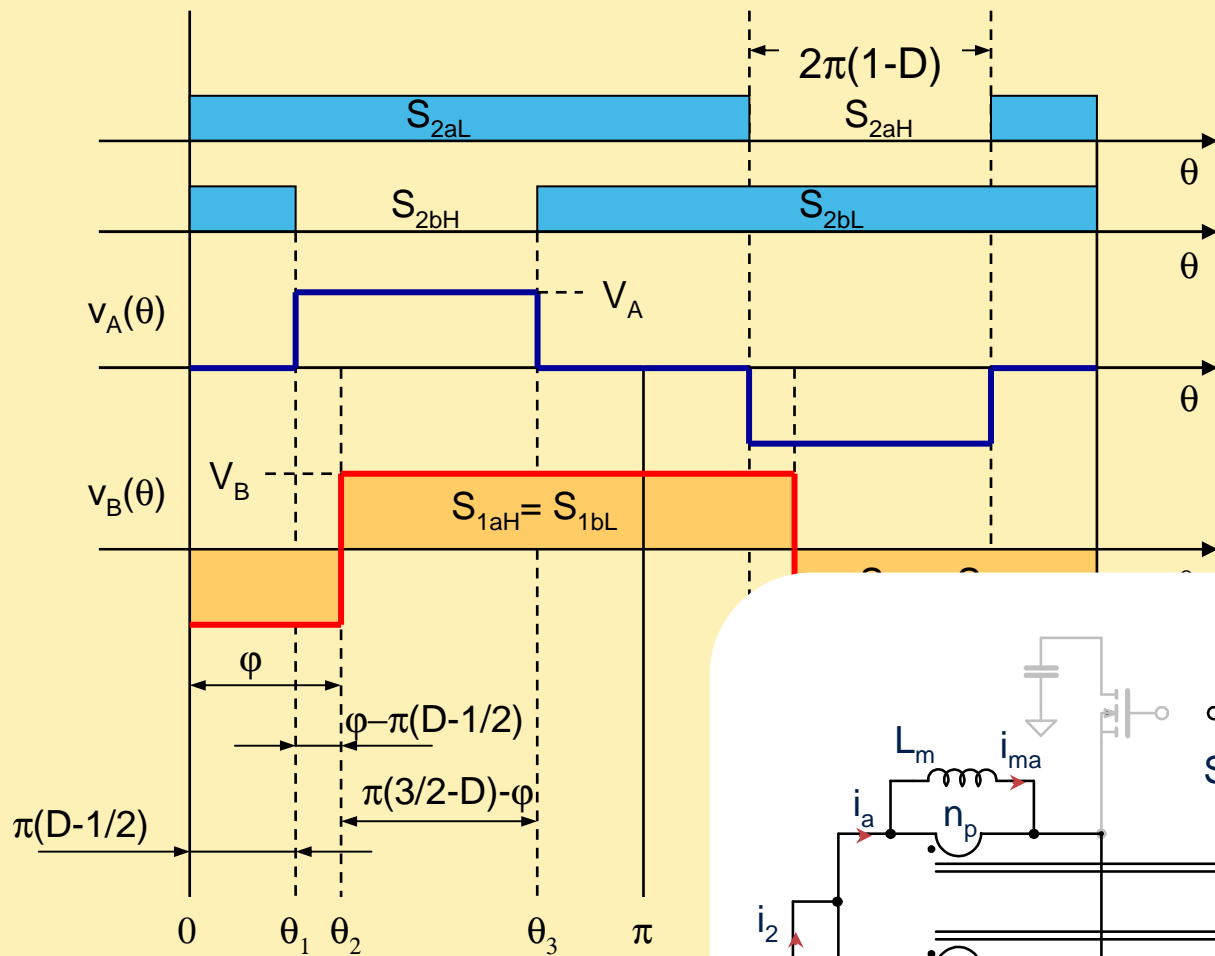
IBCI converter

$$0 \leq \theta \leq \theta_1$$

$$v_A = 0$$



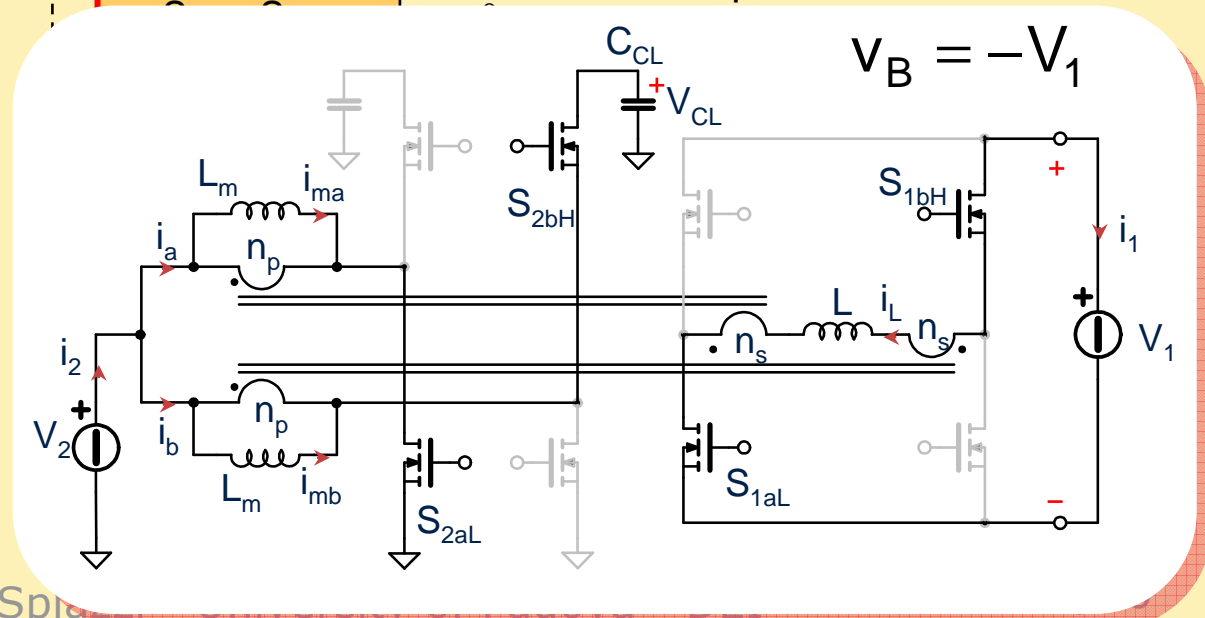
Plus Phase-Shift Modulation



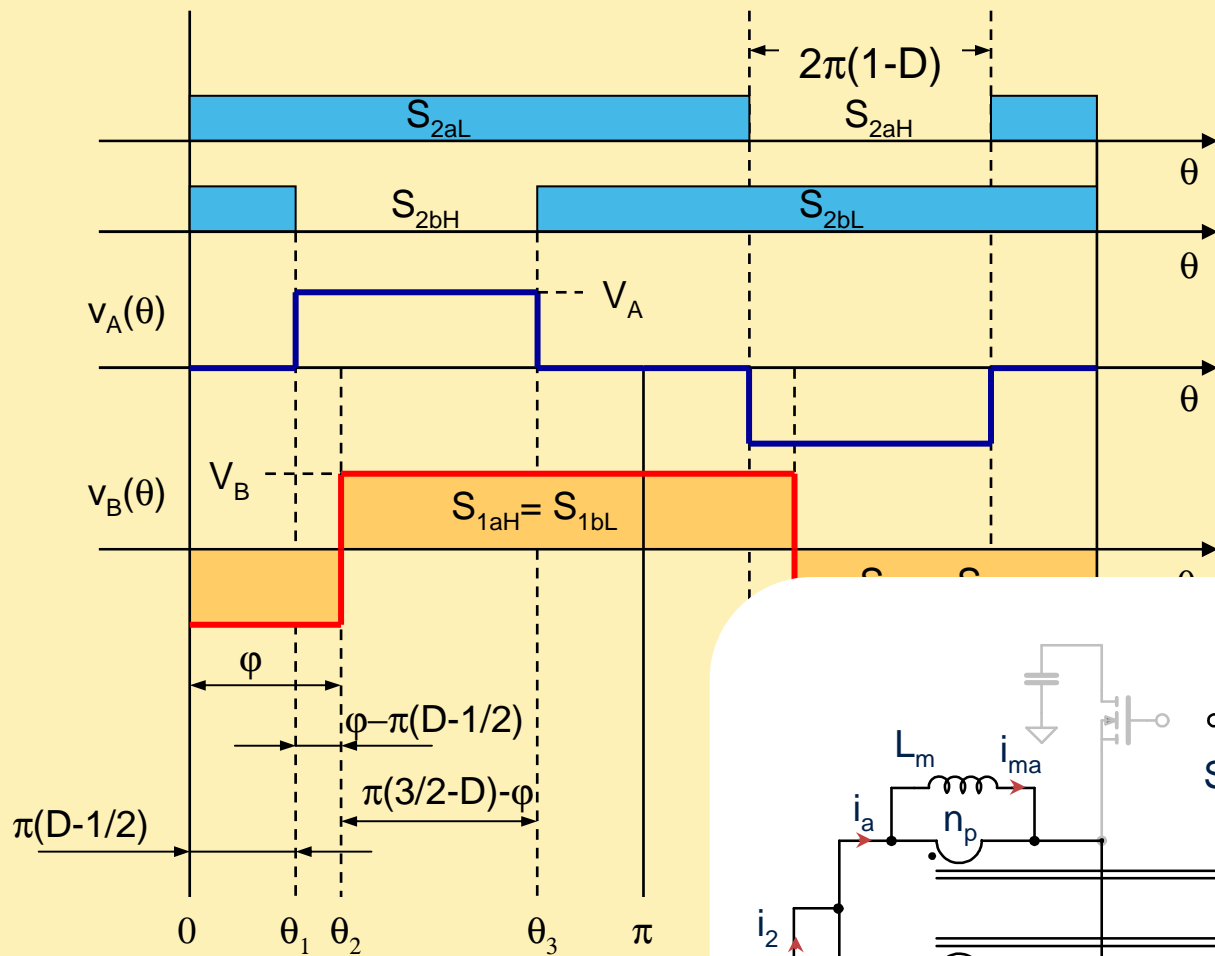
IBCI converter

$$\theta_1 \leq \theta \leq \theta_2$$

$$\begin{aligned}
 v_A &= \frac{n_s}{n_p} (V_g - V_g + V_{CL}) \\
 &= \frac{n_s}{n_p} V_{CL} = V_A
 \end{aligned}$$



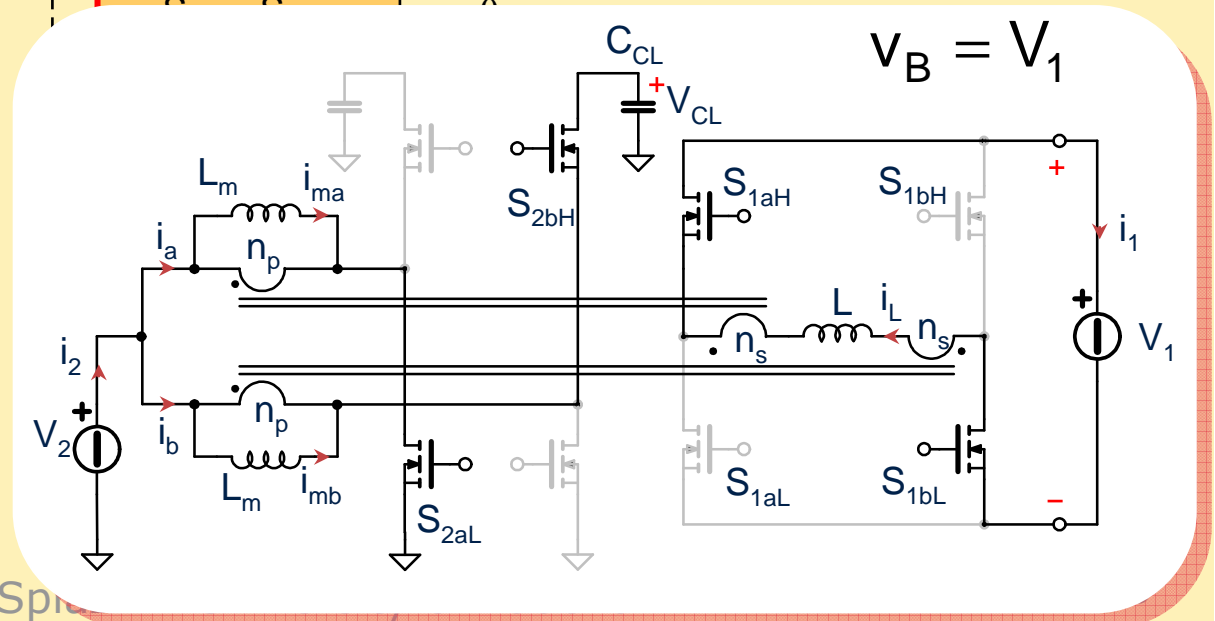
Plus Phase-Shift Modulation



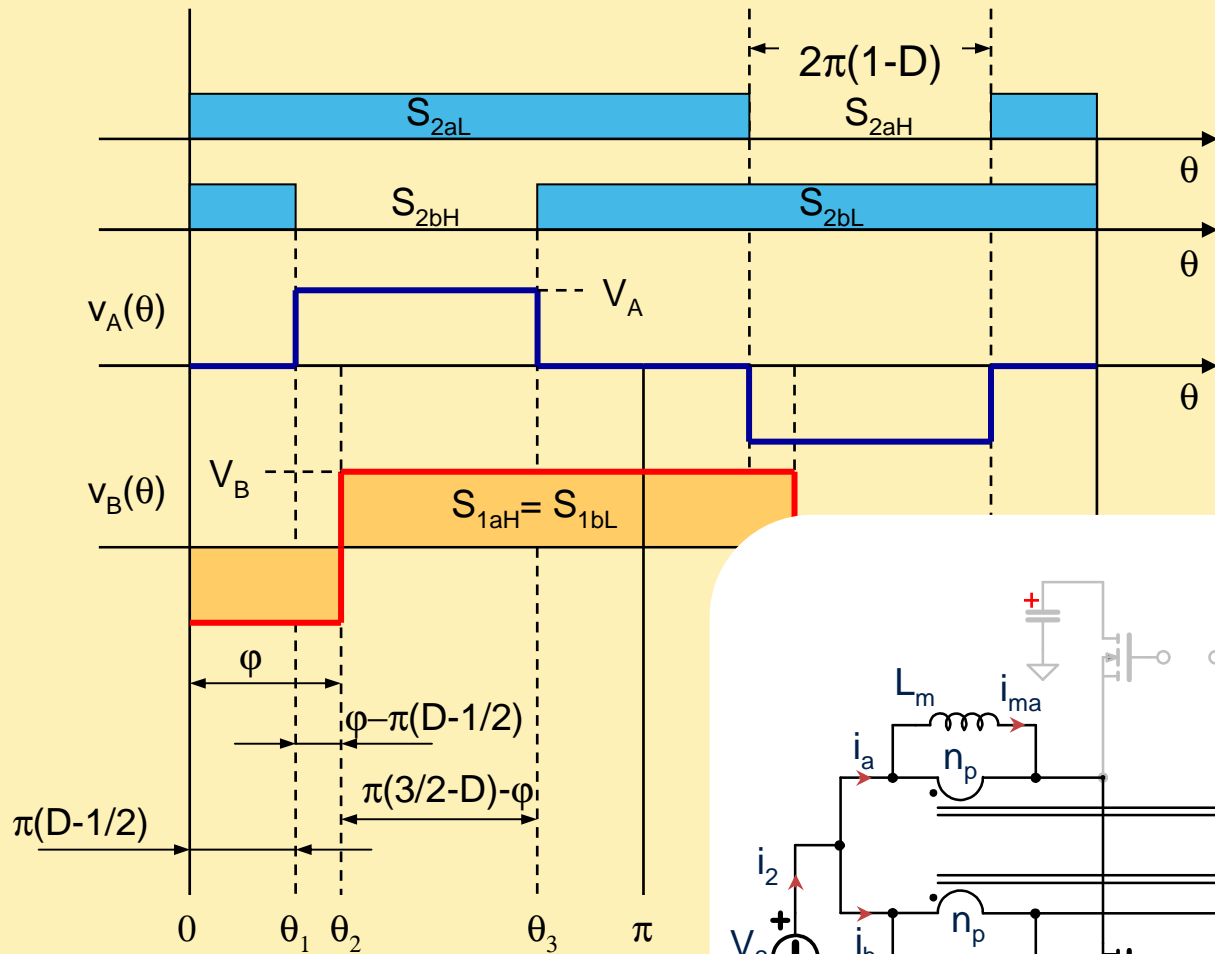
IBCI converter

$$\theta_2 \leq \theta \leq \theta_3$$

$$\begin{aligned}
 v_A &= \frac{n_s}{n_p} (V_g - V_g + V_{CL}) \\
 &= \frac{n_s}{n_p} V_{CL} = V_A
 \end{aligned}$$



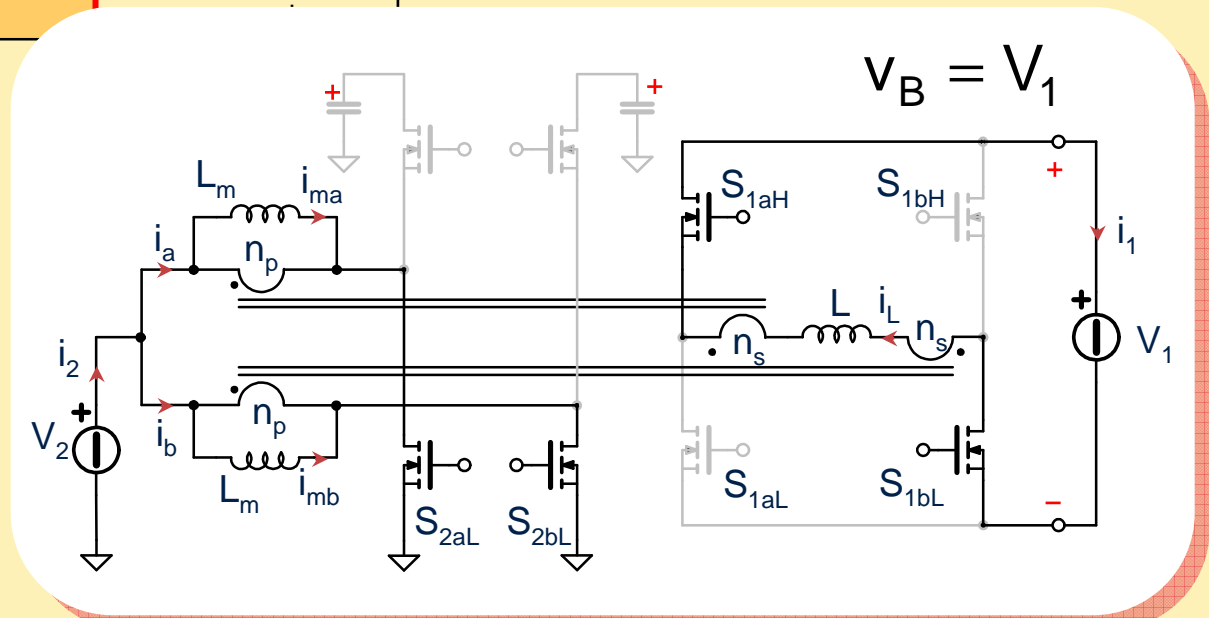
Plus Phase-Shift Modulation



IBCI converter

$$\theta_3 \leq \theta \leq \pi$$

$$V_A = 0$$



Plus Phase-Shift Modulation

- v_A is a three-level voltage of amplitude V_A while v_B is a square wave voltage of amplitude V_B

$$V_g D = (V_{CL} - V_g)(1 - D) \quad \Rightarrow \quad V_{CL} = \frac{V_g}{1 - D}$$

$$V_A = \frac{n_s}{n_p} V_{CL} = \frac{n_s}{n_p} \frac{V_2}{1 - D} \quad V_B = V_1$$

- The duty-cycle of port 2 switches is controlled so as to obtain the condition $V_A = V_B$
- The power transfer between ports 1 and 2 is controlled through the phase-shift angle φ

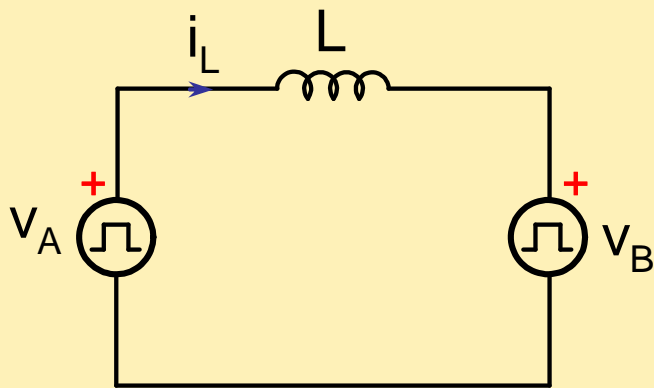
$$0 \leq \varphi \leq \pi/2 \quad \begin{array}{c} P \\ v_A \rightarrow v_B \end{array}$$

$$-\pi/2 \leq \varphi \leq 0 \quad \begin{array}{c} P \\ v_A \leftarrow v_B \end{array}$$

- Inductor current has a piecewise linear behavior

Plus Phase-Shift Modulation

IBCI converter

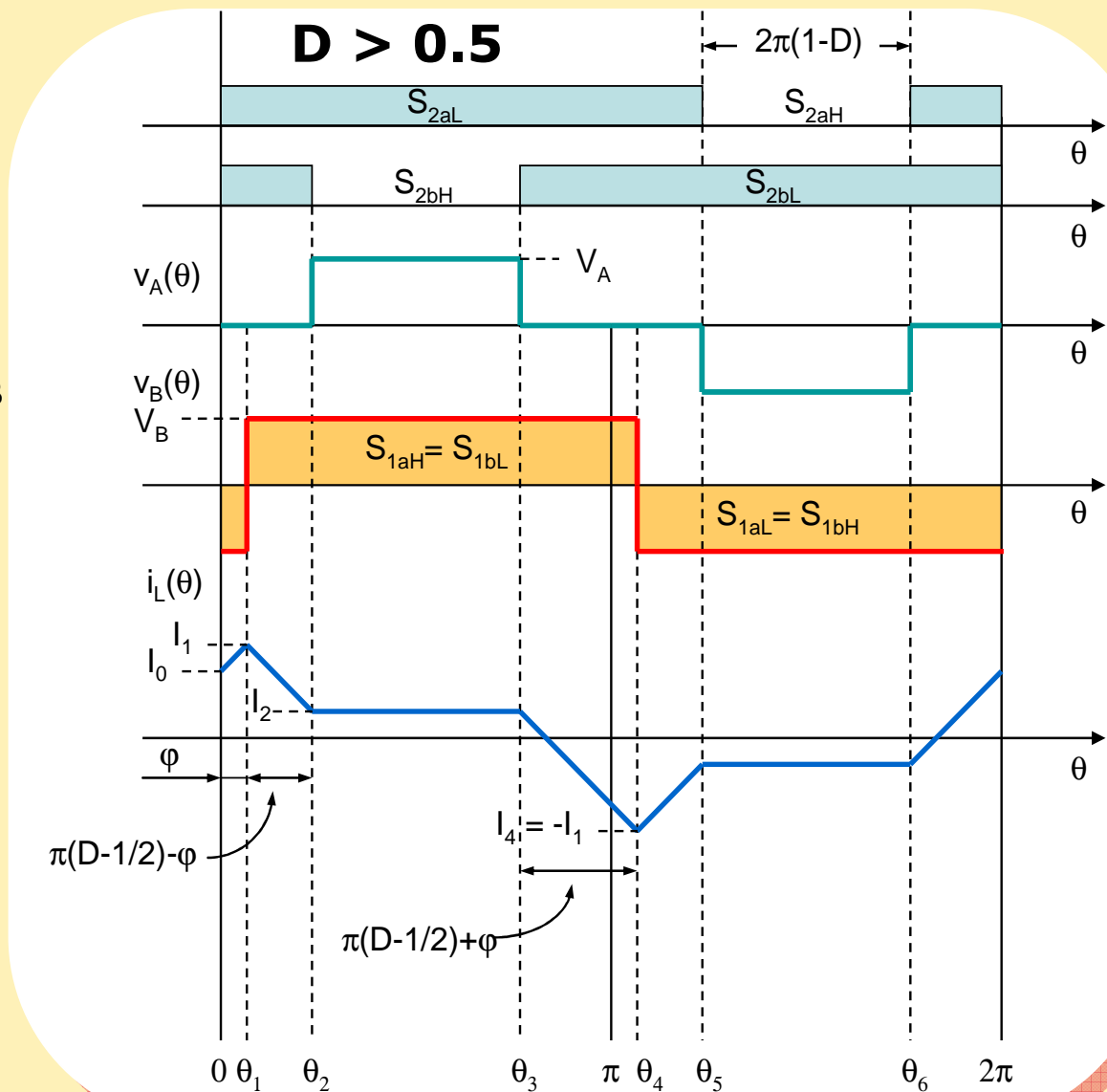


Power flow 

Case A:

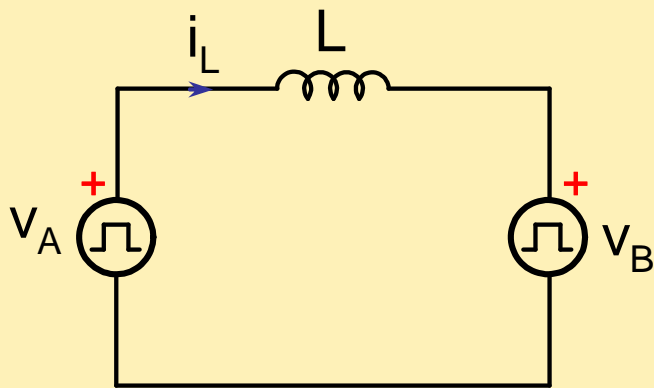
$$0 < \varphi < \pi(D-1/2)$$

$$(V_A = V_B)$$



Plus Phase-Shift Modulation

IBCI converter

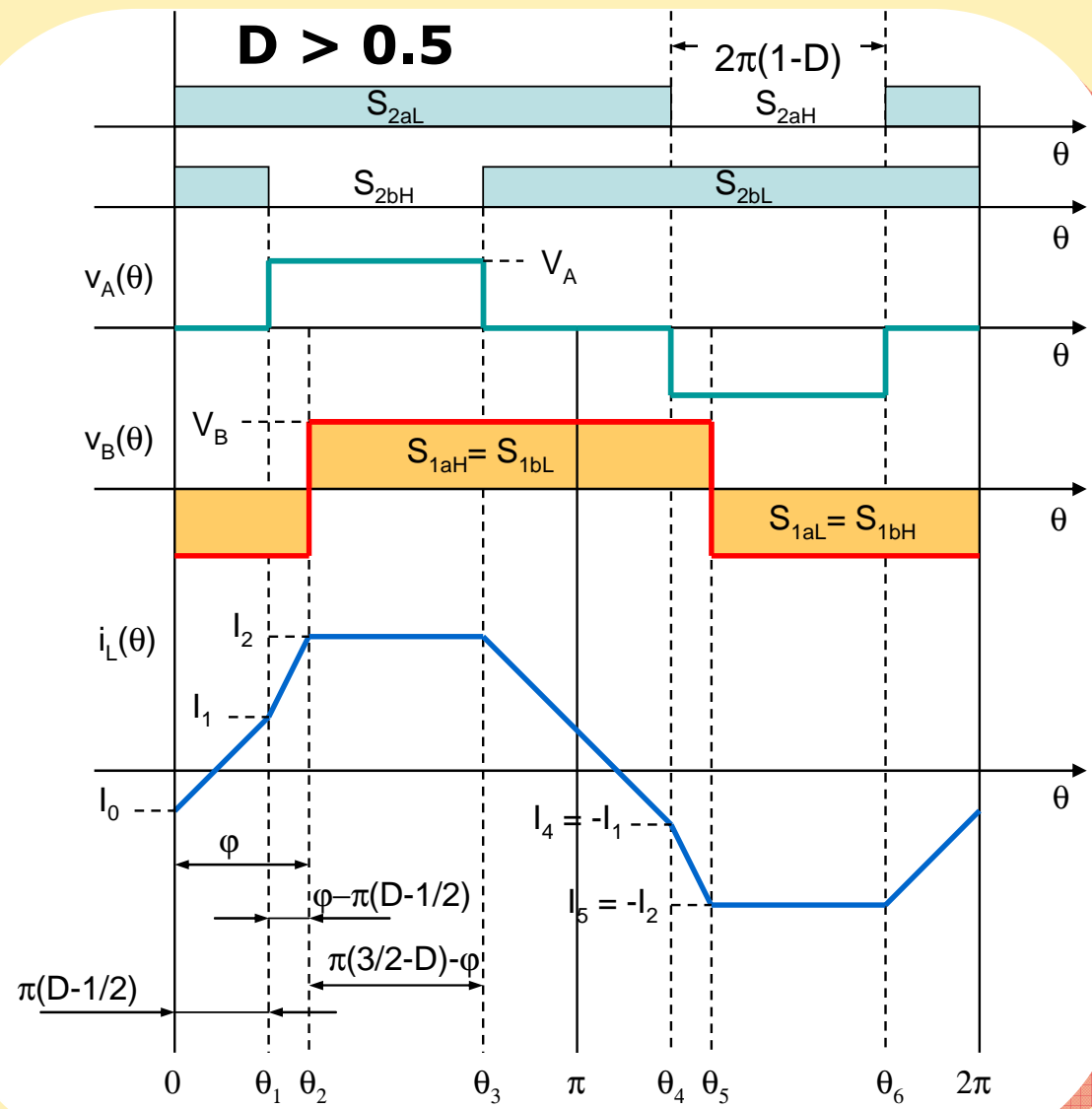


Power flow 

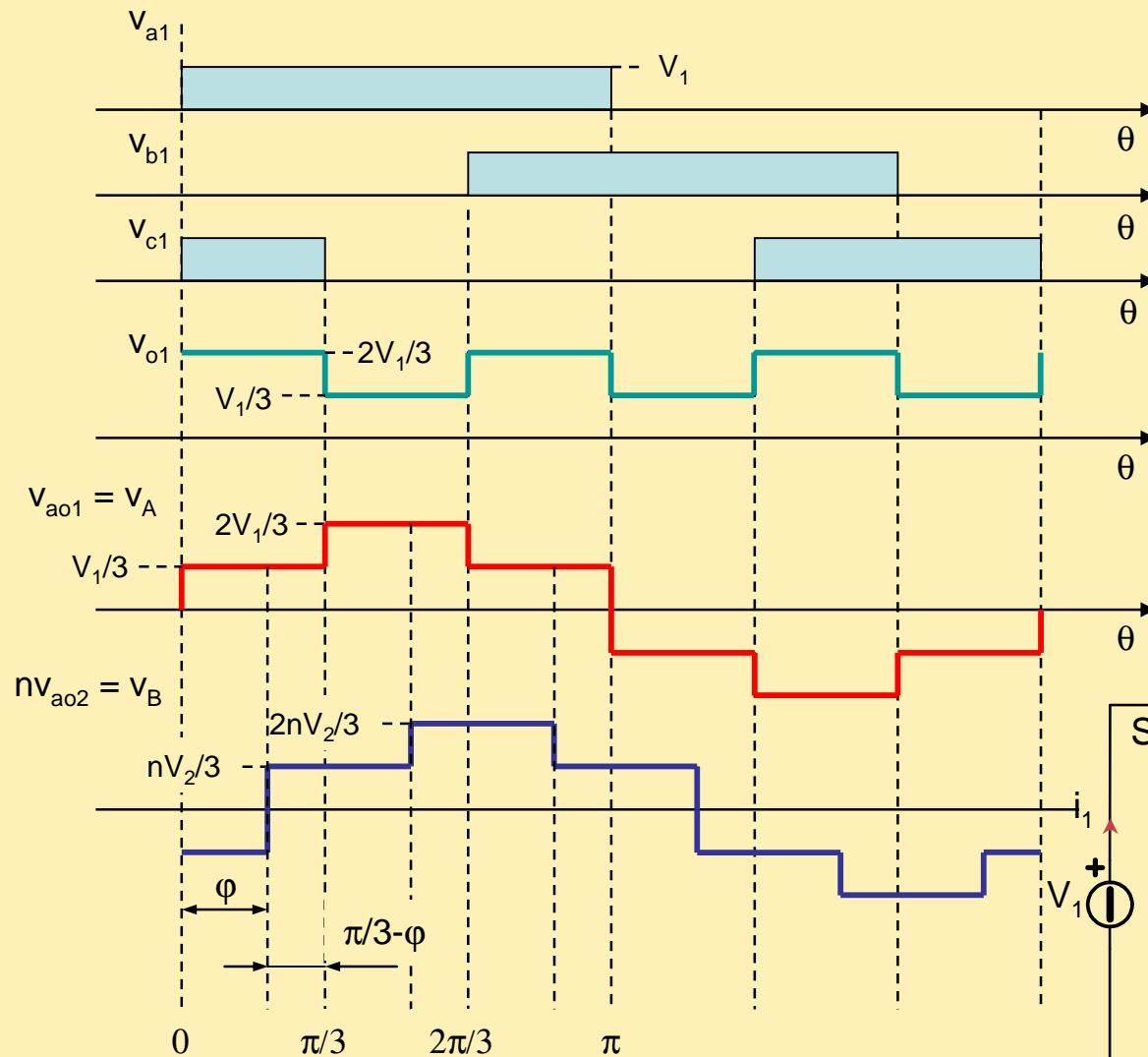
Case B:

$$\pi(D-1/2) < \varphi < \pi/2$$

$$(V_A = V_B)$$

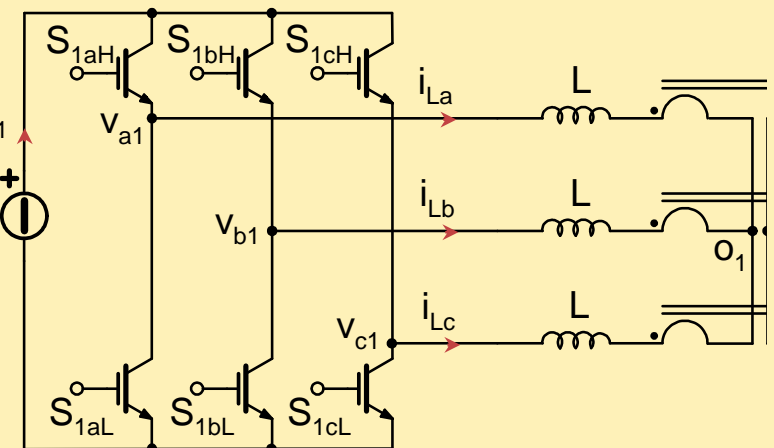


Phase-Shift Modulation in Three-Phase Converters



Phase **a** voltages
(H_p : symmetry)

$$\varphi < \pi/3$$



Phase-Shift Modulation in Three-Phase Converters

- Decoupled phase behavior (perfect symmetry)
- v_A and v_B are six-step voltage waveforms
- The power transfer between ports 1 and 2 is controlled through the phase-shift angle φ

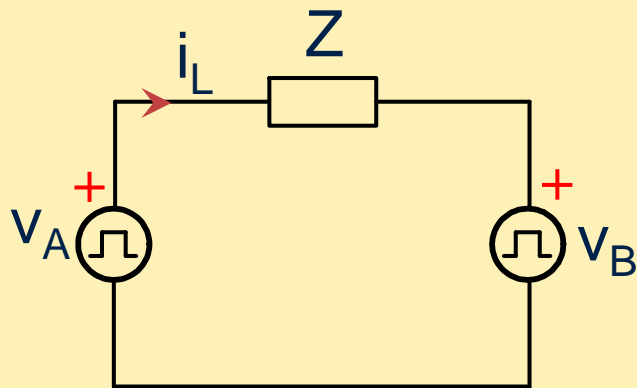
$$0 \leq \varphi \leq \pi/2 \quad v_A \xrightarrow{P} v_B$$
$$-\pi/2 \leq \varphi \leq 0 \quad v_A \xleftarrow{P} v_B$$

- Two different situations (power from port 1 to port 2):
 - $0 \leq \varphi \leq \pi/3$
 - $\pi/3 \leq \varphi \leq \pi/2$
- Inductor current has a piecewise linear behavior (DAB)

Systematic Steady-State Analysis

- The analytical determination of the current waveforms requires a systematic method for complex topologies (e.g. three-phase resonant DAB).
- The outcome is the mathematical expression of phase currents as a function of phase-shift and other design parameters.
- In addition, soft switching conditions can be analyzed in detail by extracting current values at the moment of switch commutations.

General case:



$$v = \frac{V_Z}{V_N} = \frac{V_A - V_B}{V_N}$$

$$k = \frac{V_B}{V_A}$$

Systematic Steady-State Analysis

The half switching period is divided into m subintervals. For each subinterval ($i = 1 \div m$), the values of the current shaping impedance state variables \mathbf{x}_i at the end of the interval are calculated, in normalized form, as a function of their value \mathbf{x}_{i-1} at the beginning, i.e.:

$$\mathbf{x}_i = \mathbf{M}_i \mathbf{x}_{i-1} + \mathbf{N}_i \mathbf{v}_i$$

We can iterate, obtaining

$$\mathbf{x}_m = \mathbf{M}_{m,1} \mathbf{x}_0 + \left(\sum_{i=1}^{m-1} \mathbf{M}_{m,i+1} \mathbf{N}_i \mathbf{v}_i \right) + \mathbf{N}_m \mathbf{v}_m = \mathbf{M}_{m,1} \mathbf{x}_0 + \mathbf{F}$$

where
$$\mathbf{M}_{j,i} = \prod_{k=i}^j \mathbf{M}_k \quad j \geq i$$

and \mathbf{v} is a column vector containing m \mathbf{v}_i elements, i.e.

$$\mathbf{v} = [\mathbf{v}_1 \quad \mathbf{v}_2 \quad \cdots \quad \mathbf{v}_m]^T$$

Exploiting the waveform symmetry, we can write:

$$\mathbf{x}_m = \mathbf{M}_{m,1} \mathbf{x}_0 + \mathbf{F} = -\mathbf{x}_0$$

from which the initial state variable values are found:

$$\mathbf{x}_0 = \left(-\mathbf{I} - \mathbf{M}_{m,1} \right)^{-1} \mathbf{F}$$

that can be used to derive the current waveform expressions and to discuss soft-switching conditions

Example: IBCI

Base variables:

- **Base voltage:** $V_N = V_A$
- **Base impedance:** $Z_N = \omega_{sw} L$
- **Base current:** $I_N = V_N / Z_N$
- **Base power:** $P_N = V_N^2 / Z_N$

The half switching period is subdivided into 4 subintervals (**m = 4**).

Two situations has to be considered:

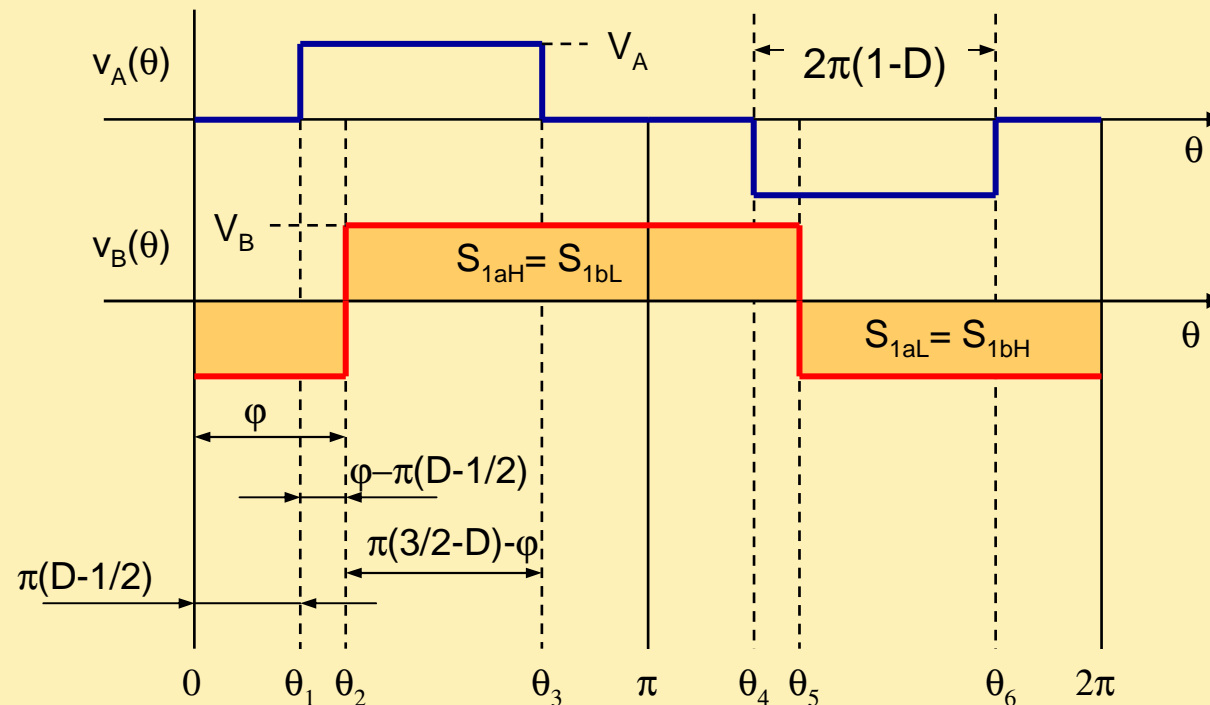
Case A: $0 < \varphi < \pi(D-1/2)$

Case B: $\pi(D-1/2) < \varphi < \pi/2$

Example: IBCI

$i =$	1		2		3		4	
	v	δ	v	δ	v	δ	v	δ
A	k	φ	$-k$	$\pi\left(D - \frac{1}{2}\right) - \varphi$	$1-k$	$2\pi(1-D)$	$-k$	$\pi\left(D - \frac{1}{2}\right)$
B	k	$\pi\left(D - \frac{1}{2}\right)$	$1+k$	$\varphi - \pi\left(D - \frac{1}{2}\right)$	$1-k$	$\pi\left(\frac{3}{2} - D\right) - \varphi$	$-k$	$\pi\left(D - \frac{1}{2}\right)$

Case B




Example: IBCI

Defining $j(\theta) = i(\theta)/I_N$ as the normalized inductor current:

$$J_i = J_{i-1} + v_i \delta_i \quad \text{for } i = 1, \dots, m$$

Comparing with: $\mathbf{x}_i = \mathbf{M}_i \mathbf{x}_{i-1} + \mathbf{N}_i v_i$


$$\begin{cases} \mathbf{M}_i = 1 \\ \mathbf{N}_i = \delta_i \end{cases} \quad \text{for } i = 1, \dots, m$$

Example: IBCI

From: $\mathbf{x}_0 = (-\mathbf{I} - \mathbf{M}_{m,1})^{-1} \mathbf{F}$

the normalized initial inductor current value is:

$$J_0 = -\frac{1}{2} \sum_{i=1}^4 v_i \delta_i$$

For both cases A and B we have:

$$J_0 = -\pi(1-D) + k \left(\frac{\pi}{2} - \varphi \right)$$

For plus phase-shift modulation **k = 1**:

$$J_0 = \pi \left(D - \frac{1}{2} \right) - \varphi$$

Example: SR-DAB

Base variables:

■ Base voltage:

$$V_N = V_A$$

■ Base impedance:

$$Z_N = Z_r$$

■ Base current:

$$I_N = V_N / Z_N$$

■ Base power:

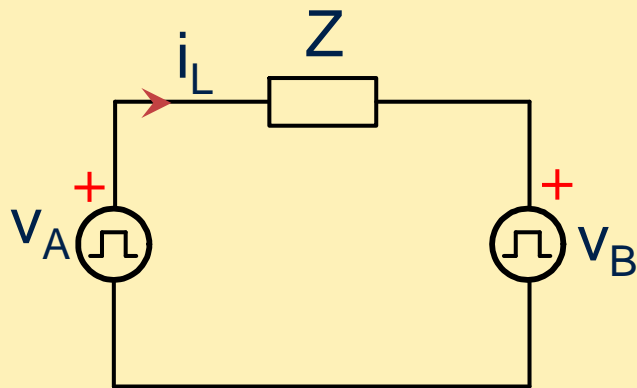
$$P_N = V_N^2 / Z_N$$

■ Base frequency:

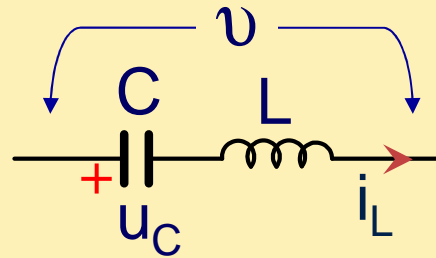
$$\omega_N = \omega_r$$

$$Z_r = \sqrt{\frac{L}{C}}$$

$$\omega_r = \frac{1}{\sqrt{LC}}$$



Example: SR-DAB



Current shaping impedance state variables:

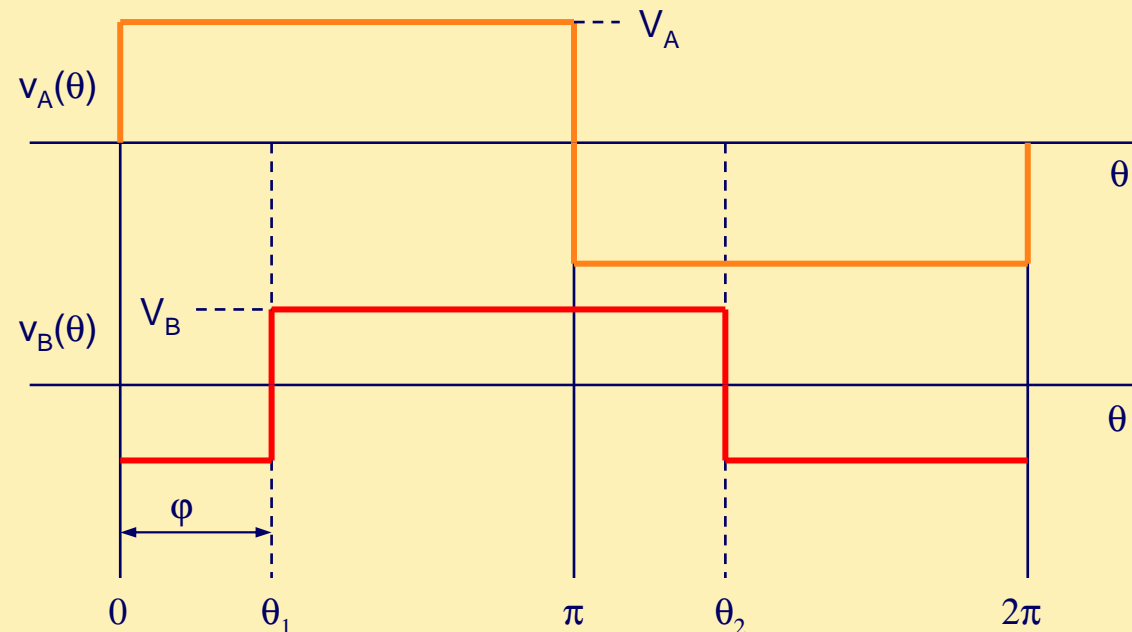
$$\begin{cases} j_L(\theta) = v \sin\left(\frac{\theta}{f_n}\right) - U_{C0} \sin\left(\frac{\theta}{f_n}\right) + J_{L0} \cos\left(\frac{\theta}{f_n}\right) \\ u_C(\theta) = v \left(1 - \cos\left(\frac{\theta}{f_n}\right)\right) + U_{C0} \cos\left(\frac{\theta}{f_n}\right) + J_{L0} \sin\left(\frac{\theta}{f_n}\right) \end{cases}$$

Normalized state variable vector: $\mathbf{x} = \begin{bmatrix} j_L \\ u_C \end{bmatrix}$

Example: SR-DAB

The half switching period is subdivided into 2 subintervals ($m = 2$).

$i = 1$		$i = 2$	
v	δ	v	δ
$1+k$	φ	$1-k$	$\pi - \varphi$



Example: SR-DAB

Current shaping impedance state variables:

$$\begin{bmatrix} J_{Li} \\ U_{Ci} \end{bmatrix} = \underbrace{\begin{pmatrix} \cos\left(\frac{\delta_i}{f_n}\right) & -\sin\left(\frac{\delta_i}{f_n}\right) \\ \sin\left(\frac{\delta_i}{f_n}\right) & \cos\left(\frac{\delta_i}{f_n}\right) \end{pmatrix}}_{\text{Matrix M}} \begin{bmatrix} J_{Li-1} \\ U_{Ci-1} \end{bmatrix} + \underbrace{\begin{pmatrix} \sin\left(\frac{\delta_i}{f_n}\right) \\ 1 - \cos\left(\frac{\delta_i}{f_n}\right) \end{pmatrix}}_{\text{Matrix N}} v_i \quad \text{for } i = 1, 2$$

Matrix M

Matrix N

$$\mathbf{M}_{m,1} = \mathbf{M}_{2,1} = \begin{pmatrix} \cos\left(\frac{\pi}{f_n}\right) & -\sin\left(\frac{\pi}{f_n}\right) \\ \sin\left(\frac{\pi}{f_n}\right) & \cos\left(\frac{\pi}{f_n}\right) \end{pmatrix}$$

Example: SR-DAB

Matrix \mathbf{F} :

$$\mathbf{F} = \begin{pmatrix} \sin\left(\frac{\pi}{f_n}\right) \\ 1 - \cos\left(\frac{\pi}{f_n}\right) \end{pmatrix} + \begin{pmatrix} \sin\left(\frac{\pi}{f_n}\right) - 2\sin\left(\frac{\pi - \varphi}{f_n}\right) \\ 2\cos\left(\frac{\pi - \varphi}{f_n}\right) - \cos\left(\frac{\pi}{f_n}\right) - 1 \end{pmatrix} \mathbf{k}$$

Initial conditions:

$$\mathbf{x}_0 = \begin{pmatrix} -1 - \cos\left(\frac{\pi}{f_n}\right) & \sin\left(\frac{\pi}{f_n}\right) \\ -\sin\left(\frac{\pi}{f_n}\right) & -1 - \cos\left(\frac{\pi}{f_n}\right) \end{pmatrix}^{-1} \mathbf{F}$$

Example: SR-DAB

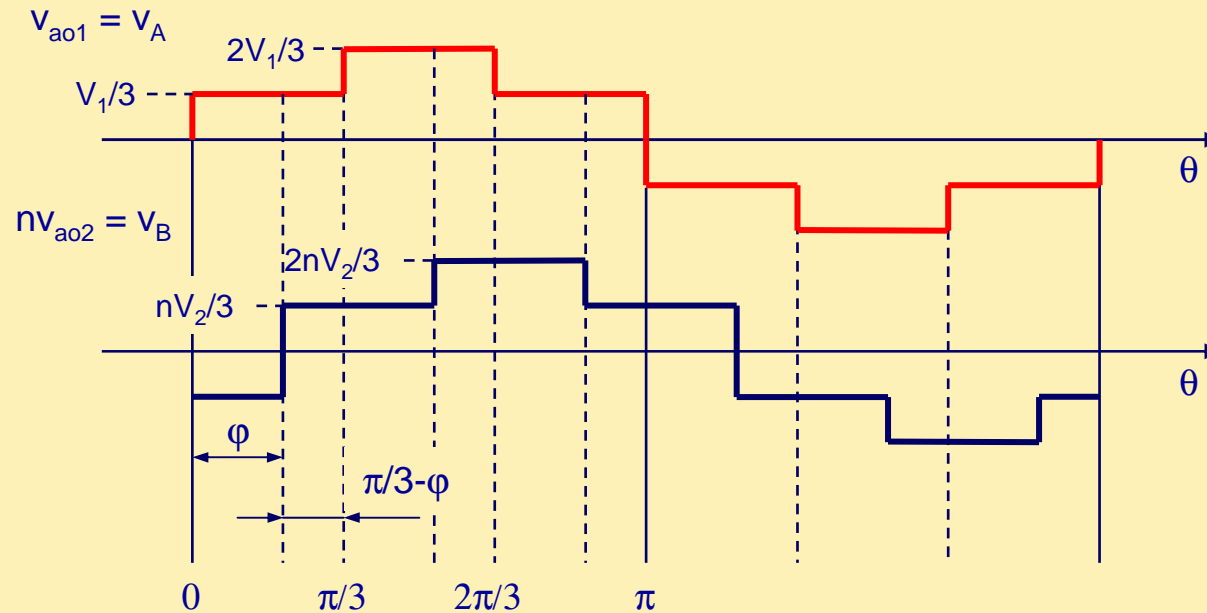
Initial conditions:

$$\mathbf{x}_0 = \frac{1}{1 + \cos\left(\frac{\pi}{f_n}\right)} \left[\begin{array}{c} \left(-\sin\left(\frac{\pi}{f_n}\right) \right) \\ 0 \end{array} + \left(\begin{array}{c} \sin\left(\frac{\pi - \varphi}{f_n}\right) - \sin\left(\frac{\varphi}{f_n}\right) \\ 1 + \cos\left(\frac{\pi}{f_n}\right) - \cos\left(\frac{\pi - \varphi}{f_n}\right) - \cos\left(\frac{\varphi}{f_n}\right) \end{array} \right) k \right]$$



$$J_{L0}(\varphi) = \frac{-\sin\left(\frac{\pi}{f_n}\right) + k \left[\sin\left(\frac{\pi - \varphi}{f_n}\right) - \sin\left(\frac{\varphi}{f_n}\right) \right]}{1 + \cos\left(\frac{\pi}{f_n}\right)}$$

Example: Three-Phase DAB

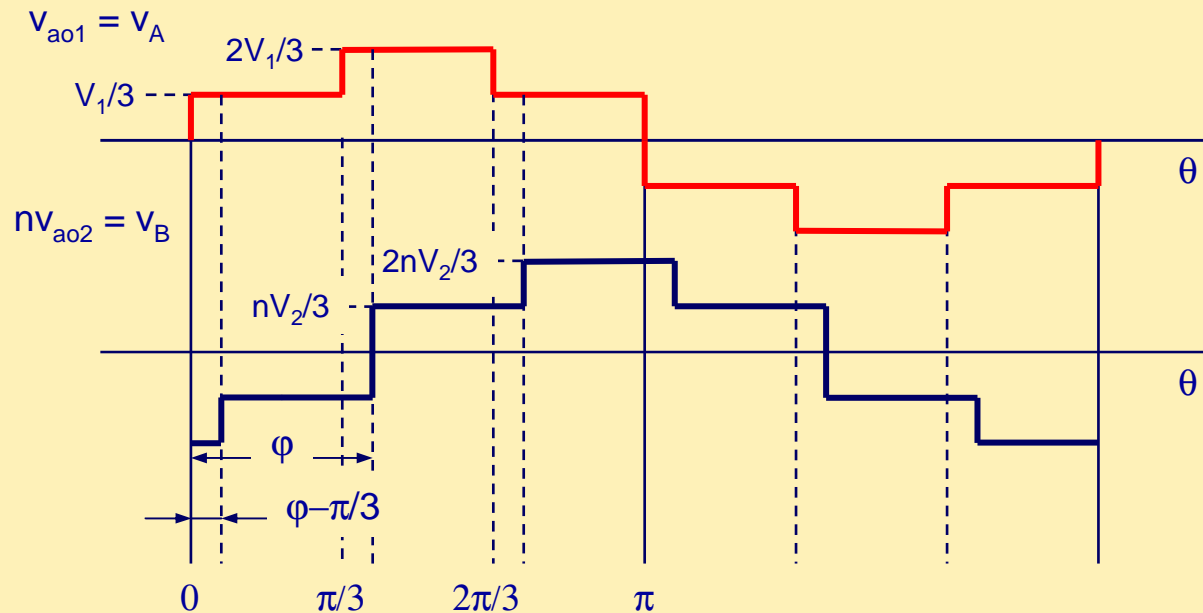


The half switching period is subdivided into 6 subintervals (**$m = 6$**).

Two situations has to be considered:

Case A: $0 < \varphi < \pi/3$

Example: Three-Phase DAB and SR-DAB



The half switching period is subdivided into 6 subintervals (**m = 6**).

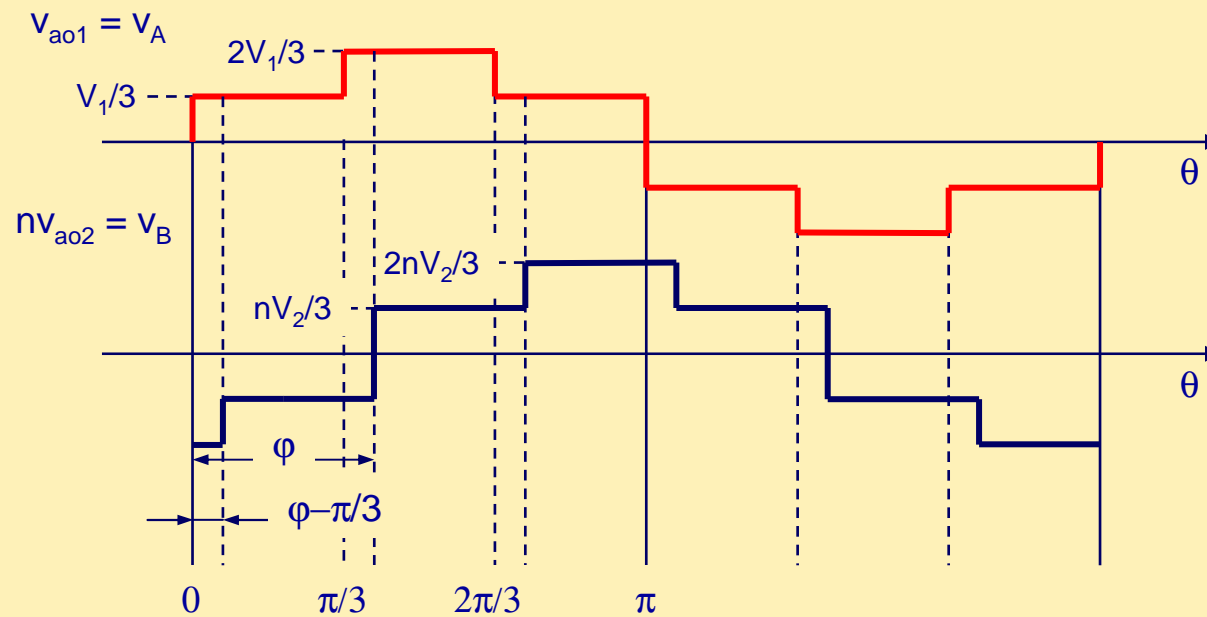
Two situations has to be considered:

Case B: $\pi/3 < \varphi < \pi/2$

Example: Three-Phase DAB and SR-DAB

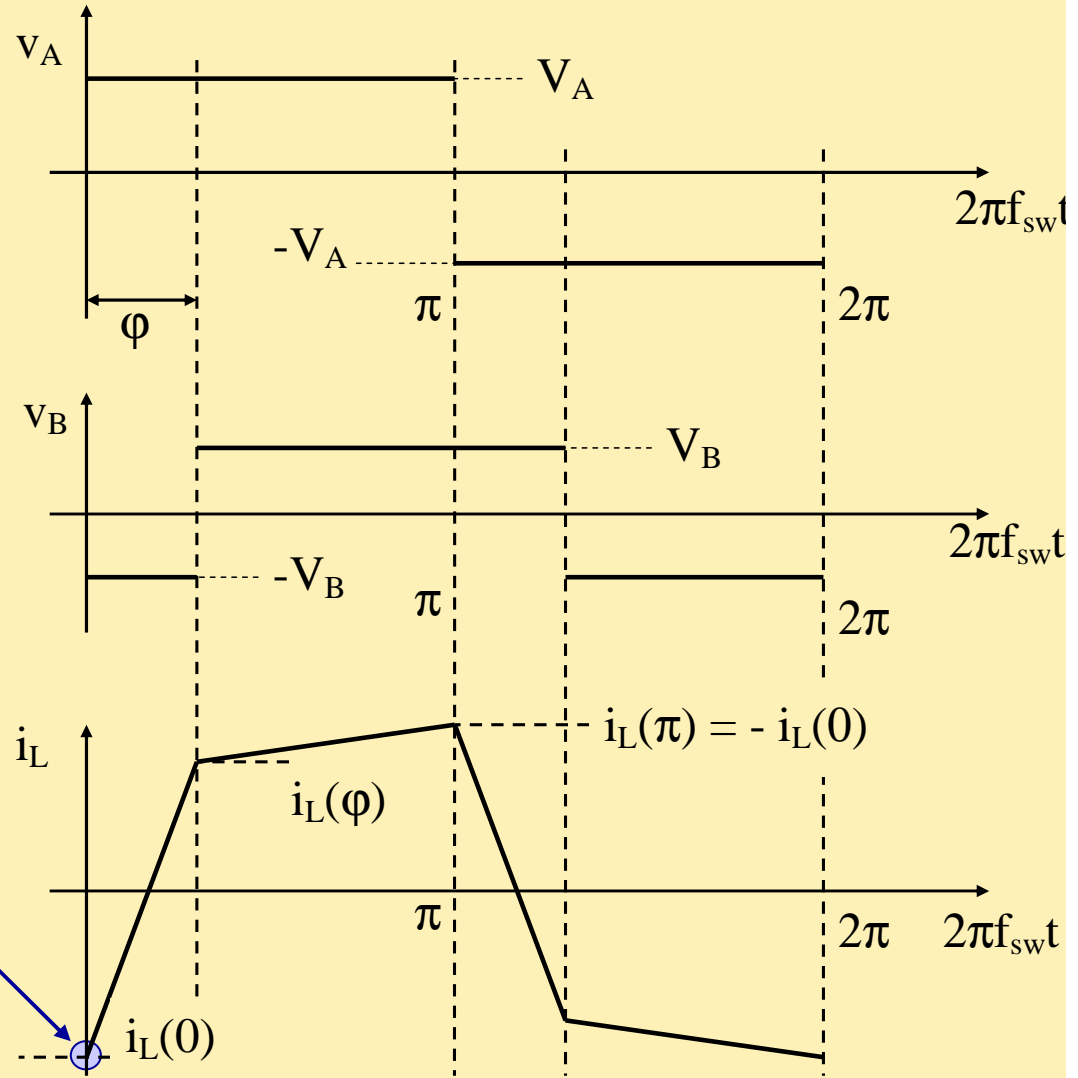
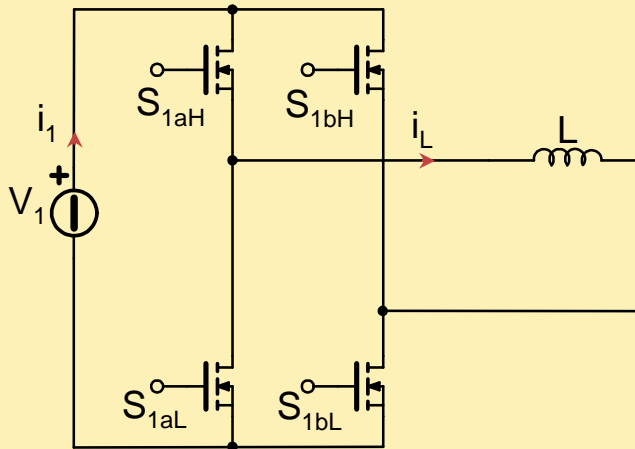
i =	1		2		3		4		5		6	
	v	δ	v	δ	v	δ	v	δ	v	δ	v	δ
A	$\frac{1+k}{3}$	φ	$\frac{1-k}{3}$	$\frac{\pi}{3}-\varphi$	$\frac{2-k}{3}$	φ	$\frac{2(1-k)}{3}$	$\frac{\pi}{3}-\varphi$	$\frac{1-2k}{3}$	φ	$\frac{1-k}{3}$	$\frac{\pi}{3}-\varphi$
B	$\frac{1+2k}{3}$	$\varphi-\frac{\pi}{3}$	$\frac{1+k}{3}$	$\frac{2\pi}{3}-\varphi$	$\frac{2+k}{3}$	$\varphi-\frac{\pi}{3}$	$\frac{2-k}{3}$	$\frac{2\pi}{3}-\varphi$	$\frac{1-k}{3}$	$\varphi-\frac{\pi}{3}$	$\frac{1-2k}{3}$	$\frac{2\pi}{3}-\varphi$

Case B



Soft-switching conditions

Single phase DAB: port 1



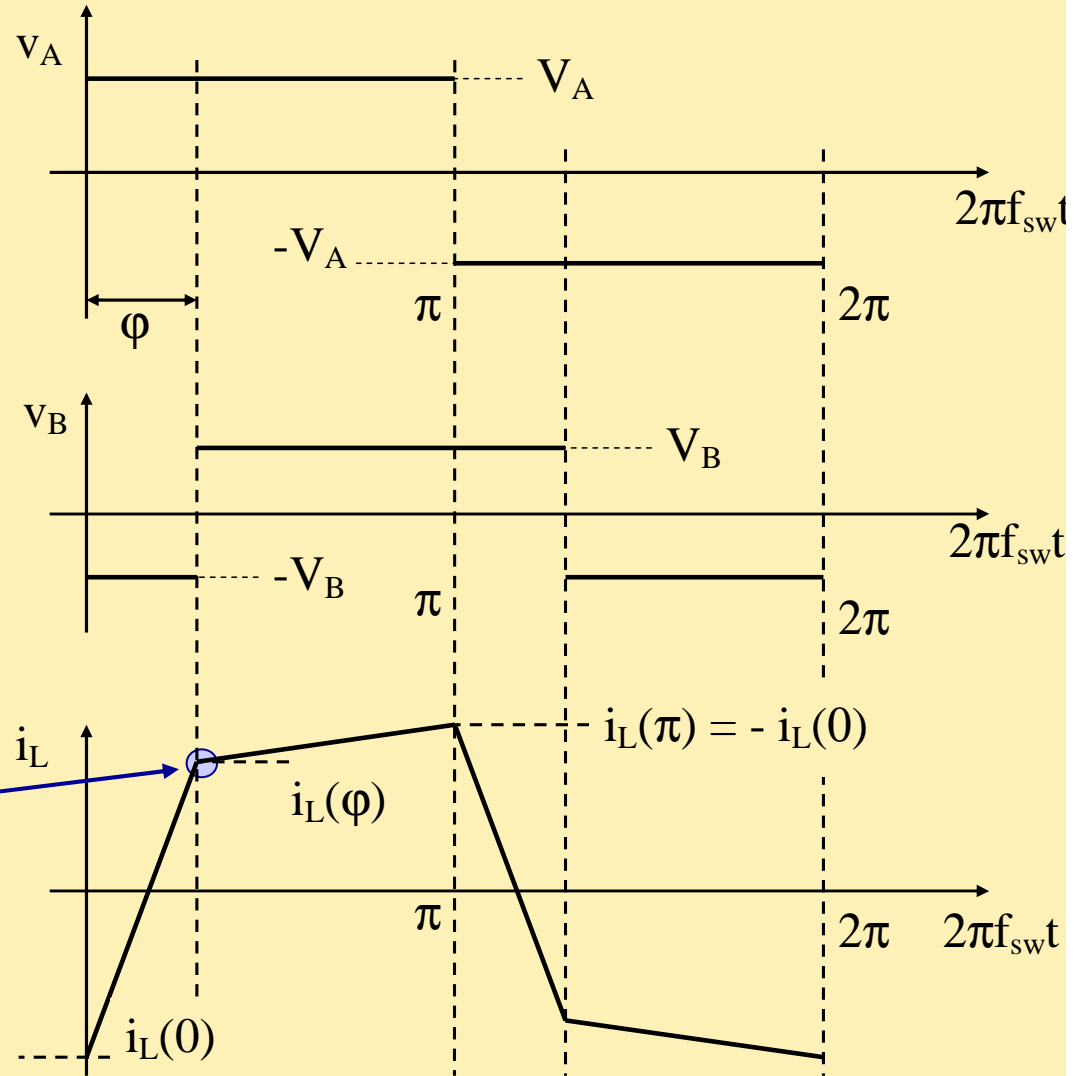
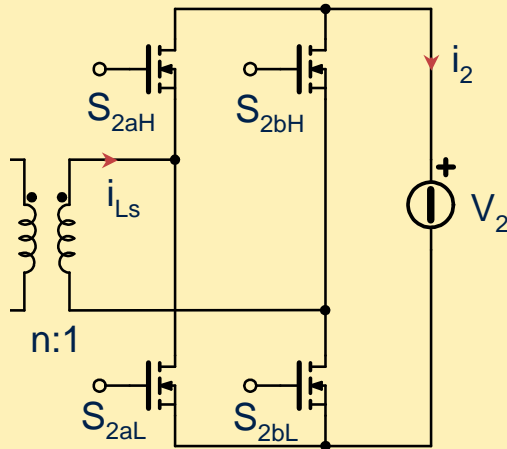
Power flow →

$$i_L(0) = -\phi \cdot k - \frac{\pi}{2}(1-k) \leq 0$$

$$\phi \geq \frac{\pi}{2} \left(\frac{1}{k} - 1 \right)$$

Soft-switching conditions

Single phase DAB: port 2



Power flow →

$$i_L(\varphi) = \varphi - \frac{\pi}{2}(1-k) \geq 0$$

$$\varphi \geq \frac{\pi}{2}(1-k)$$

Soft-switching conditions

Single phase DAB

Port 1:

$$\varphi \geq \frac{\pi}{2} \left(\frac{1}{k} - 1 \right)$$

Port 2:

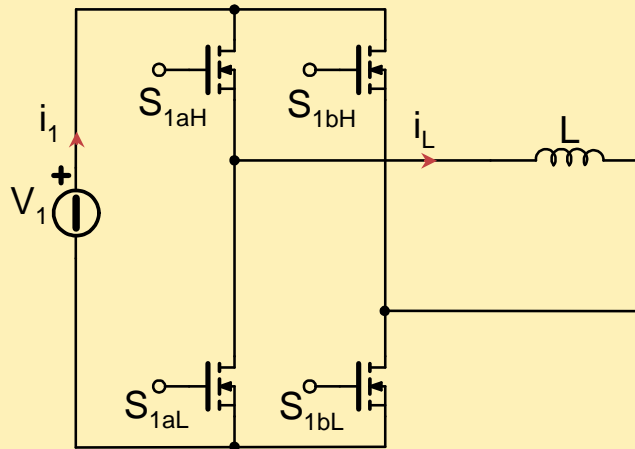
$$\varphi \geq \frac{\pi}{2} (1 - k)$$

For a power flow from port 1 to port 2 the phase-shift interval is $0 \leq \varphi \leq \pi/2$. Thus, if $k \geq 1$ the soft switching condition is satisfied for any φ value and for both bridge switches.

The same consideration holds for a power flow from port 2 to port 1 where voltages v_A and v_B are swept and $k' = 1/k$. Now, if $k' \geq 1$ ($k \leq 1$) the soft switching condition is satisfied for any φ value.

Soft-switching conditions

Single phase DAB



Port 1:

$$\varphi \geq \frac{\pi}{2} \left(\frac{1}{k} - 1 \right)$$

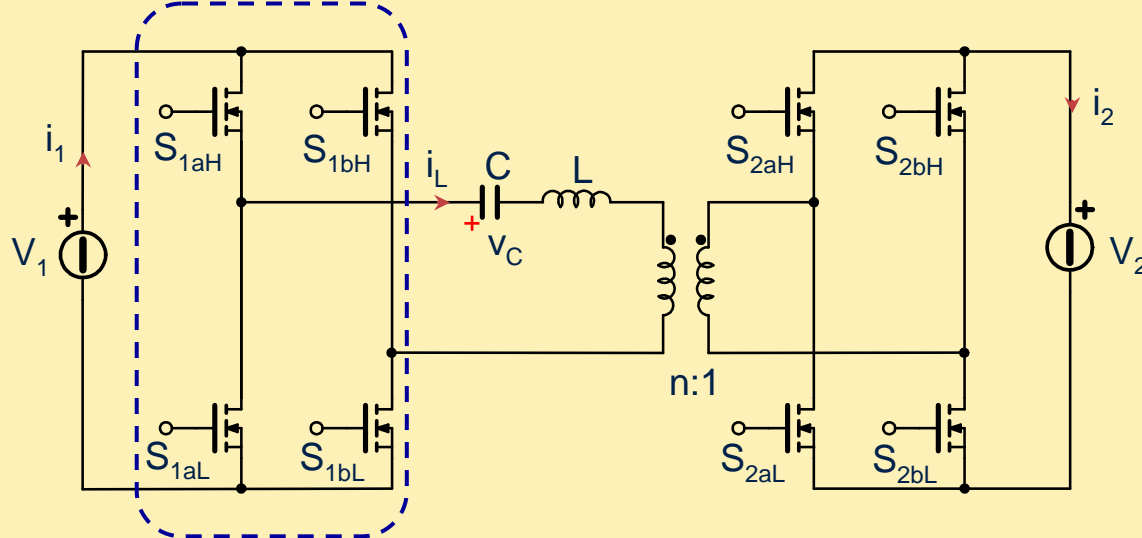
Port 2:

$$\varphi \geq \frac{\pi}{2} (1 - k)$$

For a bidirectional power flow, if $k = 1$ the soft switching condition is satisfied for any φ value between $-\pi/2$ and $\pi/2$.

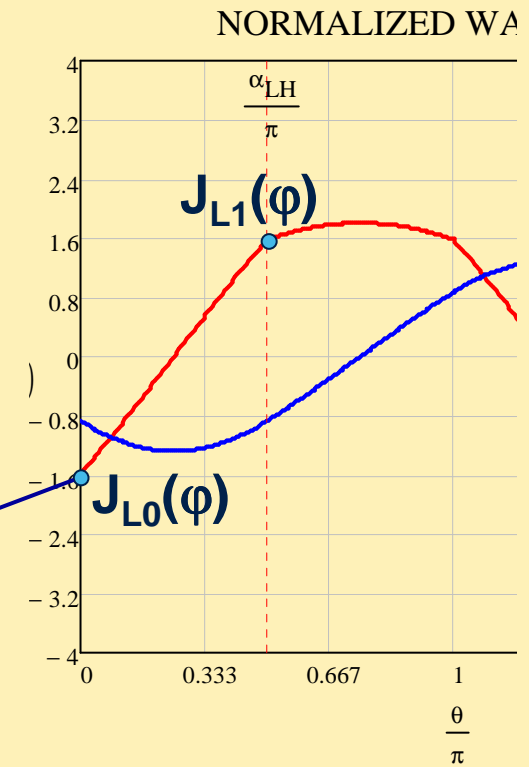
Soft-switching conditions

Single phase SR-DAB converter: port 1



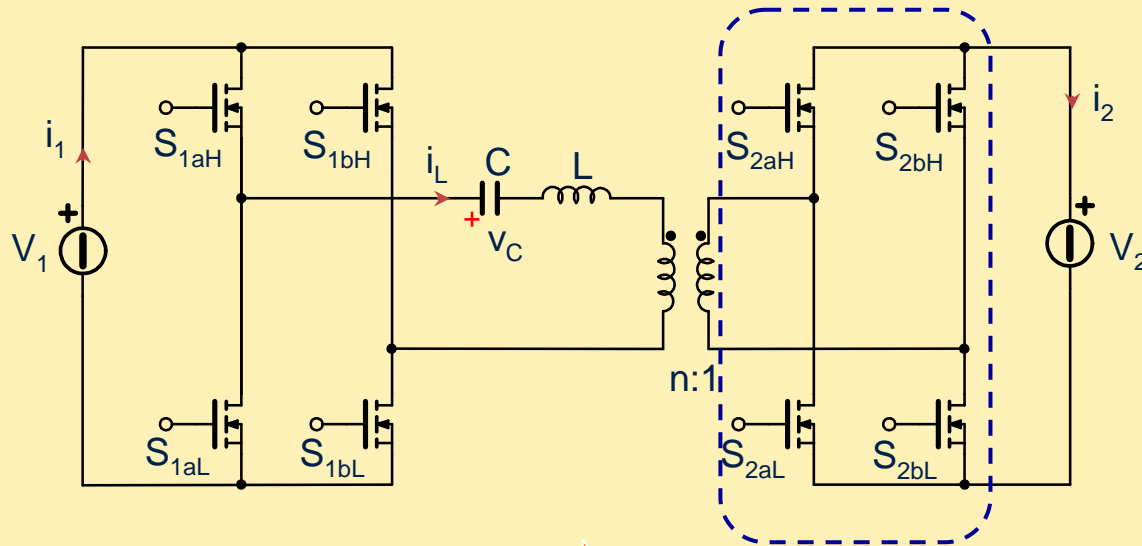
Power flow →

$$J_{L0}(\varphi) = \frac{-\sin\left(\frac{\pi}{f_n}\right) + k \left[\sin\left(\frac{\pi - \varphi}{f_n}\right) - \sin\left(\frac{\varphi}{f_n}\right) \right]}{1 + \cos\left(\frac{\pi}{f_n}\right)} < 0$$



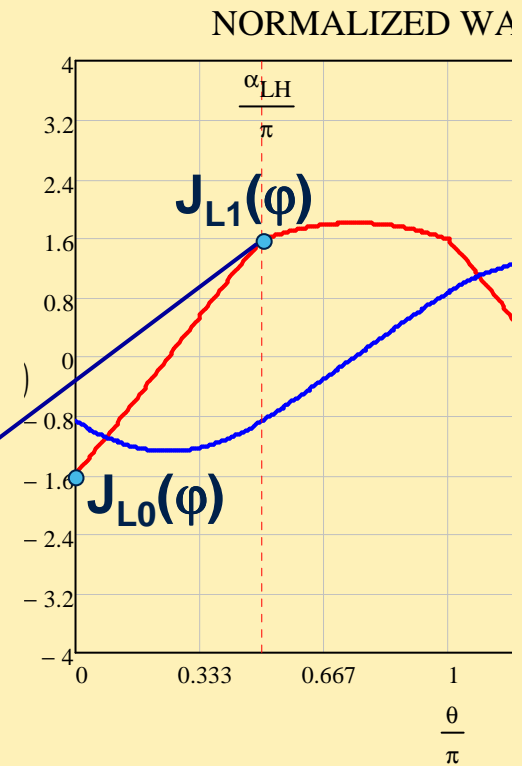
Soft-switching conditions

Single phase SR-DAB converter: port 2



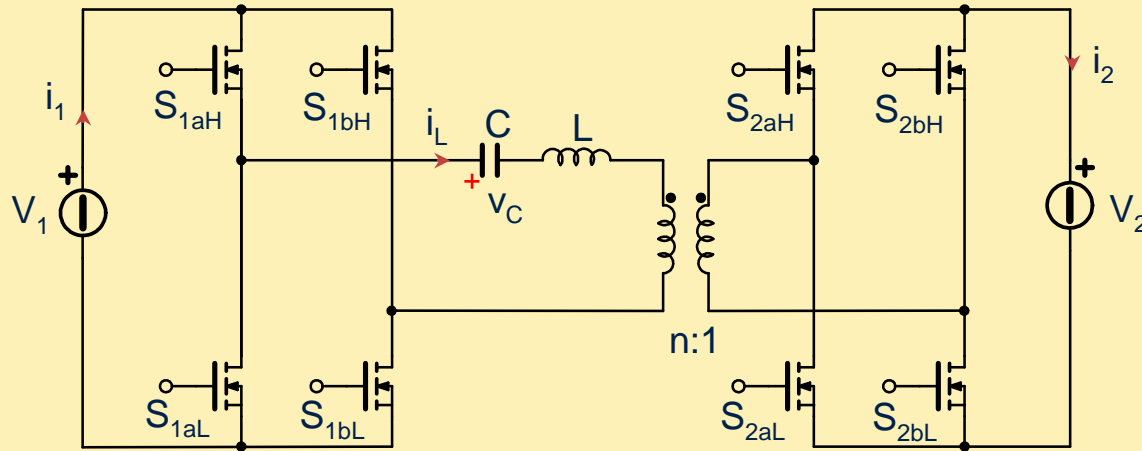
Power flow →

$$J_{L1}(\varphi) = \frac{\sin\left(\frac{\varphi}{f_n}\right) - \sin\left(\frac{\pi - \varphi}{f_n}\right)}{1 + \cos\left(\frac{\pi}{f_n}\right)} + \frac{\sin\left(\frac{\pi}{f_n}\right)}{1 + \cos\left(\frac{\pi}{f_n}\right)} \quad k > 0$$



Soft-switching conditions

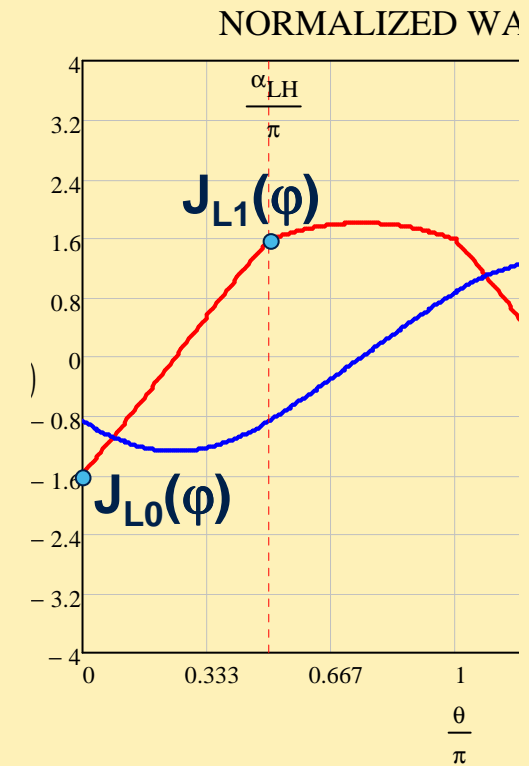
Single phase SR-DAB converter



Power flow →

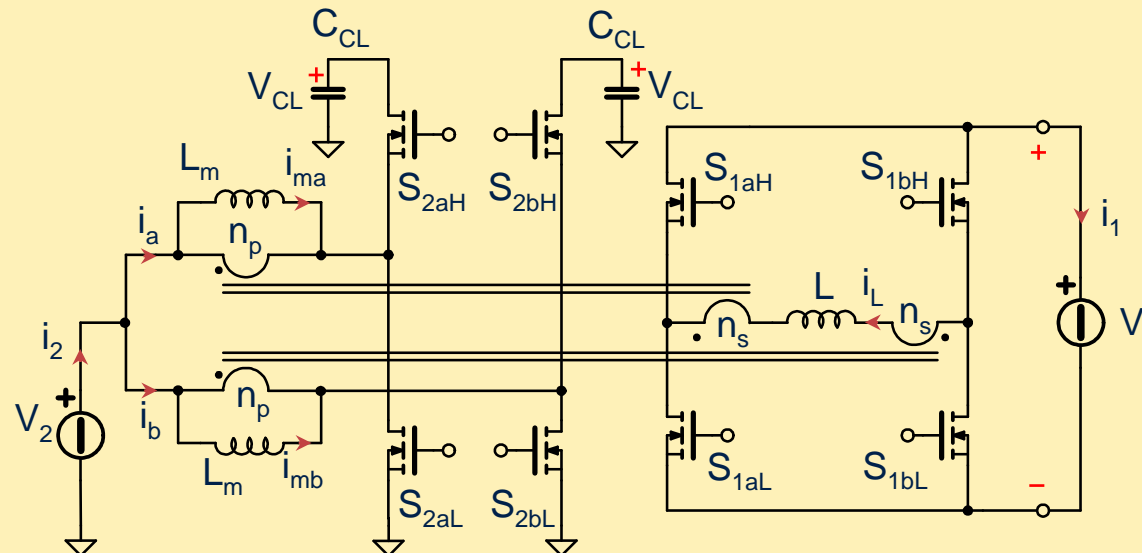
Worst case: $\varphi = 0$

$$J_{L0}(0) = J_{L1}(0) = \frac{\sin\left(\frac{\pi}{f_n}\right)}{1 + \cos\left(\frac{\pi}{f_n}\right)} (k - 1) \quad \rightarrow \quad \mathbf{K = 1}$$



Soft-switching conditions

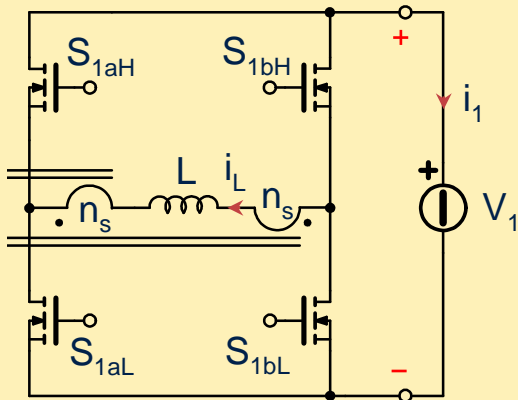
Interleaved Boost with Coupled Inductors



Power flow 

Let's analyze the **port 1** switch commutations first

Soft-switching conditions



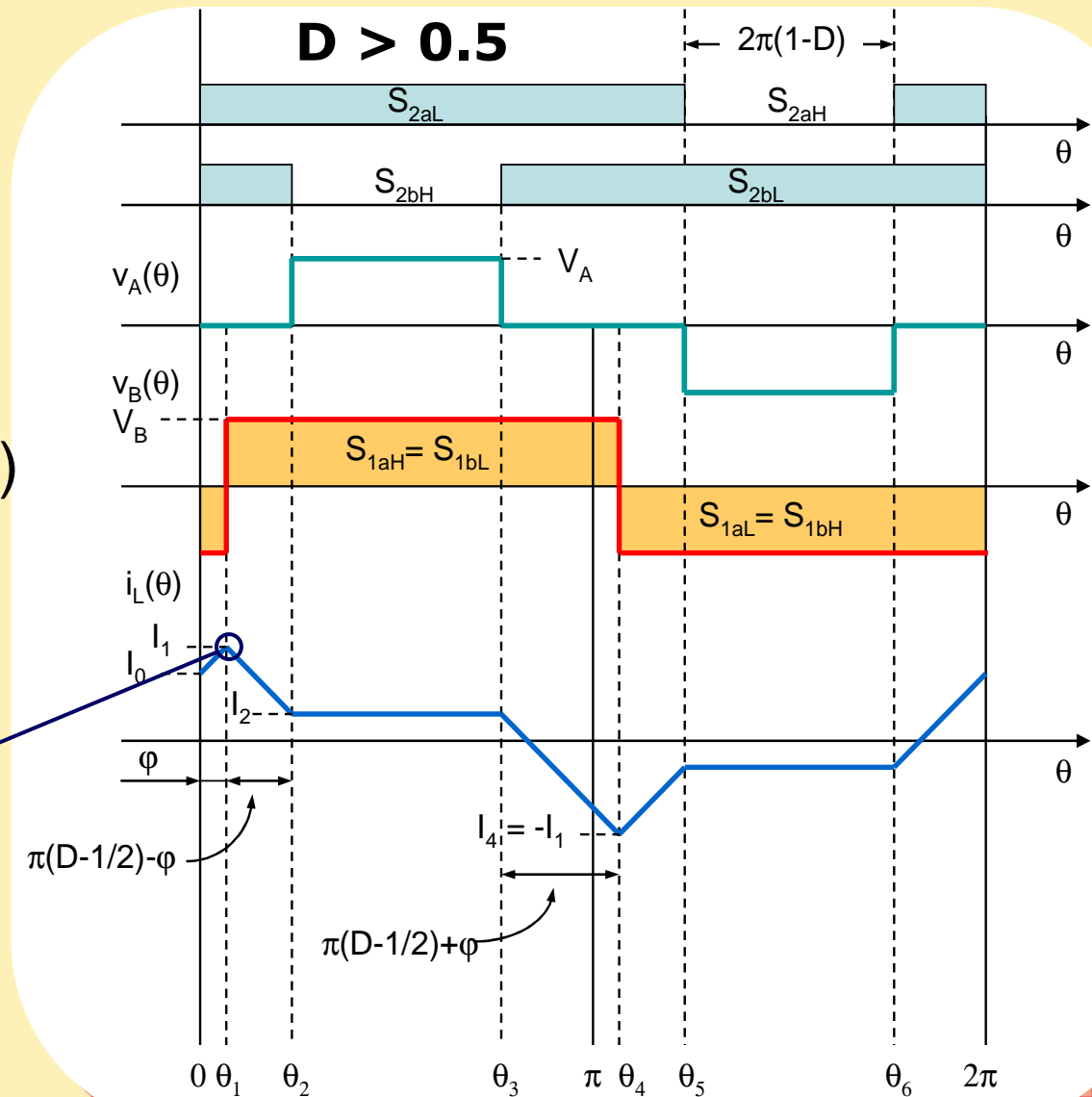
Case A: $0 < \varphi < \pi(D-1/2)$

$$J_1 = J_0 + k\varphi$$

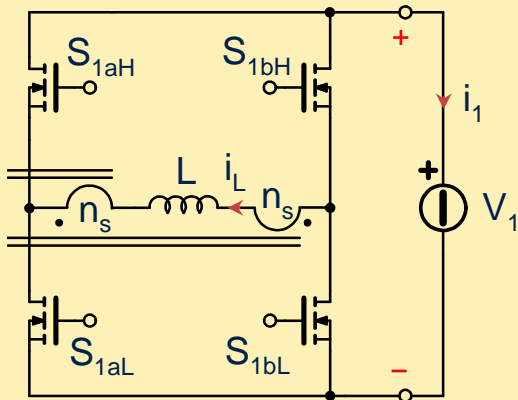
$$= -\pi(1-D) + k\frac{\pi}{2} \geq 0$$



$$k \geq 2(1-D)$$



Soft-switching conditions

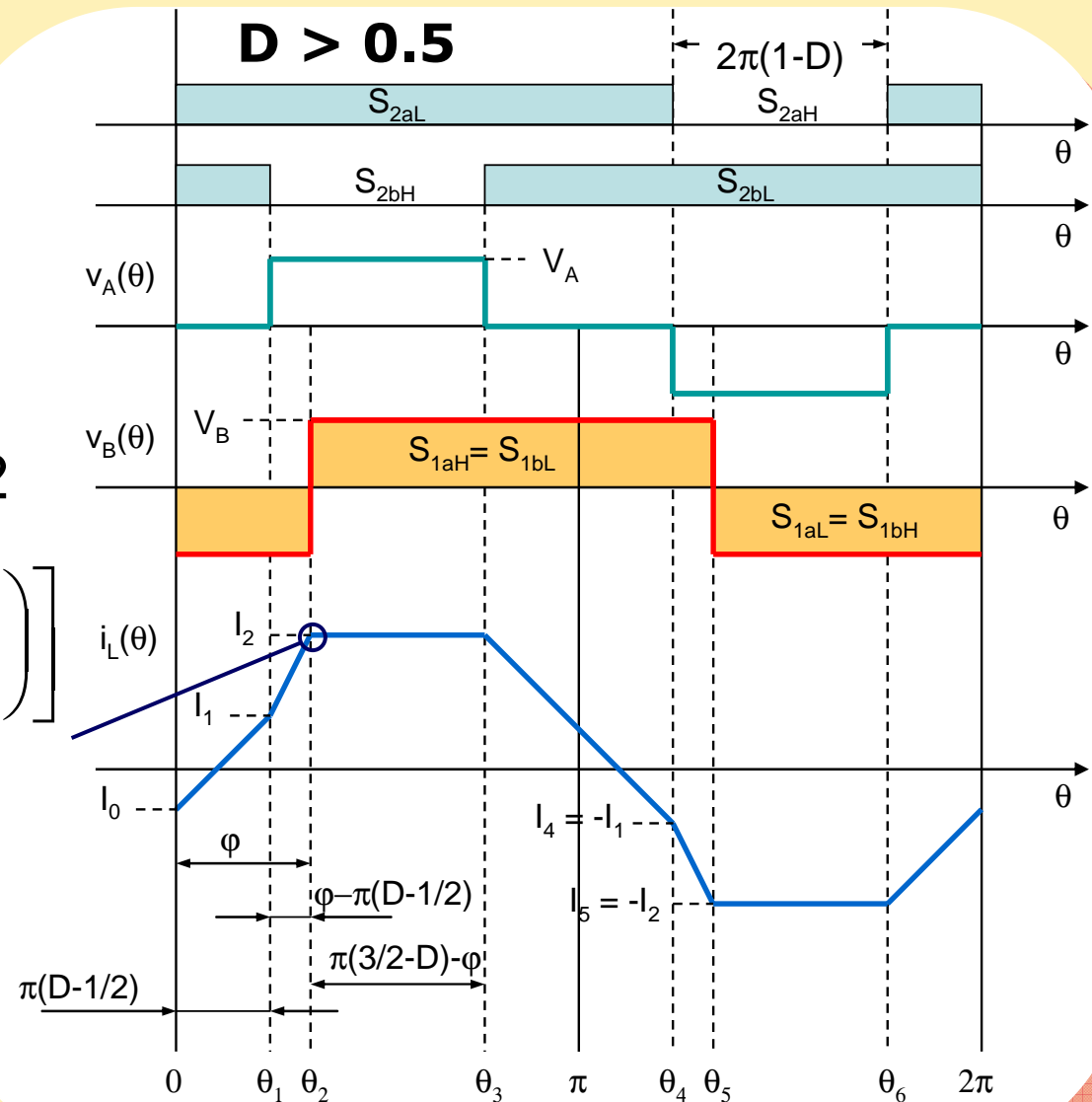


Case B: $\pi(D-1/2) < \varphi < \pi/2$

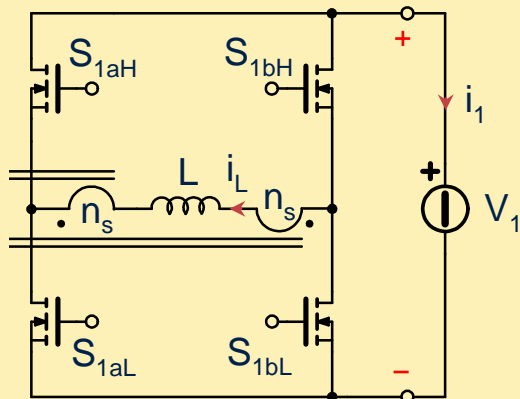
$$J_2 = J_1 + (1+k) \left[\varphi - \pi \left(D - \frac{1}{2} \right) \right]$$

$$= \varphi - \frac{\pi}{2} (1-k) \geq 0$$

$$k \geq 1 - \frac{2}{\pi} \varphi$$



Soft-switching conditions



Case A: $0 < \varphi < \pi(D-1/2)$

$$k \geq 2(1-D)$$

Case B: $\pi(D-1/2) < \varphi < \pi/2$

$$k \geq 1 - \frac{2}{\pi} \varphi$$

Case B is included in case A!

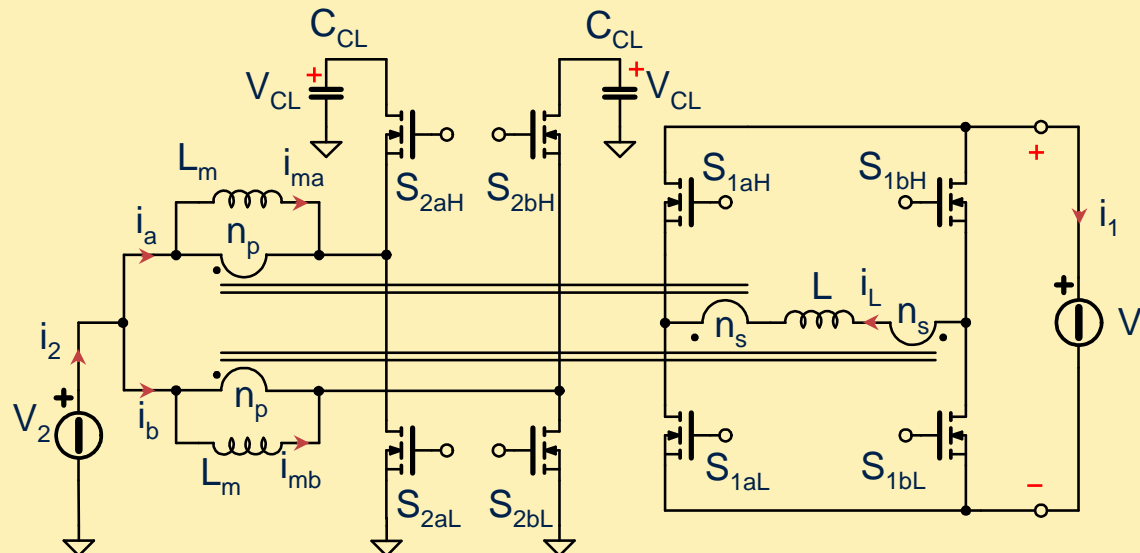
Same condition holds for reversed power flow



$k = 1$ is used to minimize the inductor current crest factor

Soft-switching conditions

Interleaved Boost with Coupled Inductors



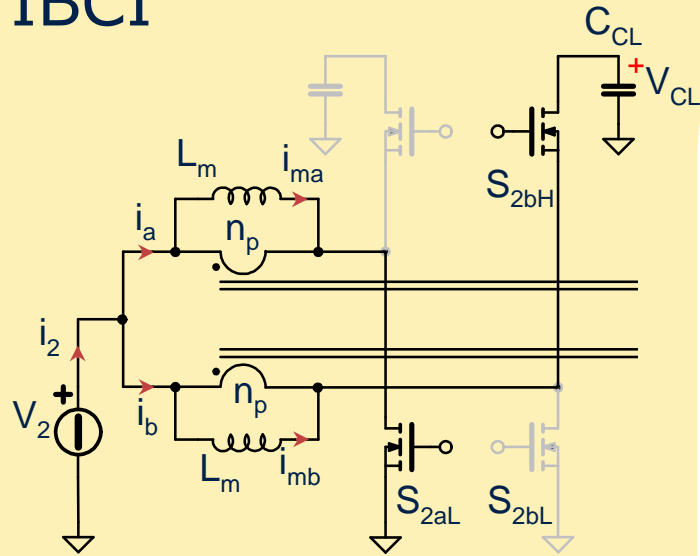
Power flow 

For **port 2** switch commutations we have to analyze the currents i_a and i_b which depend also on duty-cycle:

$$i_a = i_{ma} + \frac{n_s}{n_p} i_L \quad i_b = i_{mb} - \frac{n_s}{n_p} i_L$$

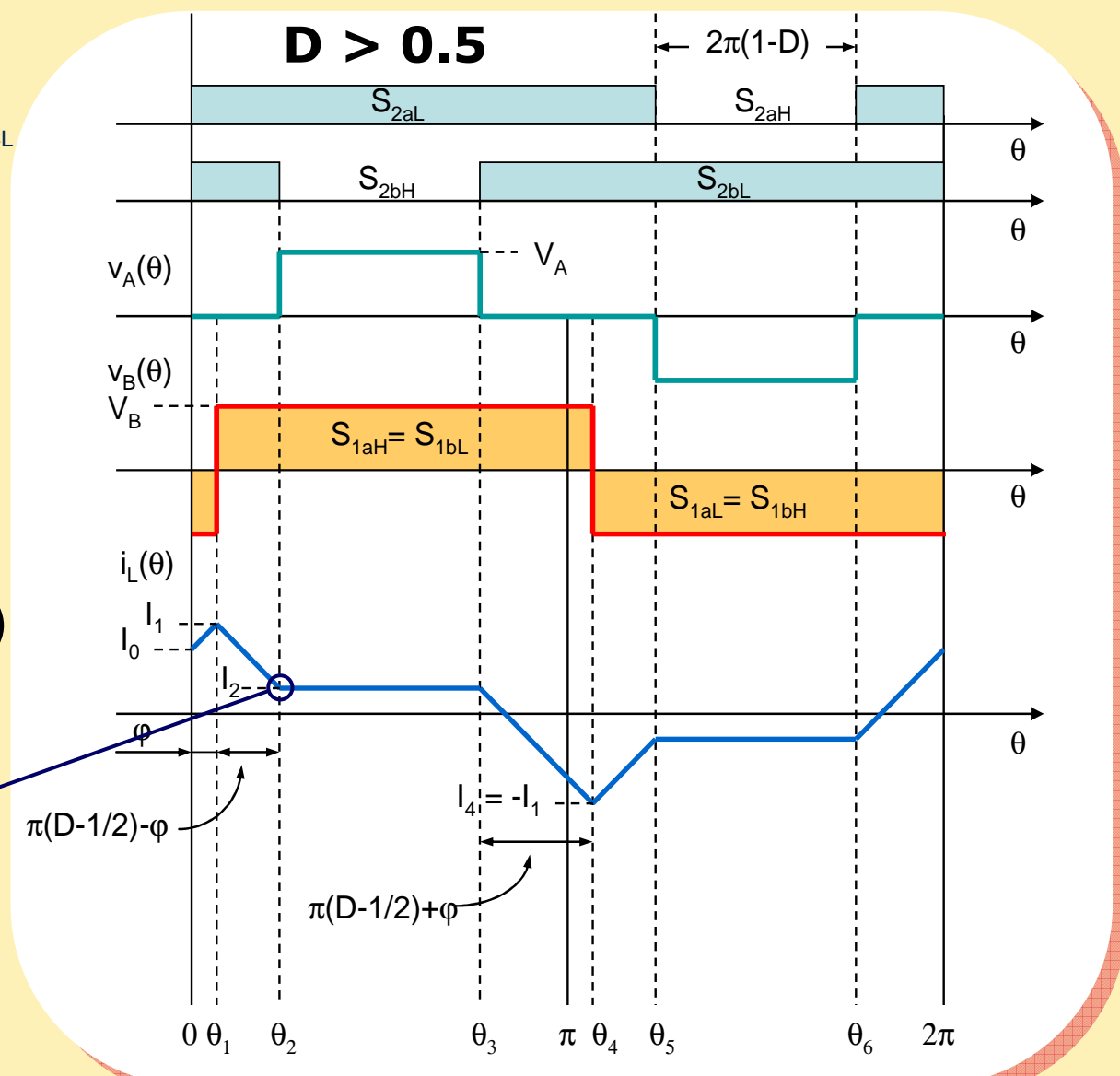
Soft-switching conditions

IBCI



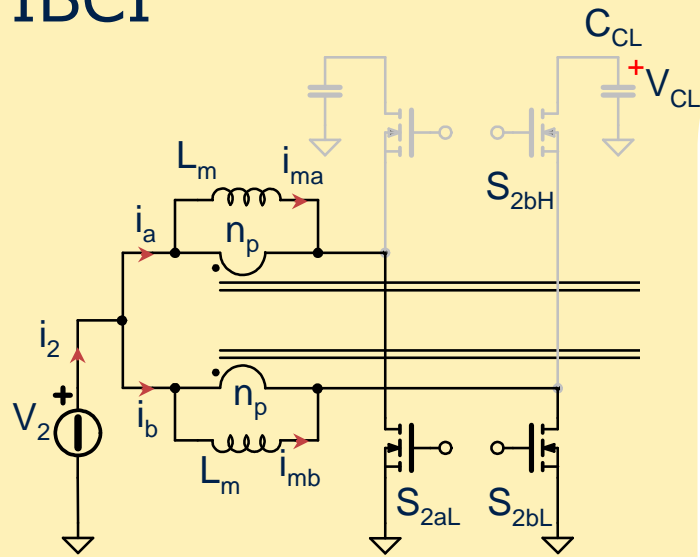
Case A: $0 < \varphi < \pi(D-1/2)$

$$i_b(\theta_2) = I_{mpk} - \frac{n_s}{n_p} I_2 \geq 0$$



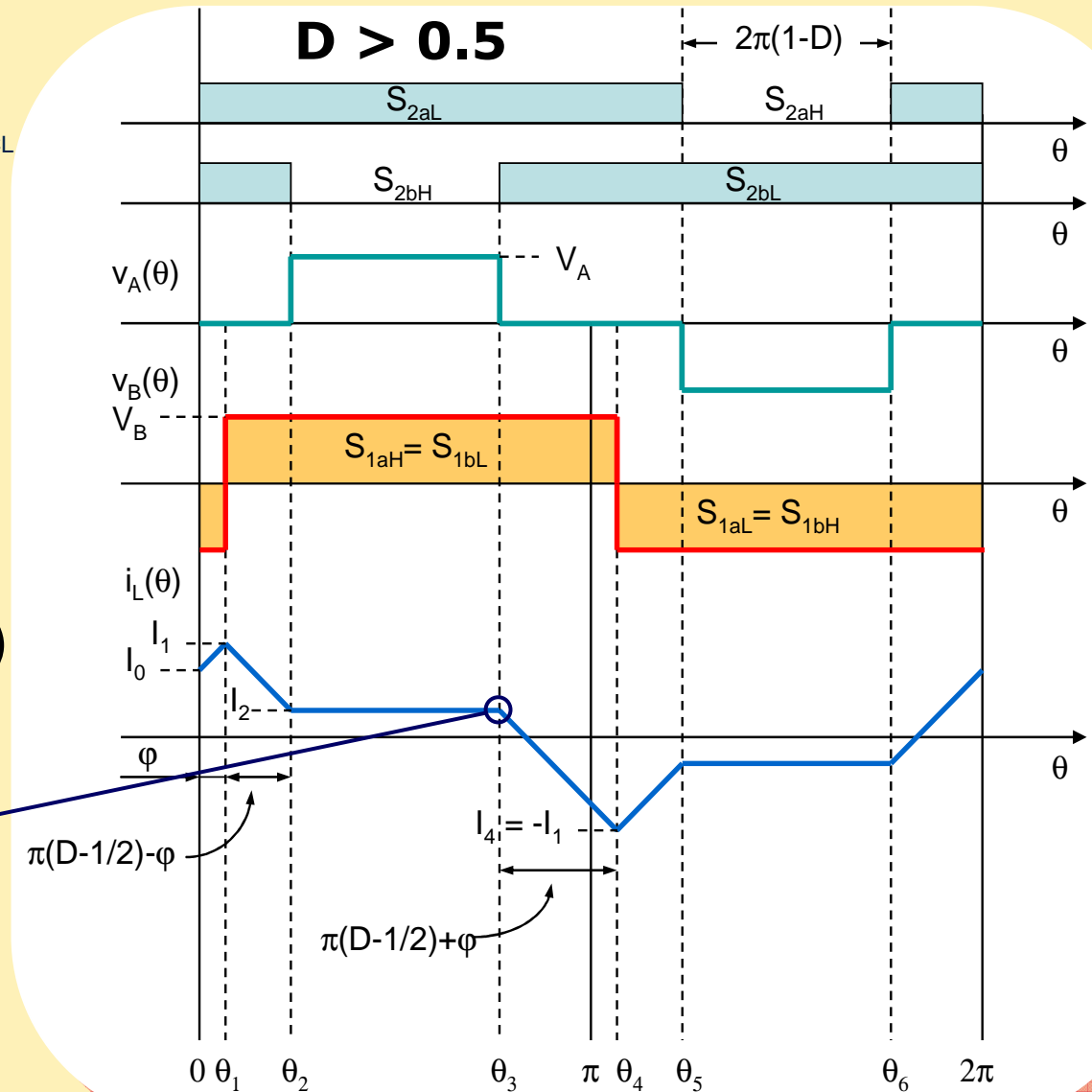
Soft-switching conditions

IBCI



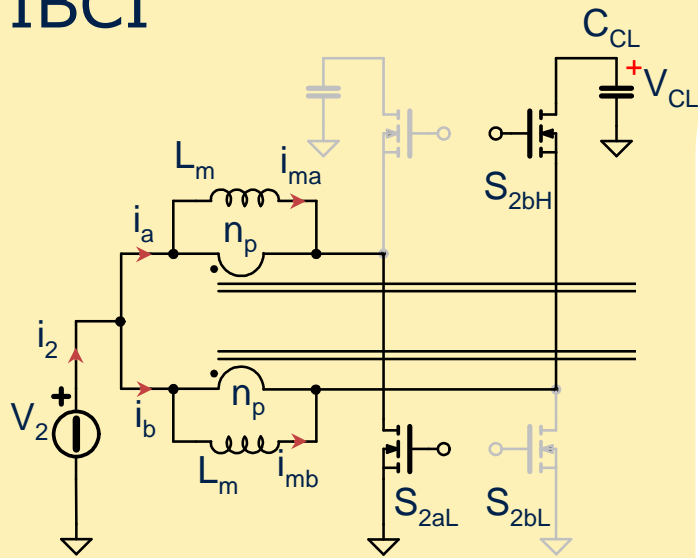
Case A: $0 < \varphi < \pi(D-1/2)$

$$i_b(\theta_3) = I_{mvl} - \frac{n_s}{n_p} I_3 \leq 0$$



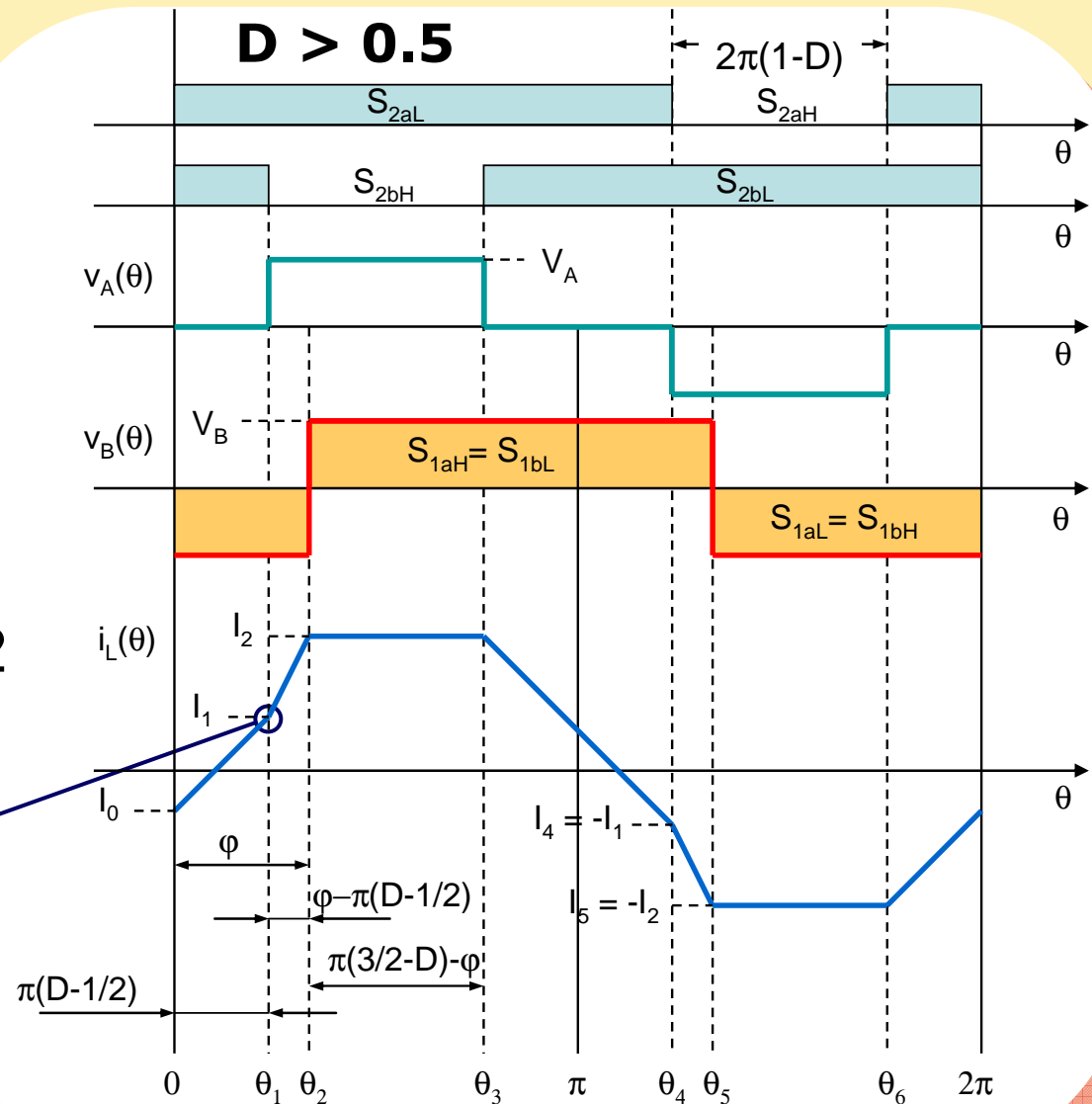
Soft-switching conditions

IBCI



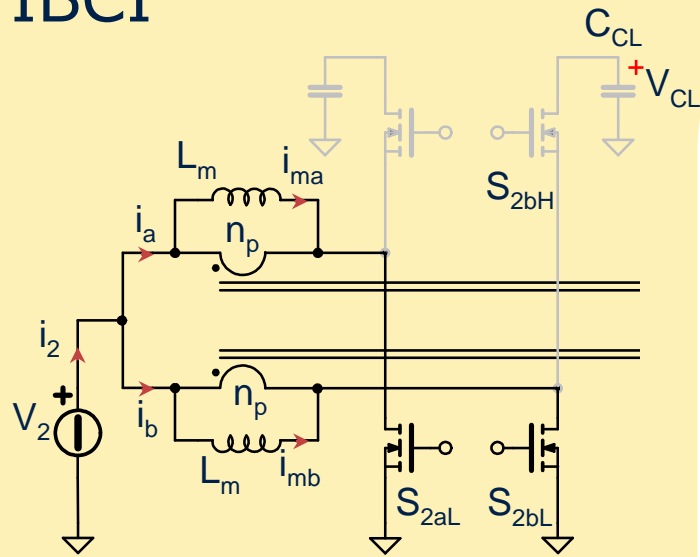
Case B: $\pi(D-1/2) < \varphi < \pi/2$

$$i_b(\theta_1) = I_{\text{mpk}} - \frac{n_s}{n_p} I_1 \geq 0$$



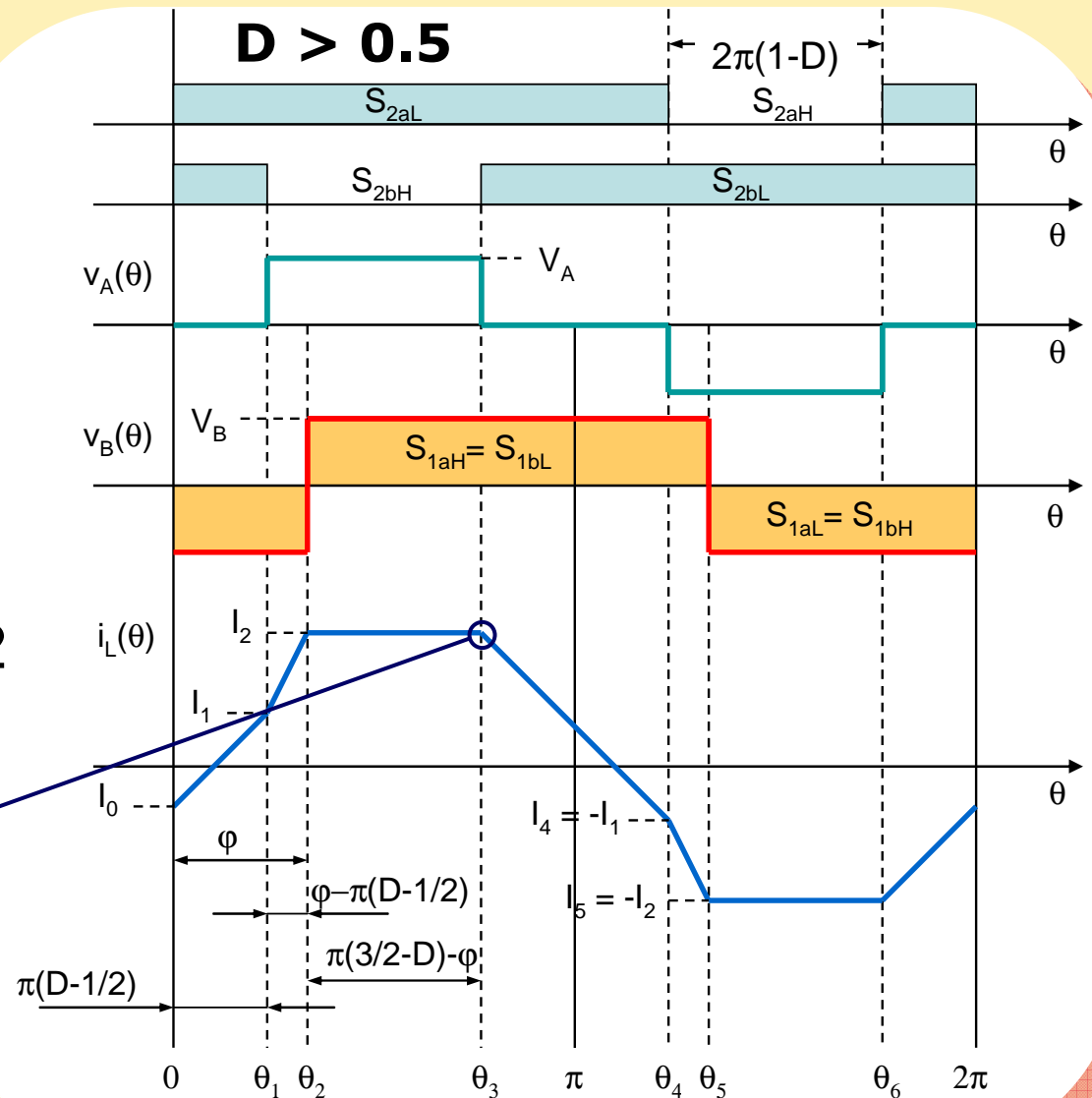
Soft-switching conditions

IBCI



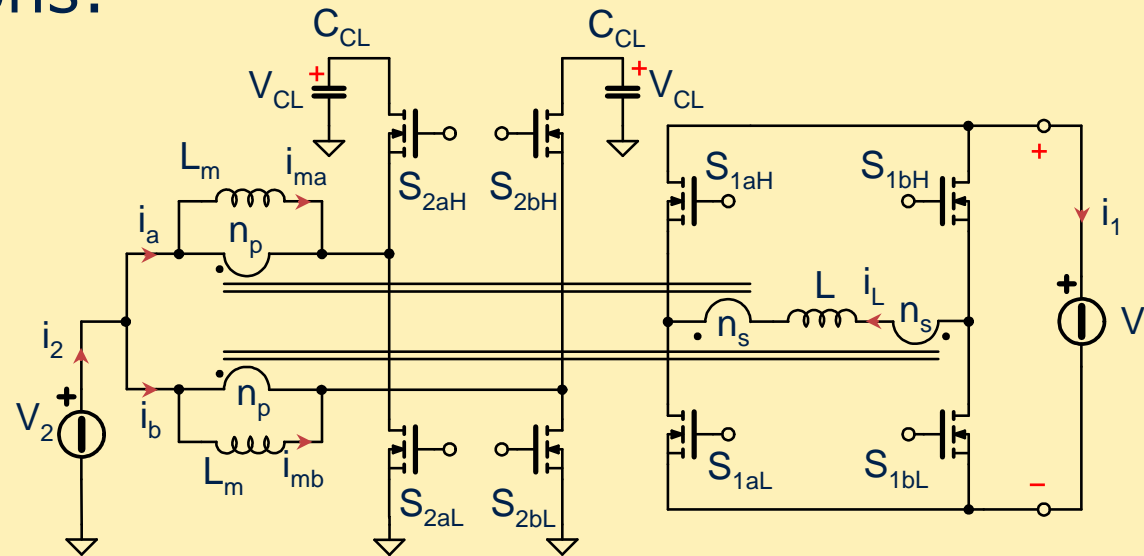
Case B: $\pi(D-1/2) < \varphi < \pi/2$

$$i_b(\theta_3) = I_{mvl} - \frac{n_s}{n_p} I_3 \leq 0$$



Soft-switching conditions

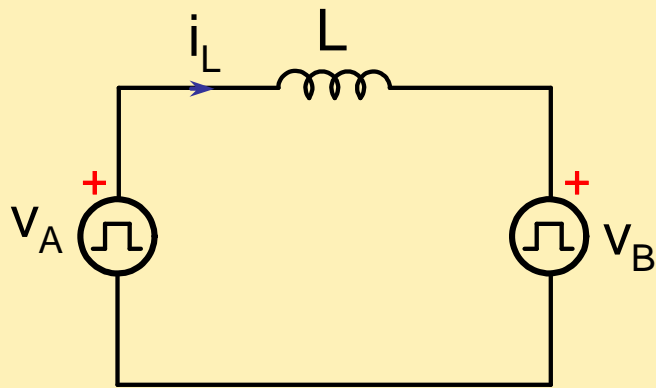
Considerations:



- **The active clamp operation requires the clamp current to have zero average value. This means that the upper switch current must reverse polarity during their conduction interval (help soft-switching)**
- **A non negligible magnetizing inductor ripple helps to satisfy the soft-switching conditions especially at low power levels**

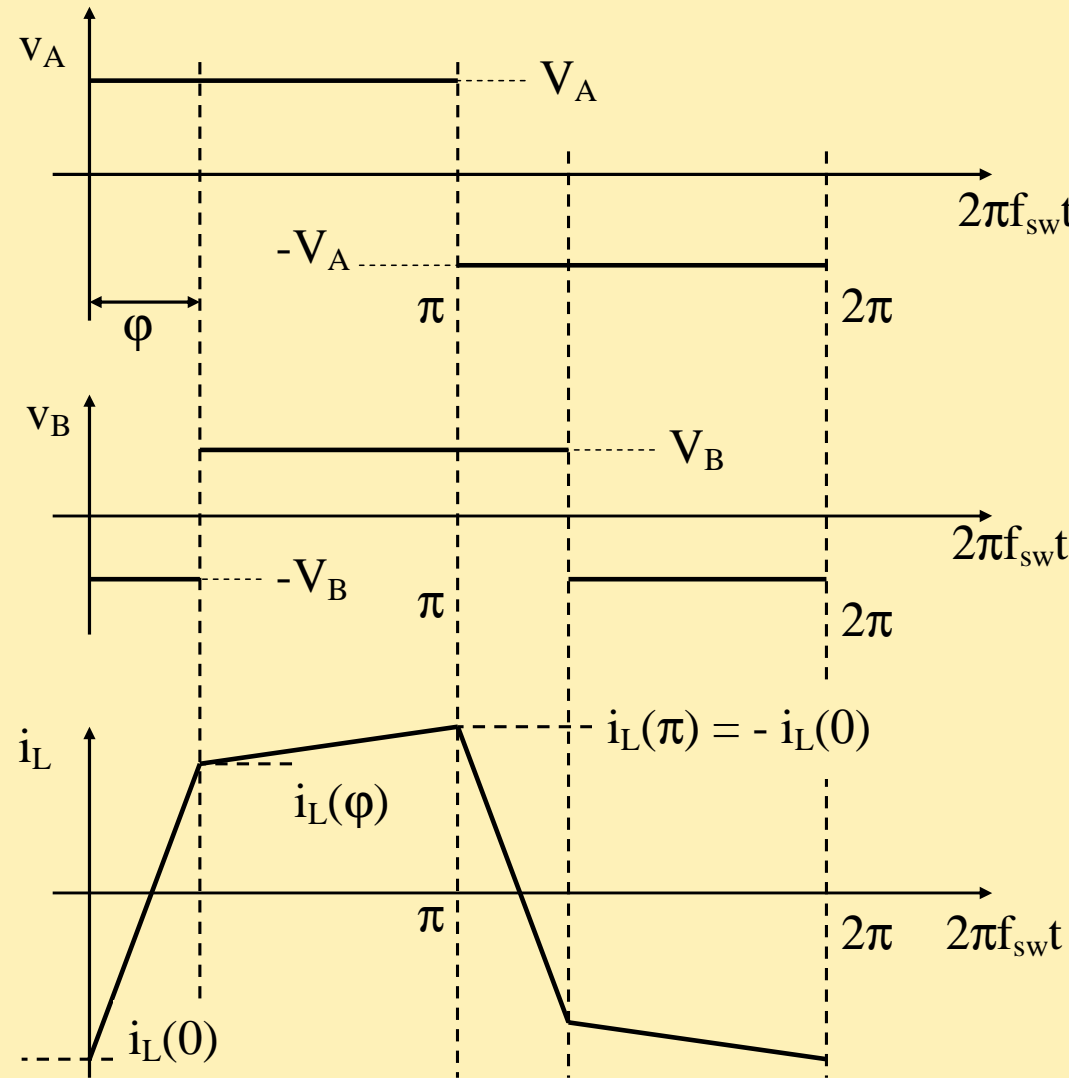
Normalized Transferred Power

Single-phase DAB



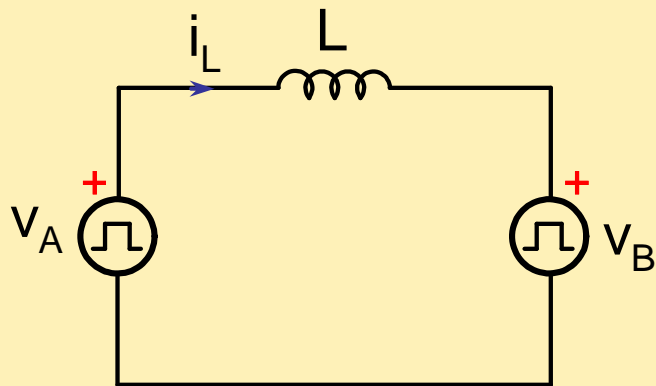
Power flow 

$$\begin{aligned}
 \Pi(\varphi) &= \frac{P(\varphi)}{P_N} \\
 &= \frac{1}{\pi} \int_0^{\pi} i_L(\theta) d\theta
 \end{aligned}$$



Normalized Transferred Power

Single-phase DAB

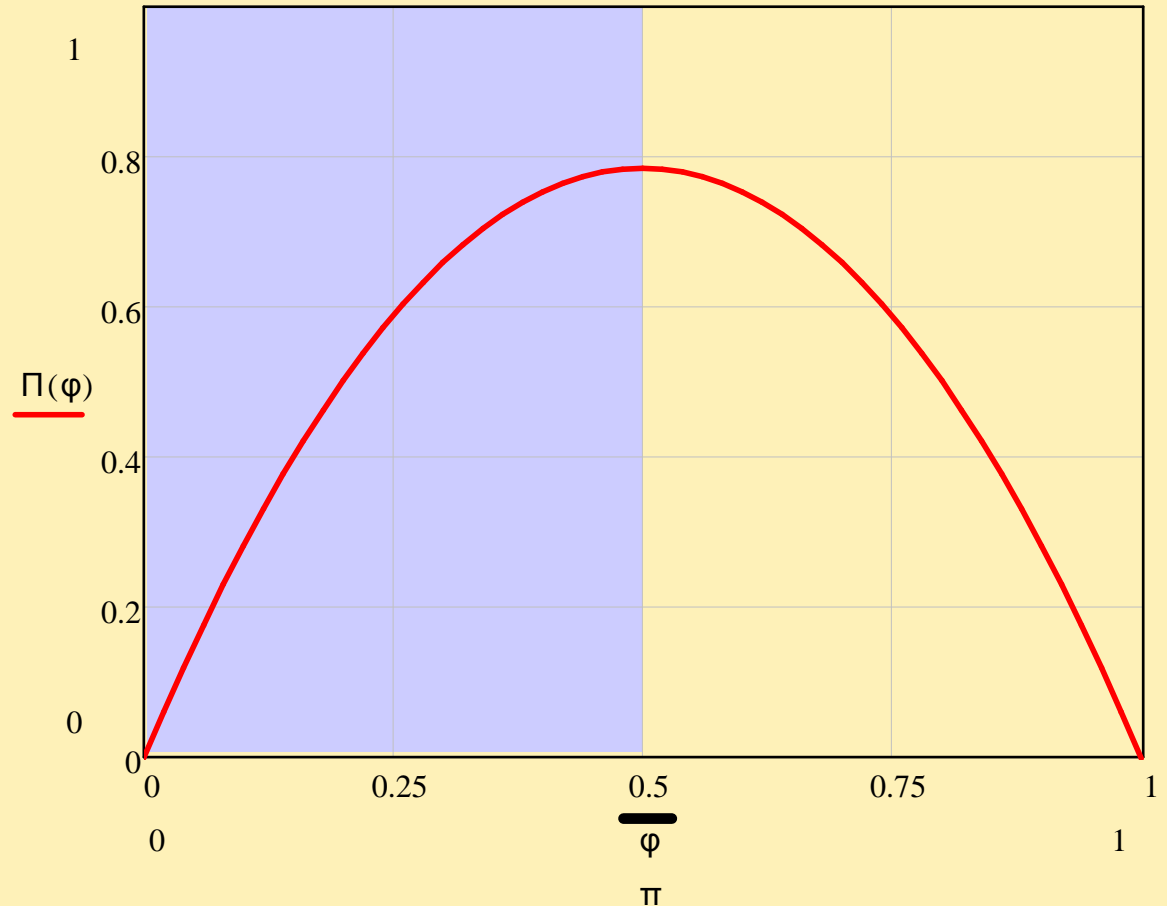


Power flow 

$$\Pi(\varphi) = k\varphi \left(1 - \frac{\varphi}{\pi} \right)$$

$$k = \frac{V_B}{V_A}$$

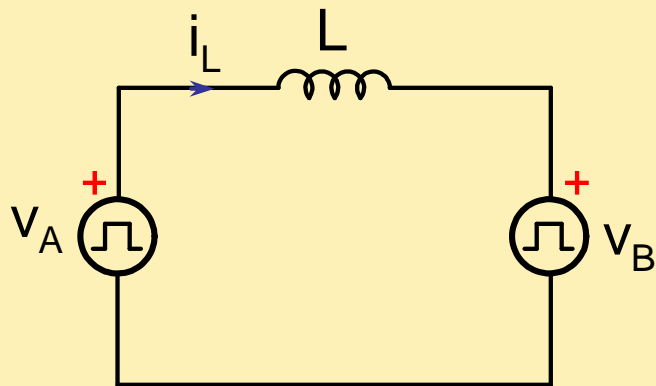
NORMALIZED TRANSFERRED POWER



NORMALIZED PHASE-SHIFT ANGLE

Normalized Transferred Power

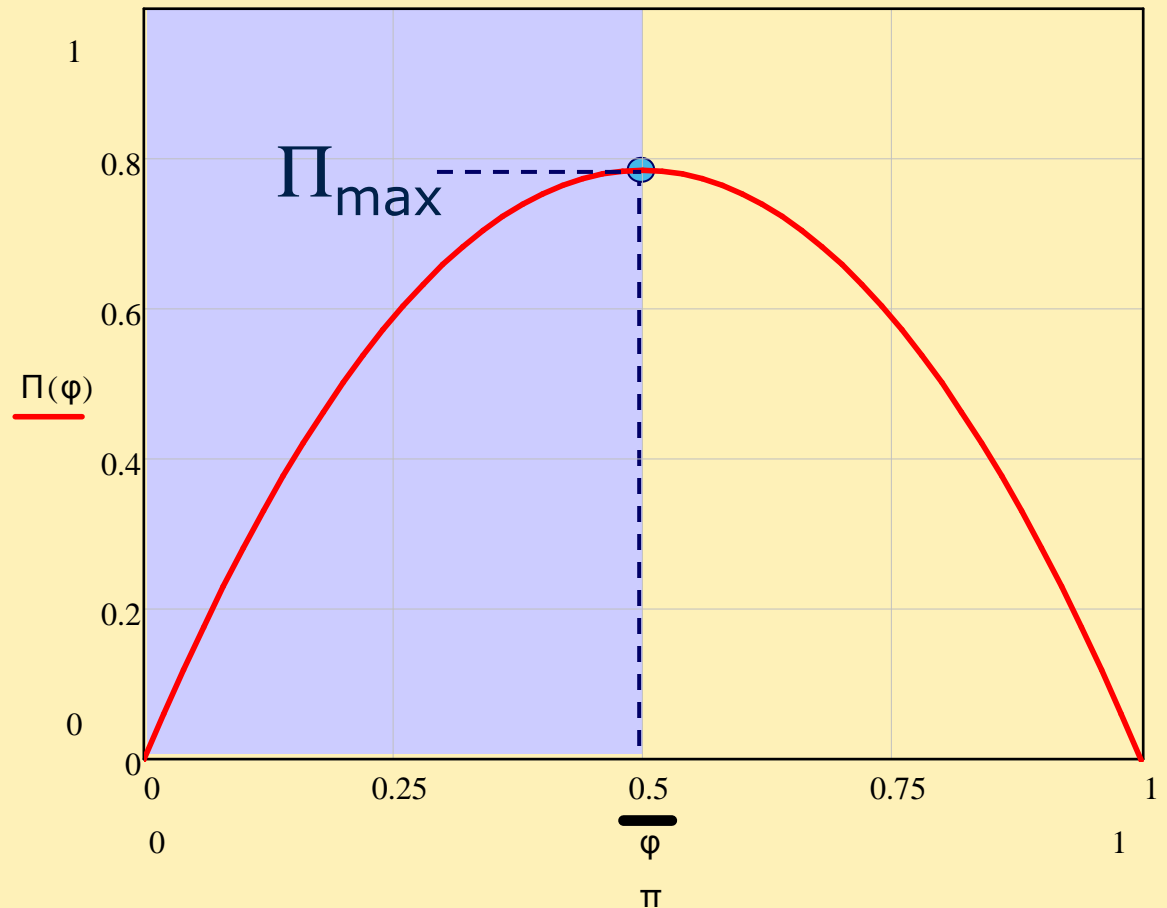
Single-phase DAB



Power flow 

$$\Pi_{\max} = \Pi\left(\frac{\pi}{2}\right) = k \frac{\pi}{4}$$

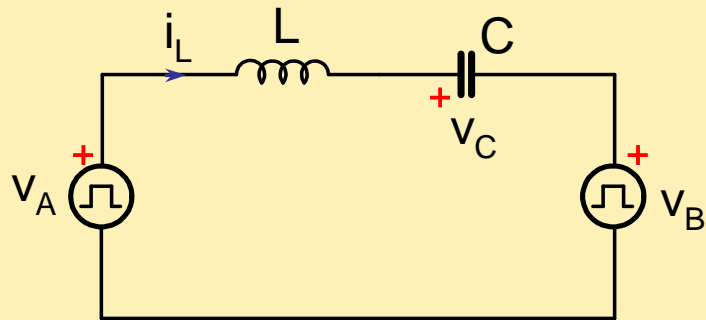
NORMALIZED TRANSFERRED POWER



NORMALIZED PHASE-SHIFT ANGLE

Normalized Transferred Power

Single-phase SR-DAB

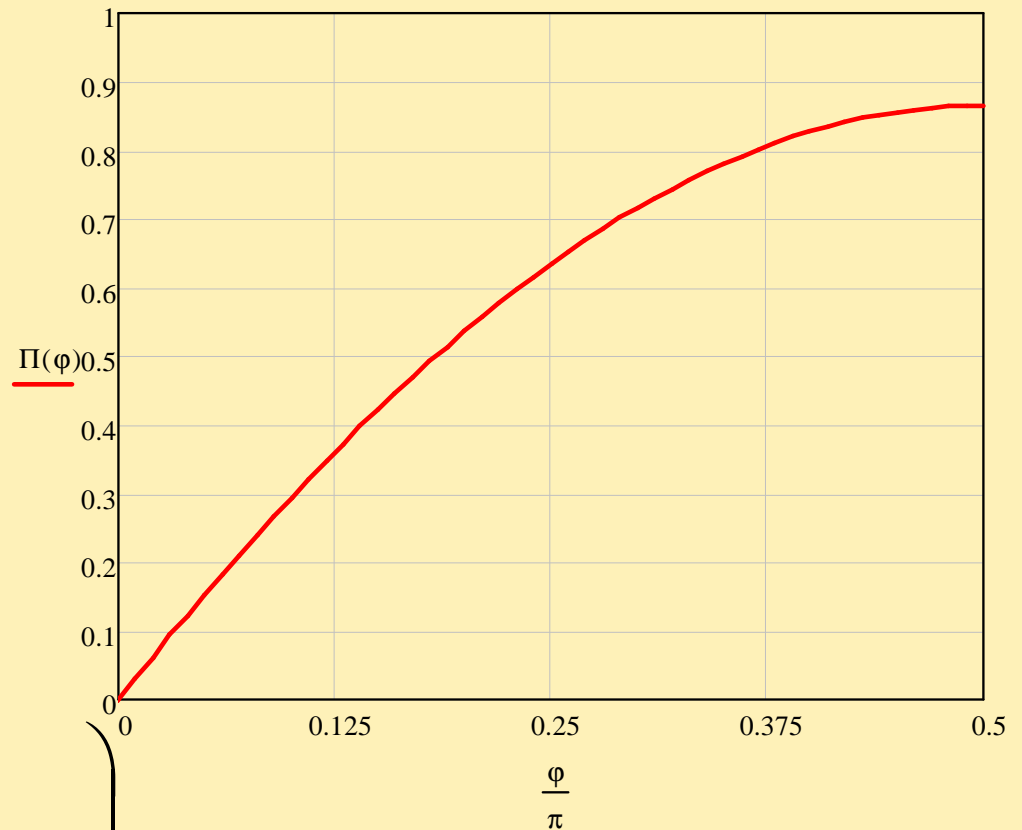


Power flow 

$$\Pi(\varphi) = \frac{P(\varphi)}{P_N}$$

$$= \frac{2kf_n}{\pi} \left(\frac{\cos\left(\frac{\pi - 2\varphi}{2f_n}\right)}{\cos\left(\frac{\pi}{2f_n}\right)} - 1 \right)$$

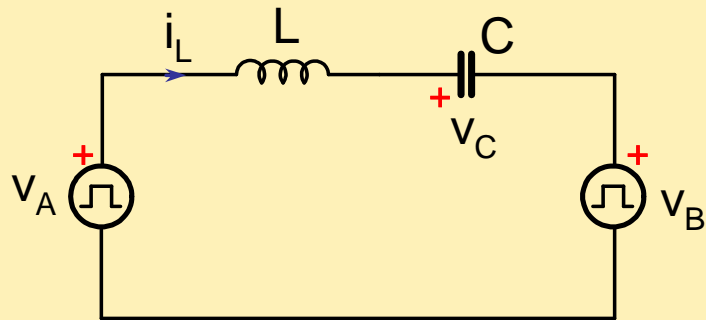
NORMALIZED TRANSFERRED POWER



NORMALIZED PHASE-SHIFT ANGLE

Normalized Transferred Power

Single-phase SR-DAB

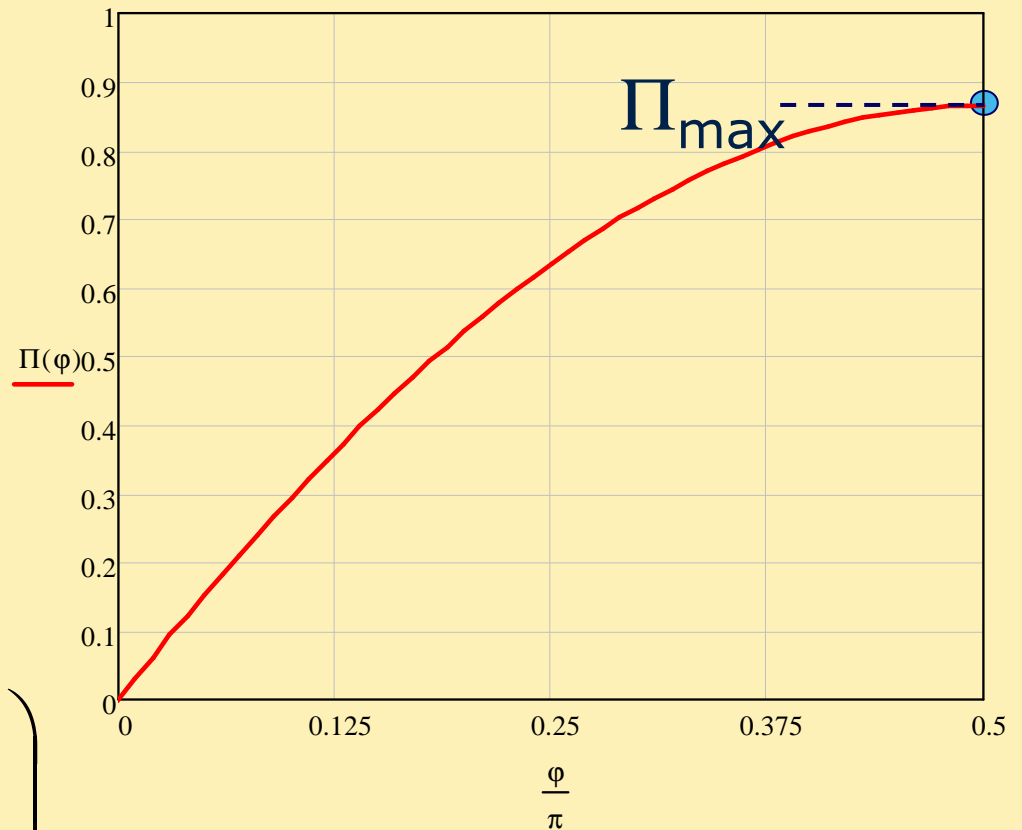


Power flow 

$$\Pi_{\max} = \Pi\left(\frac{\pi}{2}\right)$$

$$= \frac{2kf_n}{\pi} \left(\frac{1}{\cos\left(\frac{\pi}{2f_n}\right)} - 1 \right)$$

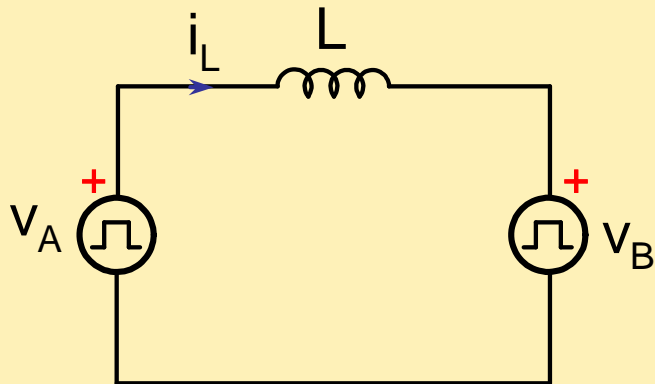
NORMALIZED TRANSFERRED POWER



NORMALIZED PHASE-SHIFT ANGLE

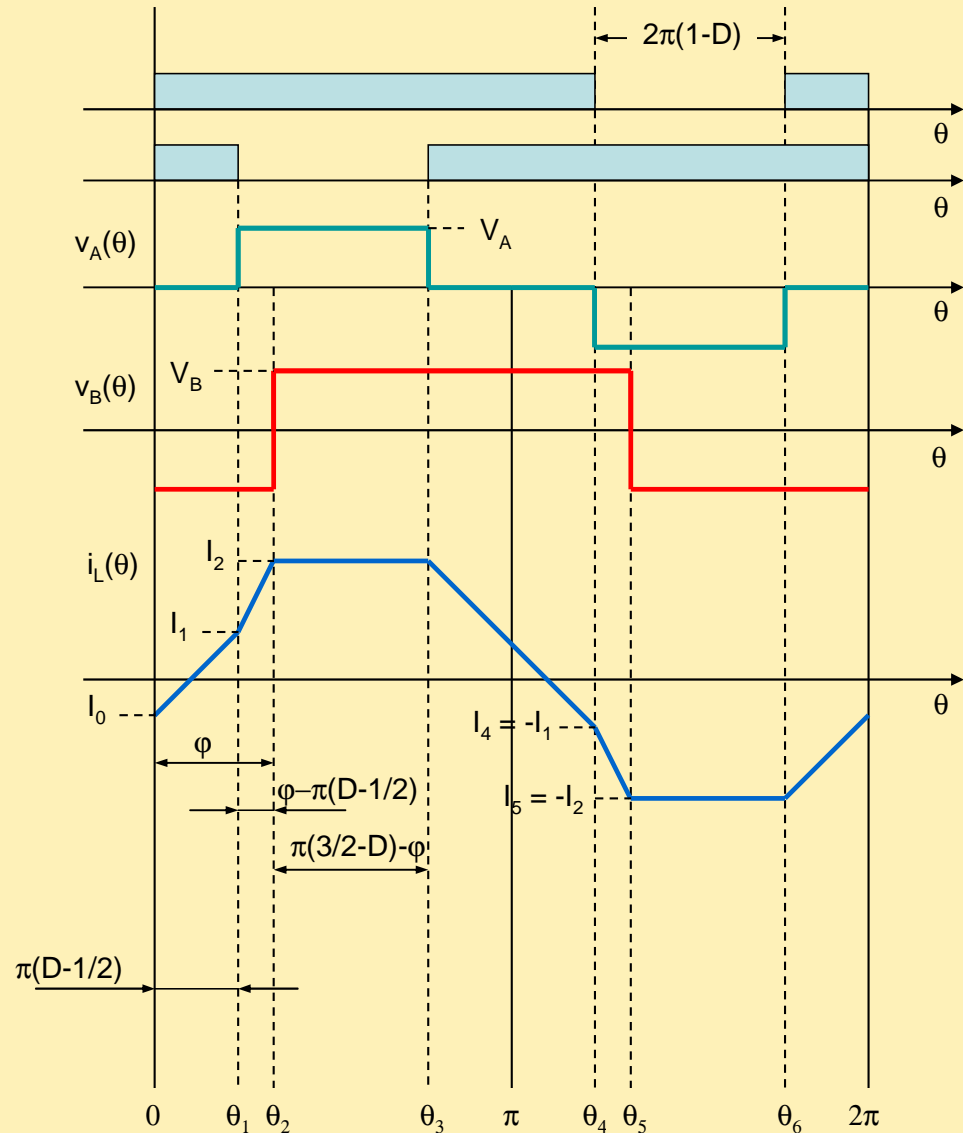
Normalized Transferred Power

IBCI



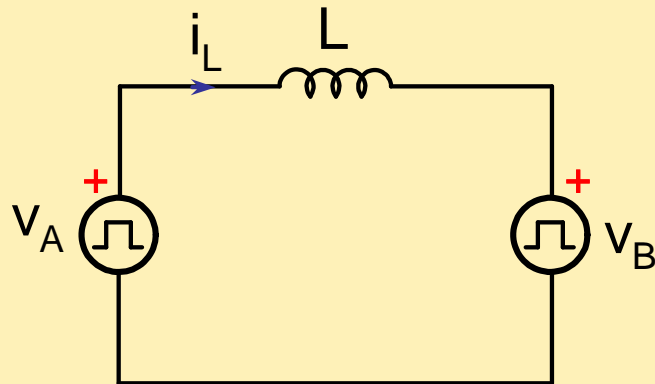
Power flow 

$$\Pi(\varphi) = \frac{1}{\pi} \int_{\theta_1}^{\theta_3} i_L(\theta) d\theta$$

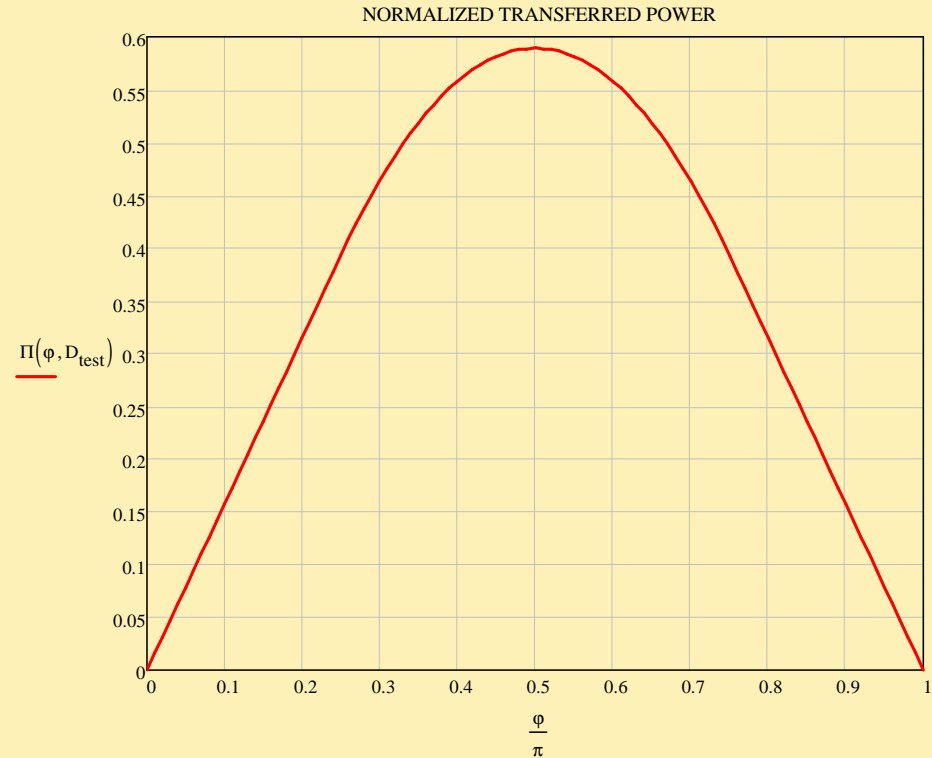


Normalized Transferred Power

IBCI



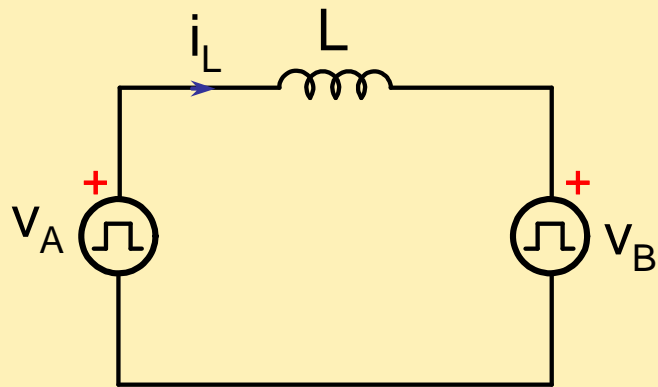
Power flow 



$$\Pi(\varphi) = \begin{cases} 2k\varphi(1-D) & \text{for } 0 \leq \varphi \leq \pi \left(D - \frac{1}{2} \right) \\ k\varphi \left(1 - \frac{\varphi}{\pi} \right) - k\pi \left(D - \frac{1}{2} \right)^2 & \text{for } \pi \left(D - \frac{1}{2} \right) \leq \varphi \leq \frac{\pi}{2} \end{cases}$$

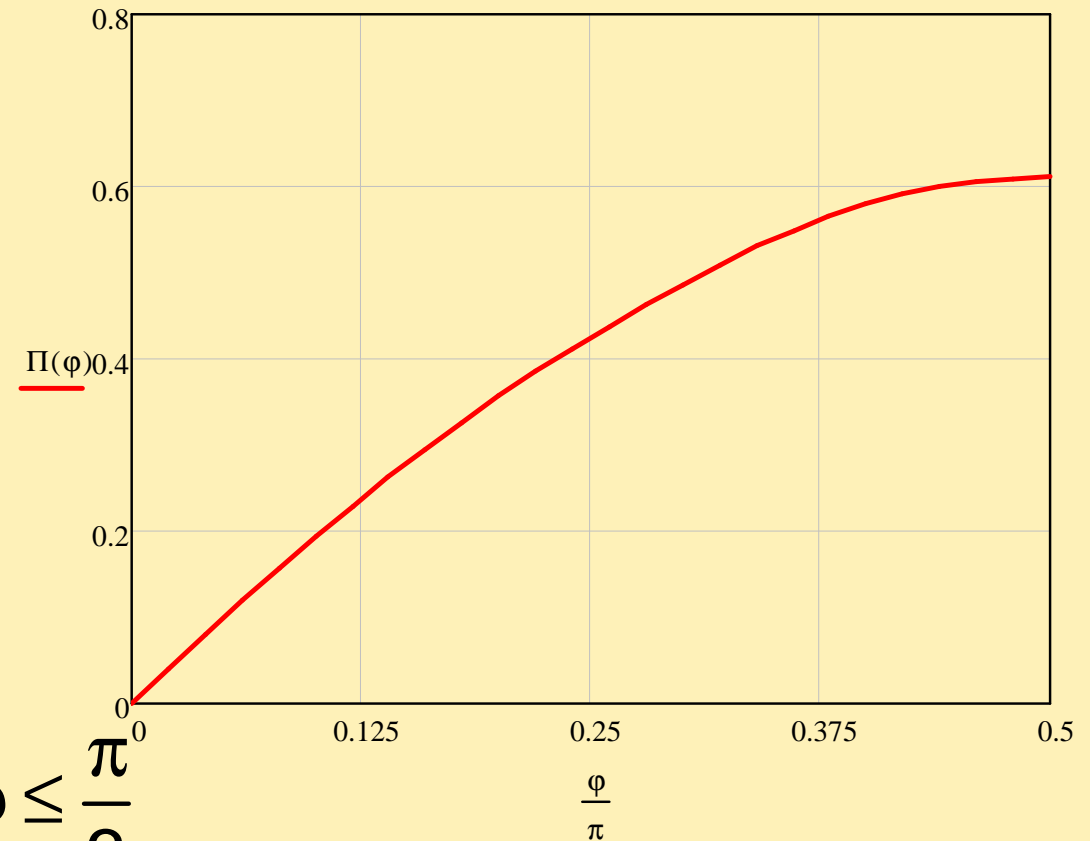
Normalized Transferred Power

Three-phase DAB



Power flow 

NORMALIZED TRANSFERRED POWER

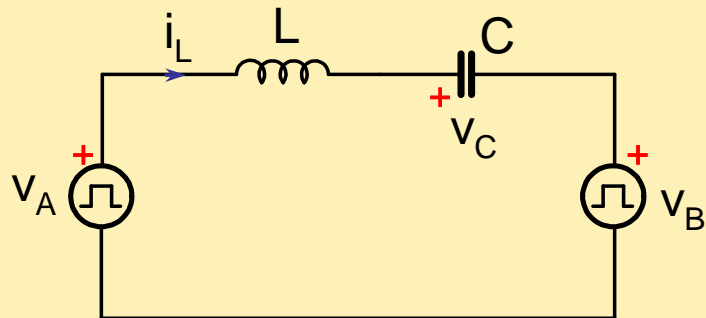


$$\Pi(\varphi) = \begin{cases} \varphi \left(\frac{2}{3} - \frac{\varphi}{2\pi} \right) & \varphi \leq \frac{\pi}{3} \\ \varphi - \frac{\varphi^2}{\pi} - \frac{\pi}{18} & \frac{\pi}{3} \leq \varphi \leq \frac{\pi}{2} \end{cases}$$

NORMALIZED PHASE-SHIFT ANGLE

Normalized Transferred Power

Three-phase SR-DAB

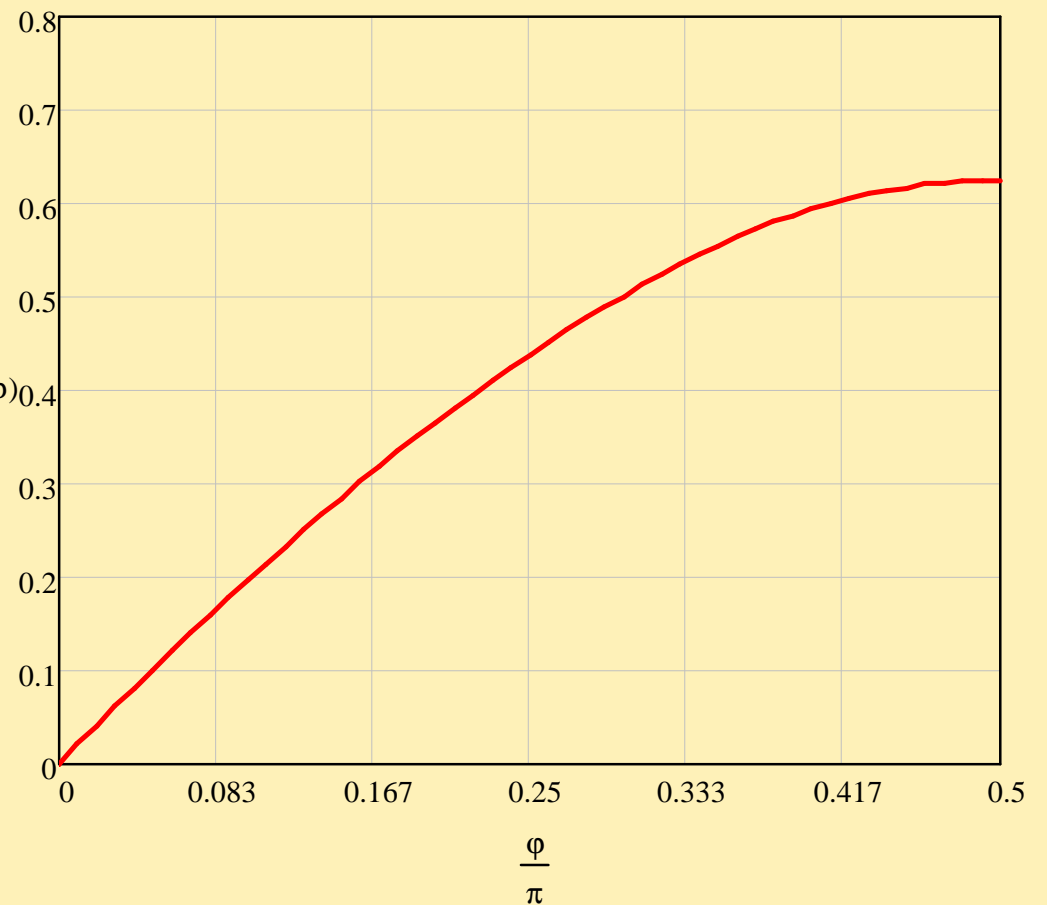


Power flow 

$\Pi_1(\varphi)$

No closed form
expression for $\Pi(\varphi)$

NORMALIZED TRANSFERRED POWER



Conclusions

- **Different isolated bidirectional topologies, belonging to the family of dual active bridge structures, have been considered**
- **A unified analysis has been carried out to calculate the steady-state current waveform responsible for the power transfer**
- **Soft-switching conditions have been investigated for each converter topology**
- **The transferred power and its relation with the phase-shift angle has been calculated**