

Smart micro-grids

Properties, trends and local control of energy sources

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Outline

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2. The potential revolution of the smart micro-grid
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10. Distributed surround control of smart micro-grids
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12. Simulation results
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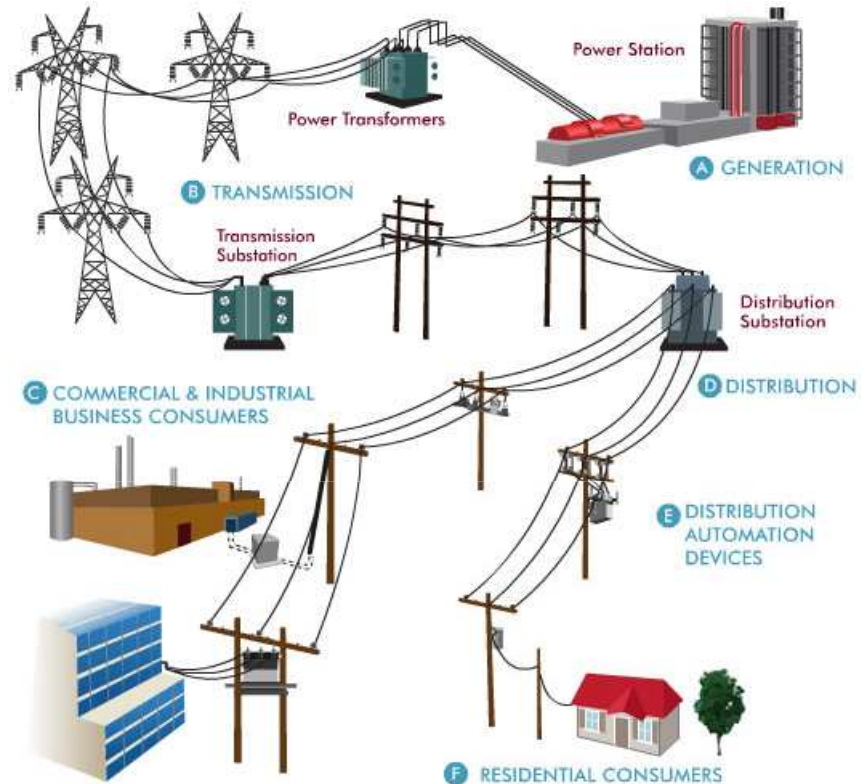
Smart micro-grids

Properties, trends and local control of energy sources

1. From the traditional grid to the smart grid

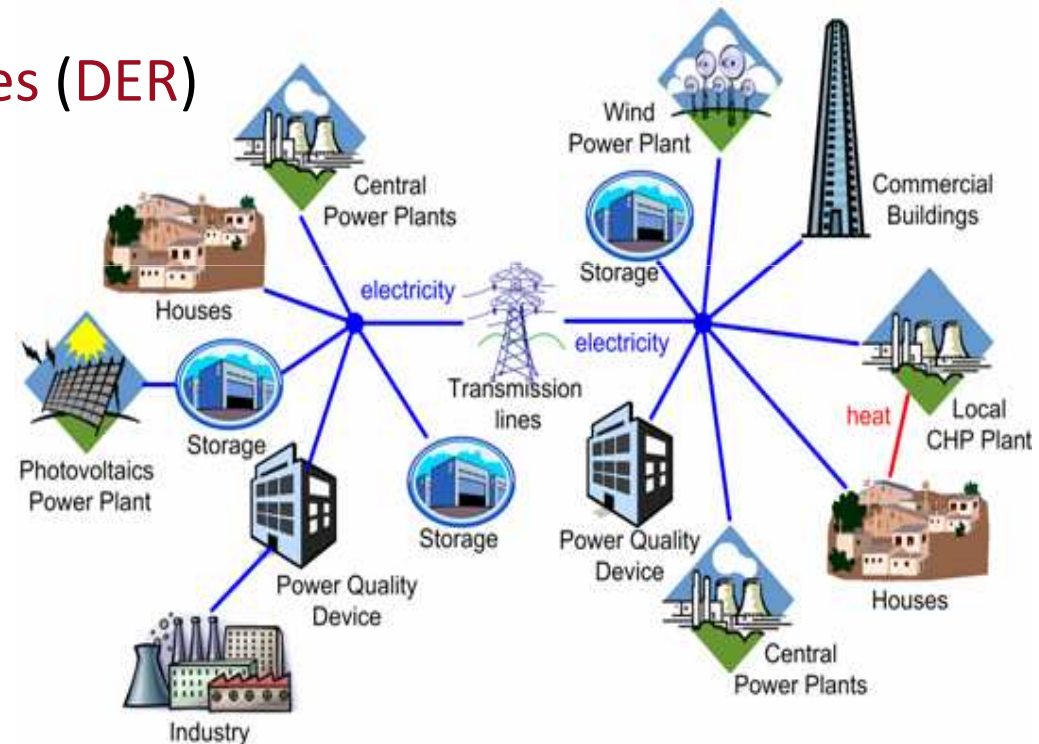
The traditional grid

- **Few large power plants** feeding large number of end-users
- Power plants located in strategic sites (cost-effective generation, safety)
- Centralized control (dispatcher)
- **Unidirectional power flow**
- **Independent operation of each apparatus** (the power grid performs nearly as an ideal voltage source with small internal impedance)
- **No customers' participation to power balance**

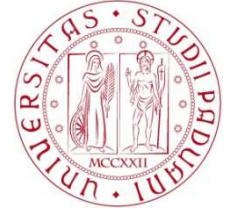


The smart grid

- Local-scale power grids which can operate in utility-connected or islanded mode
- Distributed Energy Resources (DER)
- Bidirectional power flow
- Weak grid, causing interaction of power sources and loads
- Multilateral contribution to power balance
- Intelligent and controllable electronic interfaces between energy sources and grid



Benefits of the smart grid



- **Distributed renewable resources**
 - less carbon footprint
 - energy cost reduction
- **Energy efficiency**
 - power sources close to loads
 - improved demand response
- **Improved utilization of conventional power sources**
 - less active, reactive, unbalance and distortion power flowing through the distribution lines
- **Voltage support**
 - distributed injection of active and reactive power
- **Increased hosting capacity**
 - without investments in the grid infrastructure

Challenges of the smart grid



- **Bidirectional power flow**
 - need for new control and protection strategies
 - conventional voltage stabilization techniques not applicable
- **Weak grid** (non-negligible internal impedance, especially in islanded operation)
 - voltage distortion due to nonlinear loads
 - voltage asymmetry due to unbalanced loads and single-phase DER units (PV, batteries, ...)
- **Irregular power injection** by renewable energy sources
 - need for power flow regularization and peak power shaving
 - need for energy storage devices

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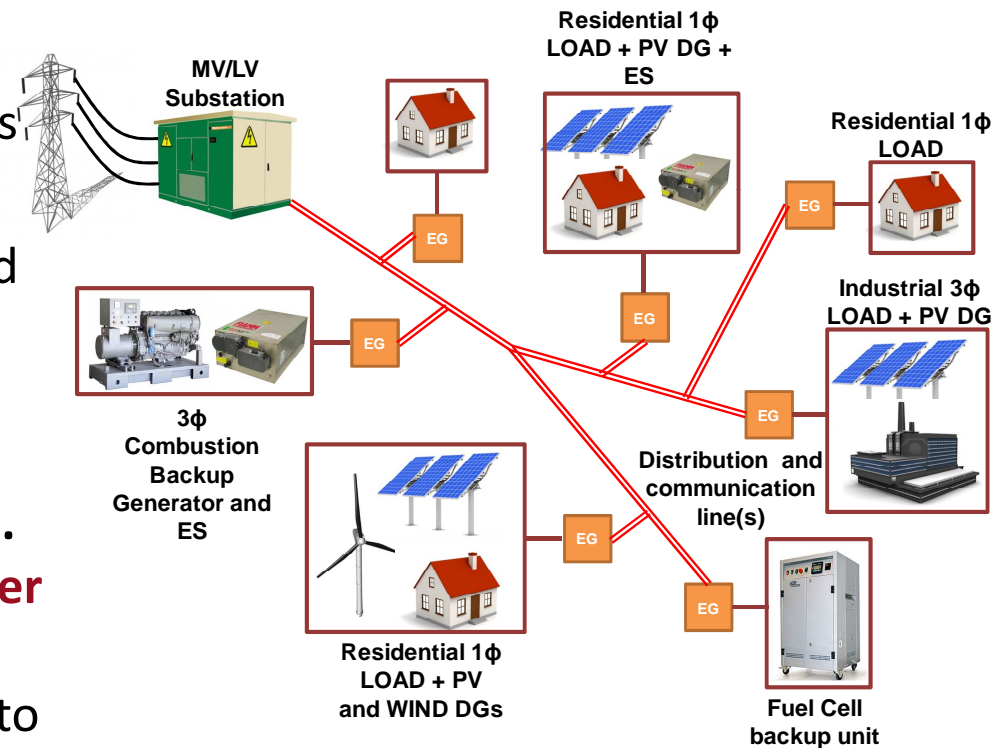
2. The potential revolution of the smart micro-grid

LV residential micro-grids

General Definition: A Smart Grid is an electrical power delivery system where power quality, efficiency and energy cost are **optimized** by pervasive use of **information and communication technology** with the aim to control **distributed energy resources**

Low-voltage Microgrid = distribution system connecting a MV/LV substation with loads & distributed energy resources (DERs).

- DERs interface with the distribution grid by electronic power processors (EPP, inverters) equipped with local measurement, control and communication (**EG = Energy Gateway**).
- EGs may implement **bidirectional power control and communicate** with other generators and loads of the micro-grid to implement cooperative operation



Expected benefits of micro-grids

Environment & savings

- Green power
- Full utilization of distributed energy resources
- Reduced distribution loss
- Increased hosting capacity
- Increased power quality even in remote locations
- Layered grid architecture

Social & economics

- Strengthen consumers role
- Develop communities of prosumers
- New functions and players in the energy market
- New arena for entrepreneurs, manufacturers and service providers
- New jobs for green collars



Technological challenges



- Exploit every available energy source
- Minimize distribution losses and non-renewable energy consumption
- Increase power quality and hosting capacity
- Implement cheap ICT architectures for distributed control and communication
- Integrate micro-grid control and domotics
- Revise accounting principles and methodologies
- Restructure network protection
- Assure data security and privacy
- Pursue flexibility and scalability (from buildings to townships)



The future of micro-grids



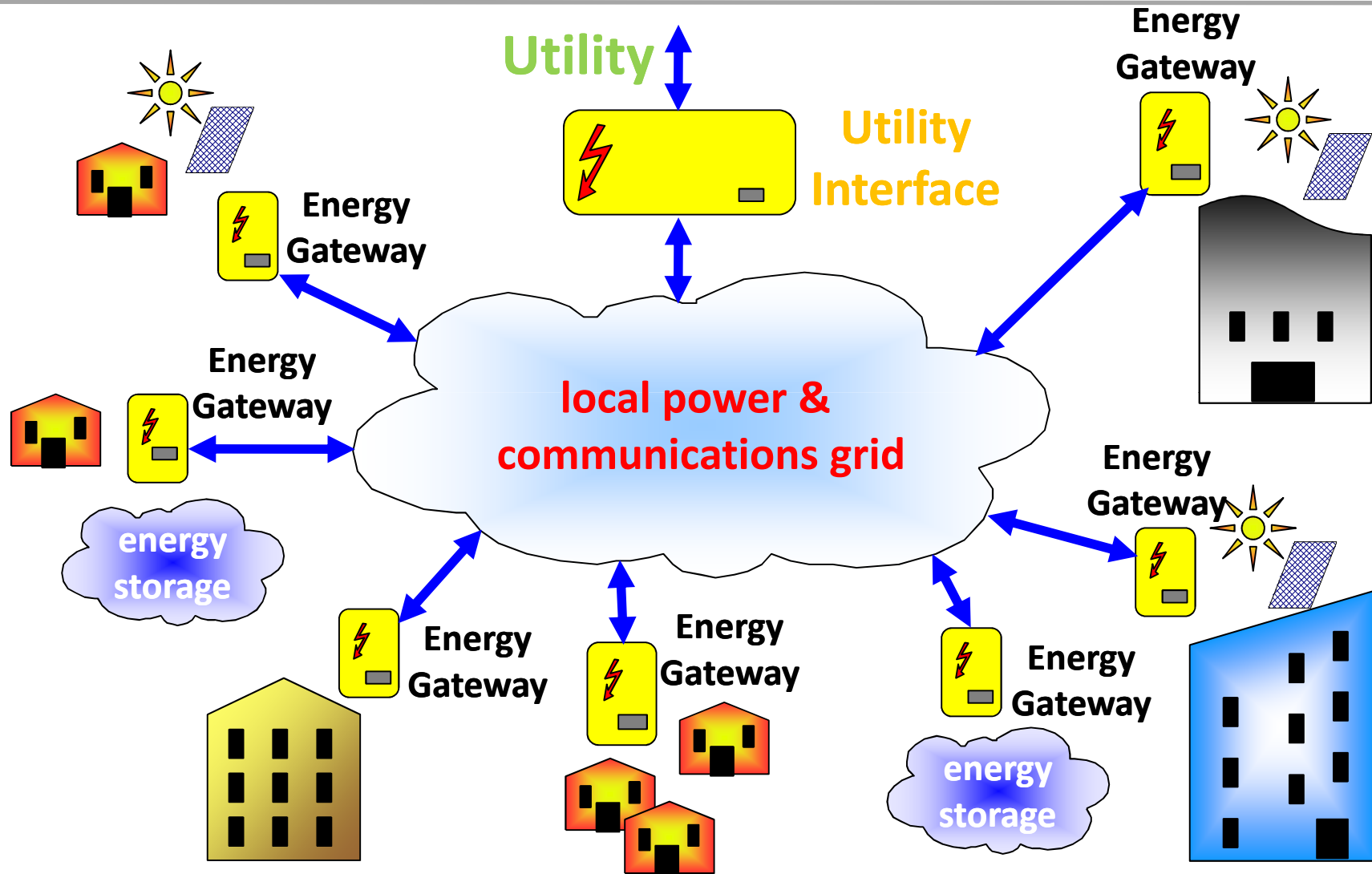
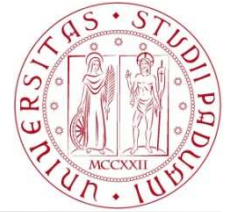
- [UE Roadmap for micro-grids](#) (CIGRE 2010)
- [Smart grid investment forecast](#) (JRC report 2011)

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3. Smart micro-grid architecture

General sketch of a micro-grid

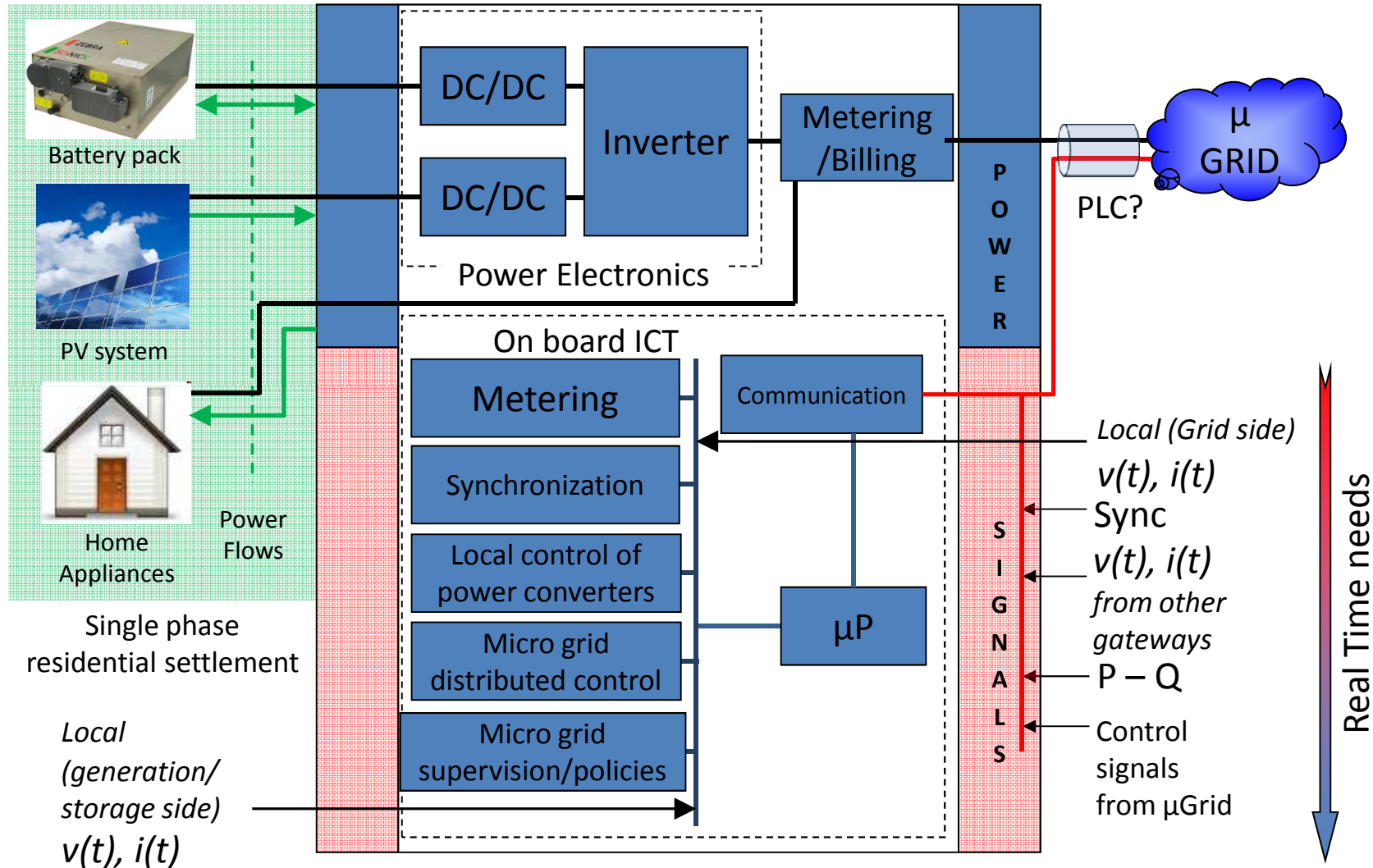


Definitions and requirements



- **Active grid nodes** correspond to *prosumers*, i.e., buildings or residential settlements equipped with distributed energy resources (DERs) and Energy Gateways (EGs)
 - *DERs* may be PV panels, wind turbines, fuel cells, batteries, flywheels, etc.
 - *EGs* include an *electronic power processor (EPP)*, capable to control the active and reactive power flow from local sources into the grid, a *local control unit (LCU)* and a *smart meter (SM)*, which provides measurement, communication and synchronization capability.
- **Passive grid nodes** correspond to traditional consumers and are assumed to be equipped with smart meters too
- **Plug & play operation** of EGs ensures *flexibility and scalability* of the micro-grid architecture
- **Distributed control and communication** allows *cooperative operation* of EGs and *synergistic utilization* of DERs

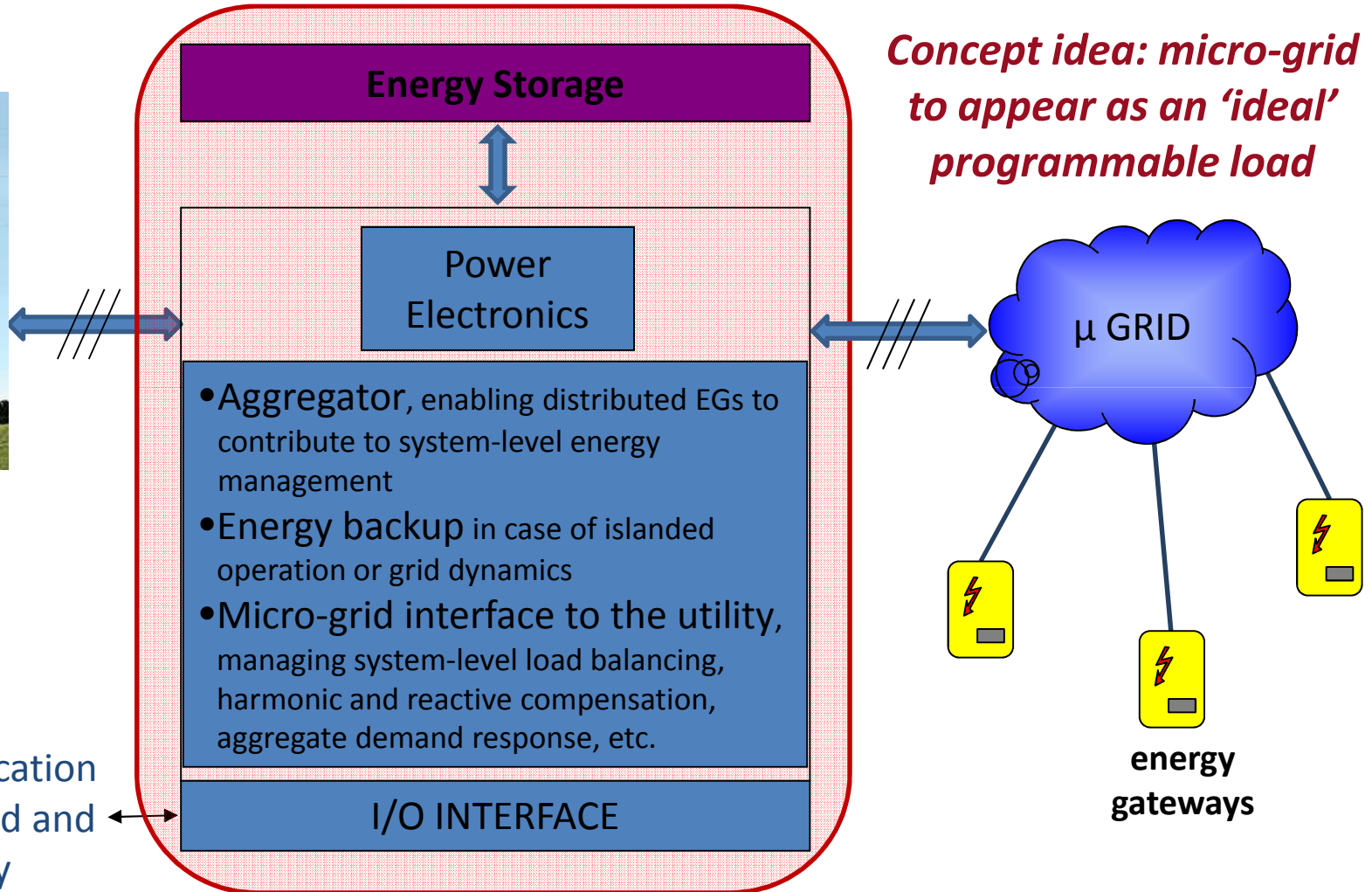
Energy Gateway – functional diagram



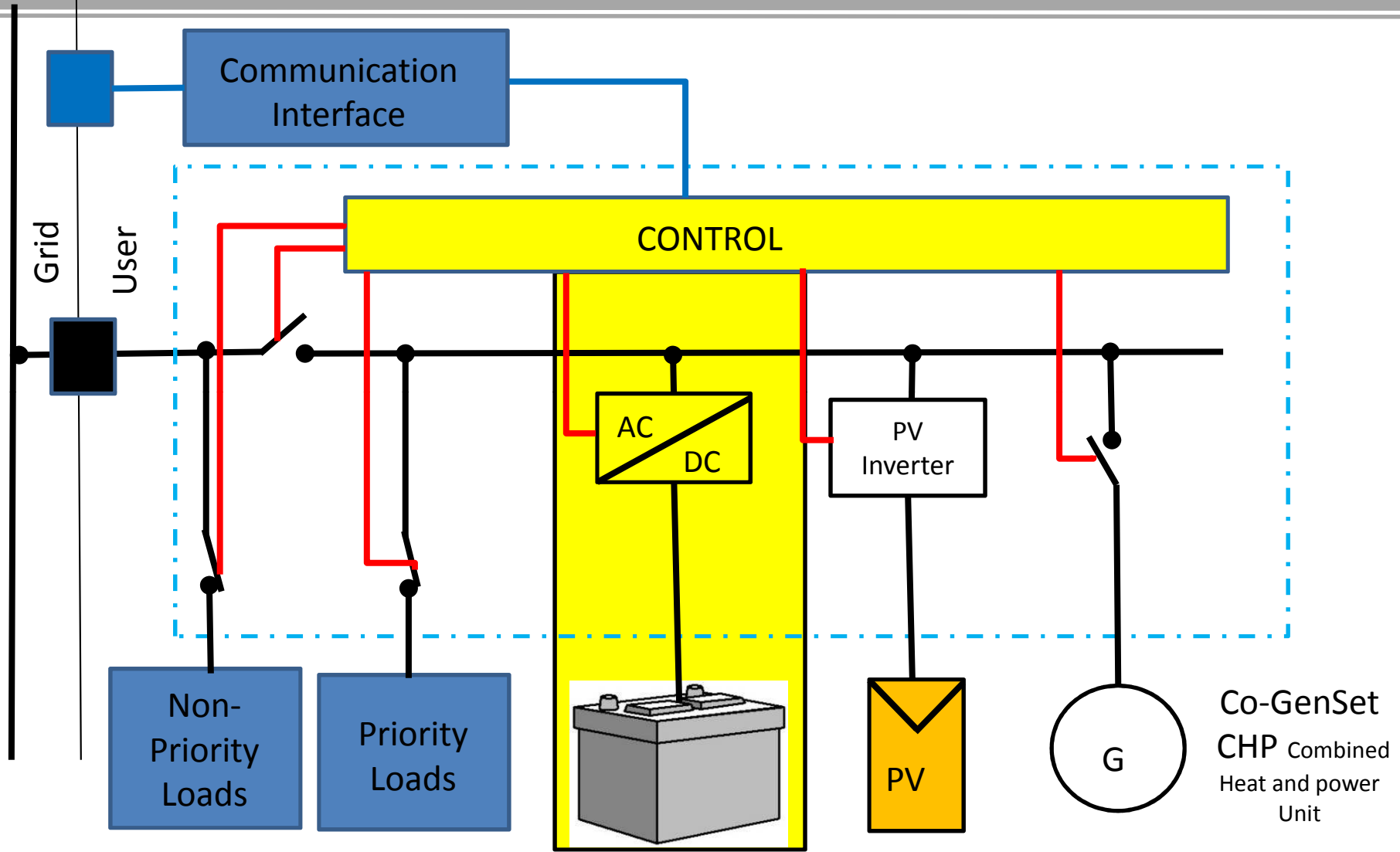
Utility Interface – functional diagram



Three phase distribution infrastructure



Retrofitting existing plants



Smart micro-grids

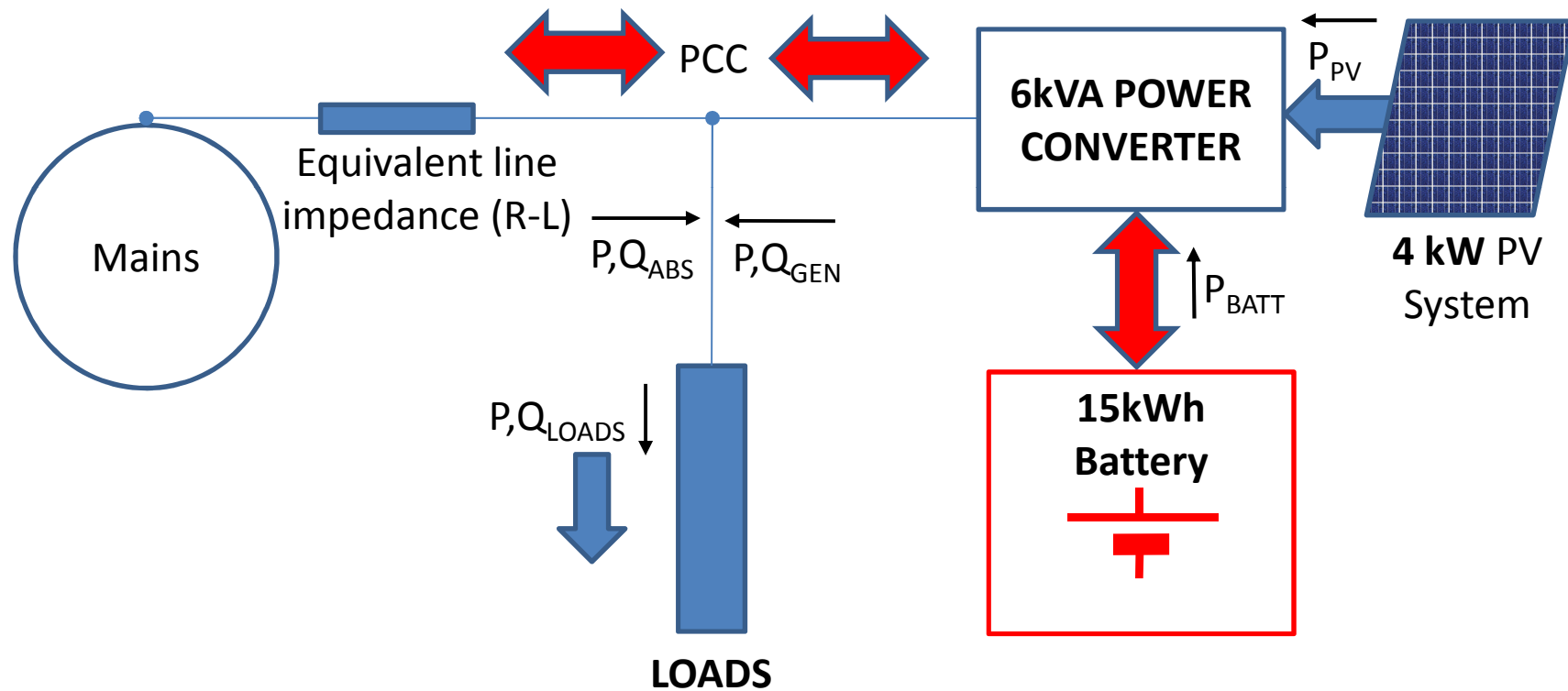
Properties, trends and local control of energy sources

4. The role of energy storage

Energy efficiency – local features

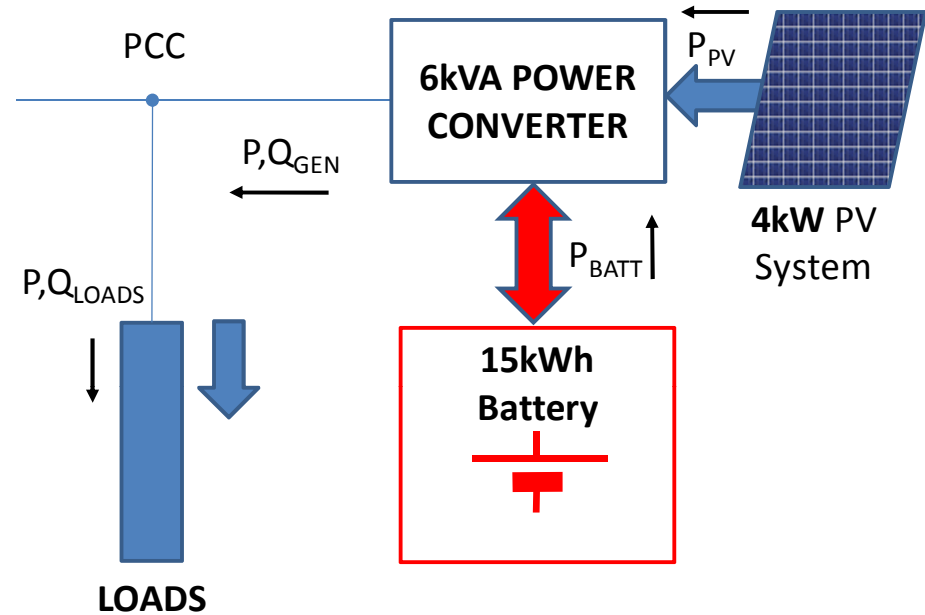
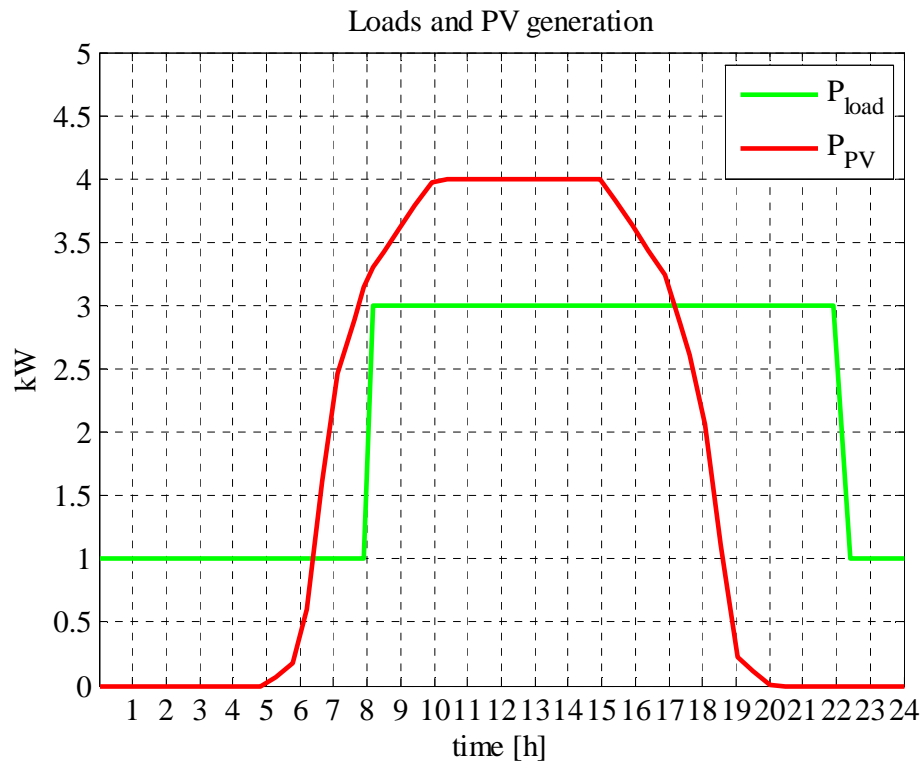


Residential settlement with loads, PV generation and energy storage connected to the mains via cabled distribution line



4. The role of energy storage

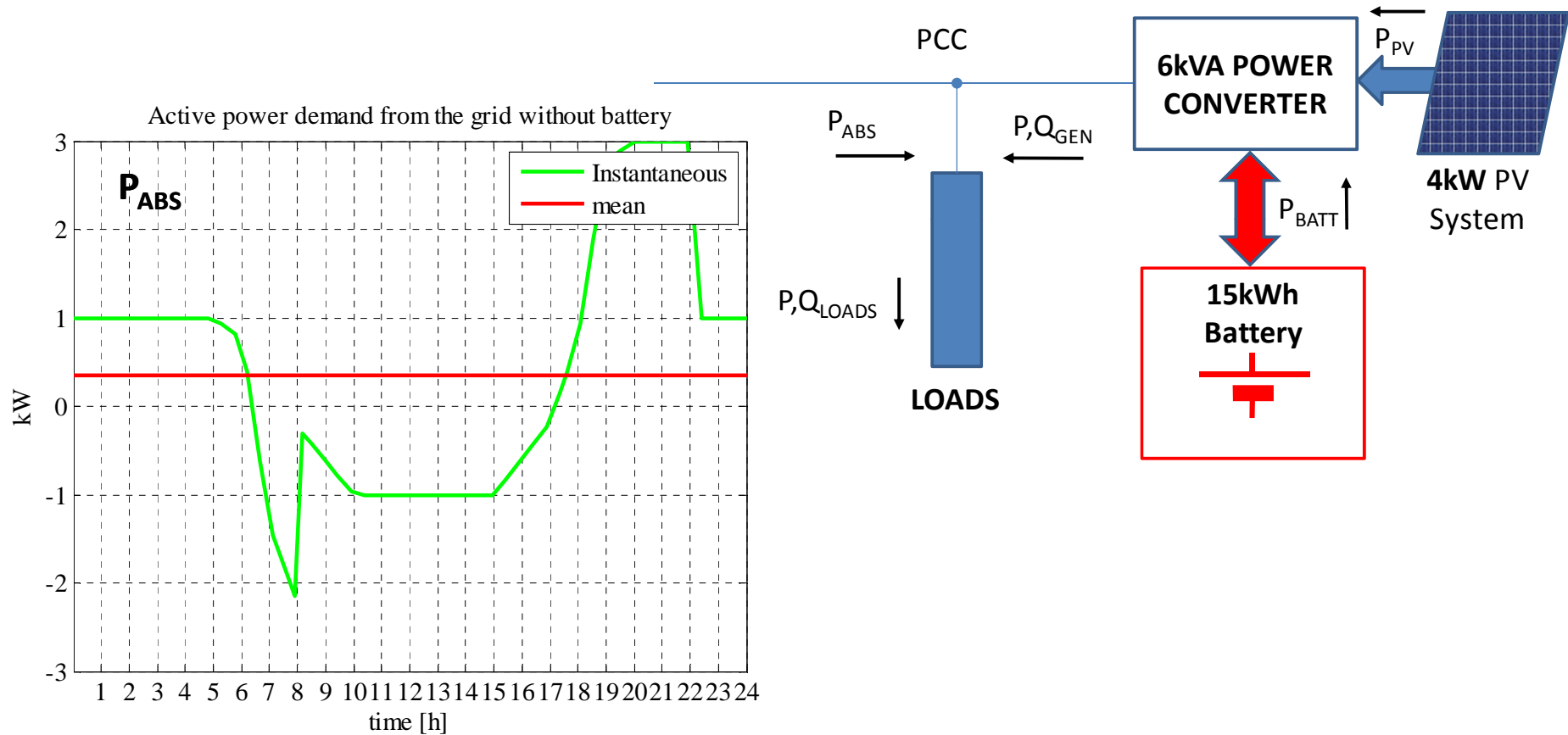
Power profiles



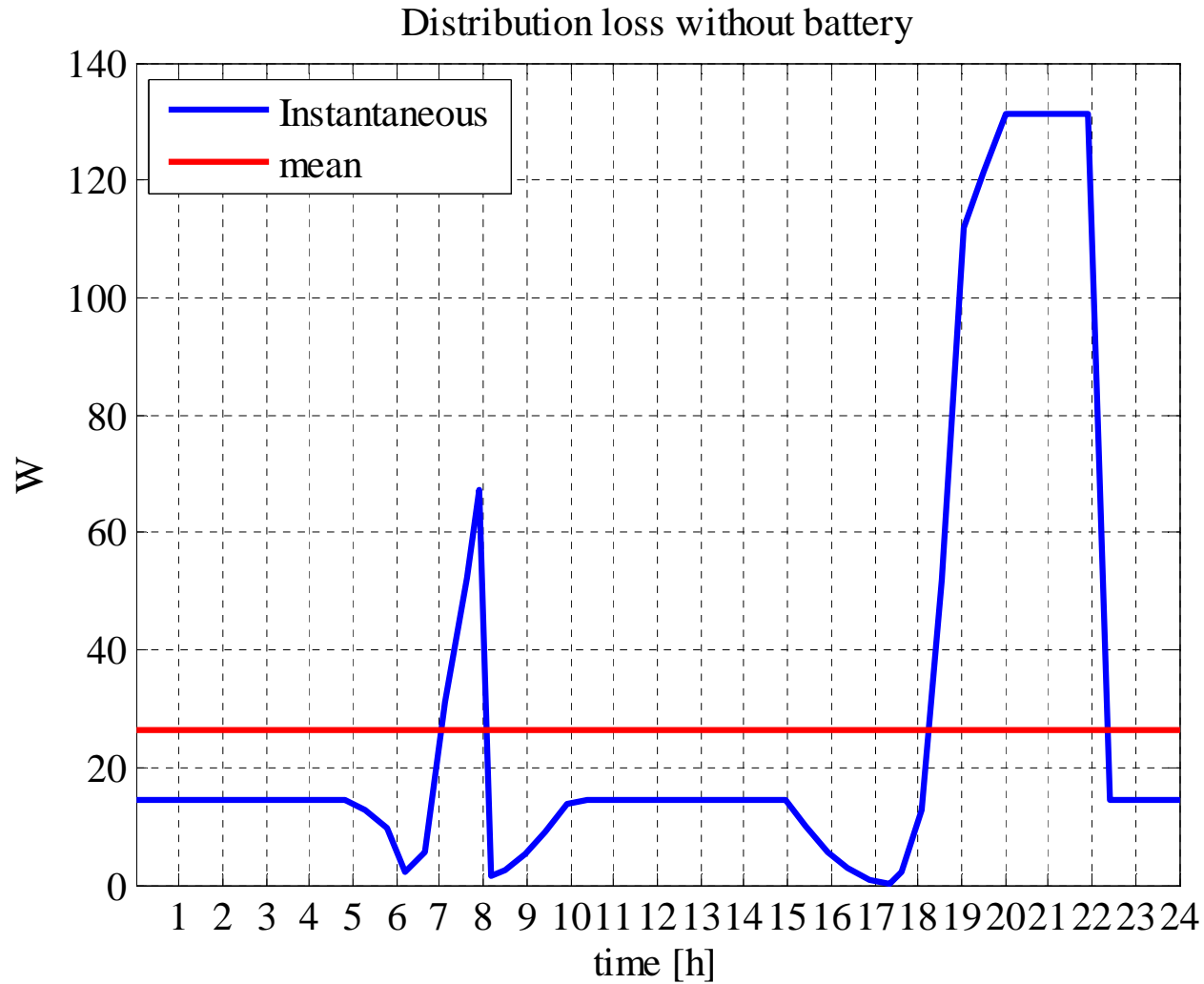
Power absorption without battery

Active power absorbed from the mains assuming

$$P_{BATT} = 0: P_{ABS} = P_{LOADS} - P_{PV}$$



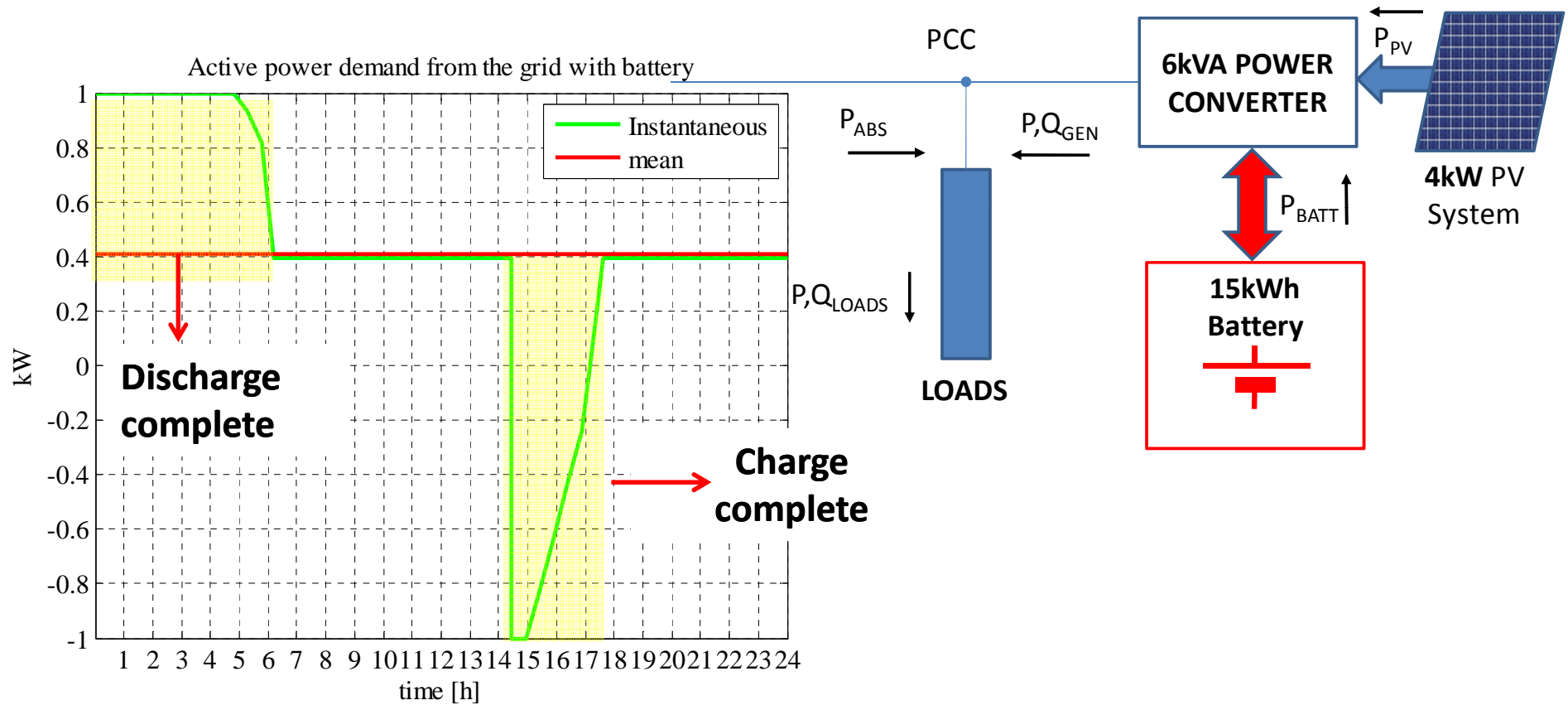
Distribution loss without battery



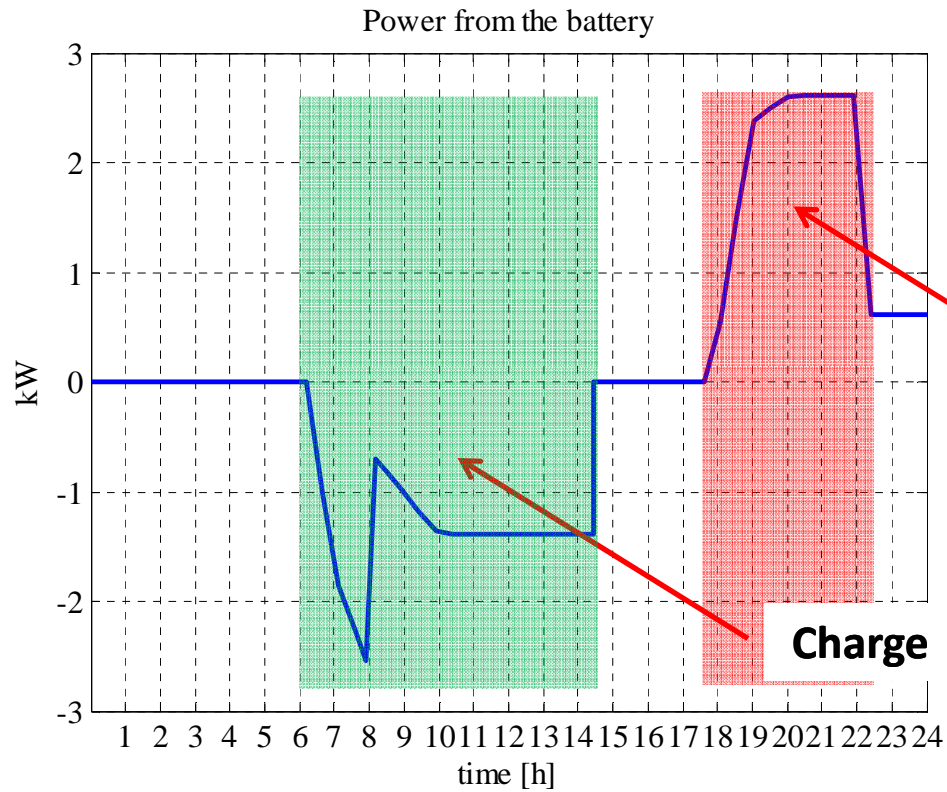
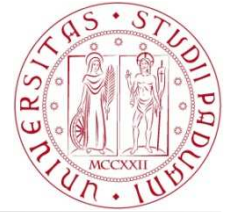
Power absorption with battery

$$P_{ABS} = P_{LOADS} - P_{PV} - P_{BATT}$$

Local control tends to enforce $P_{ABS} = P_{ABS_AVG}$ (daily average power)

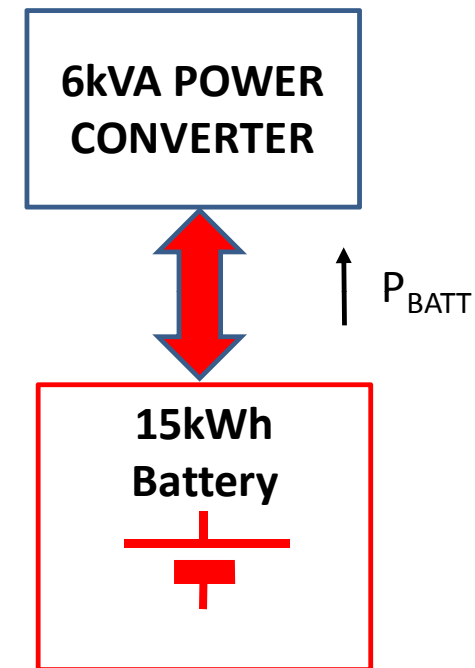


Daily power profile of battery

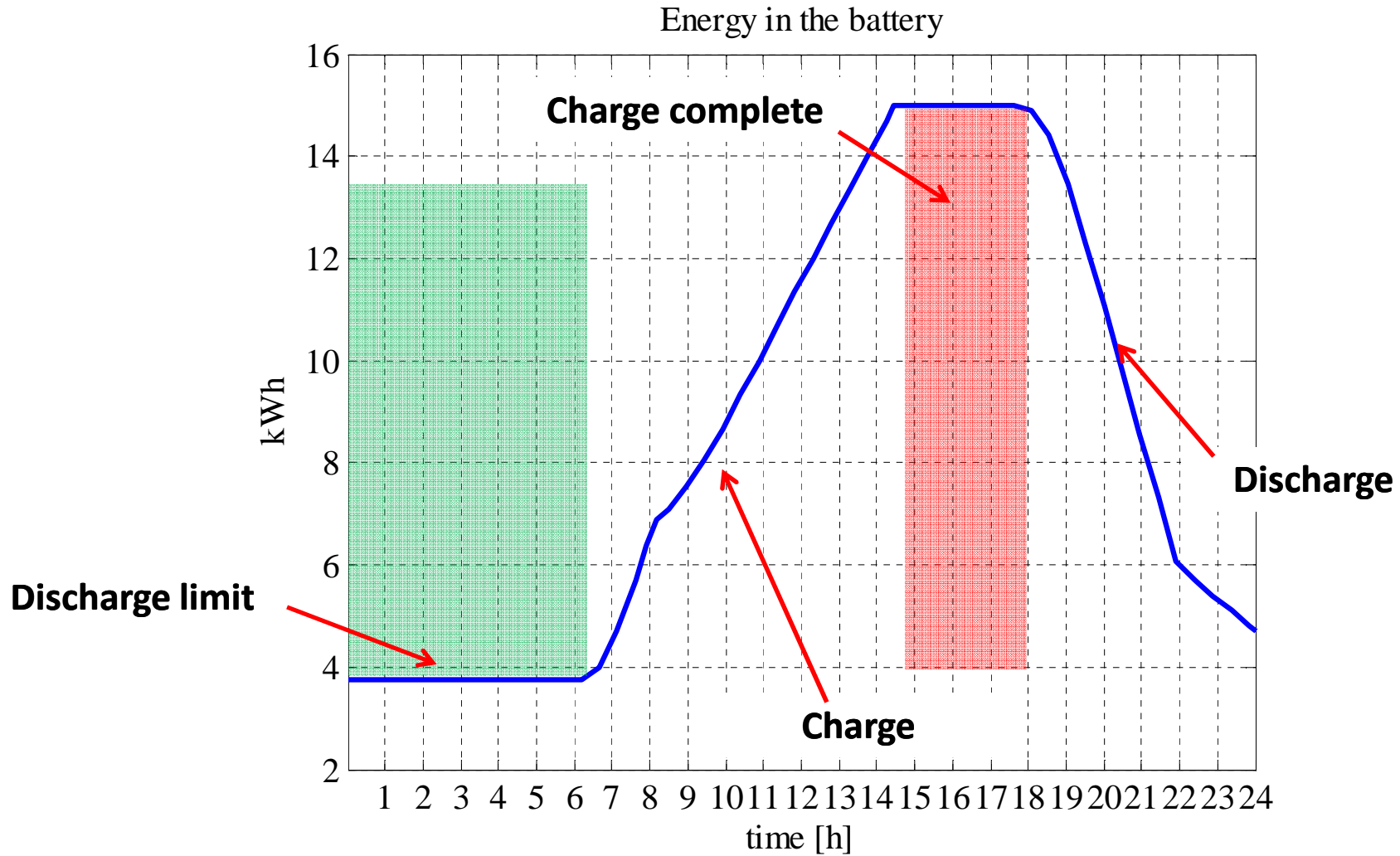


Discharge

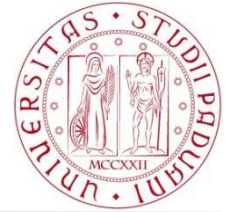
Charge



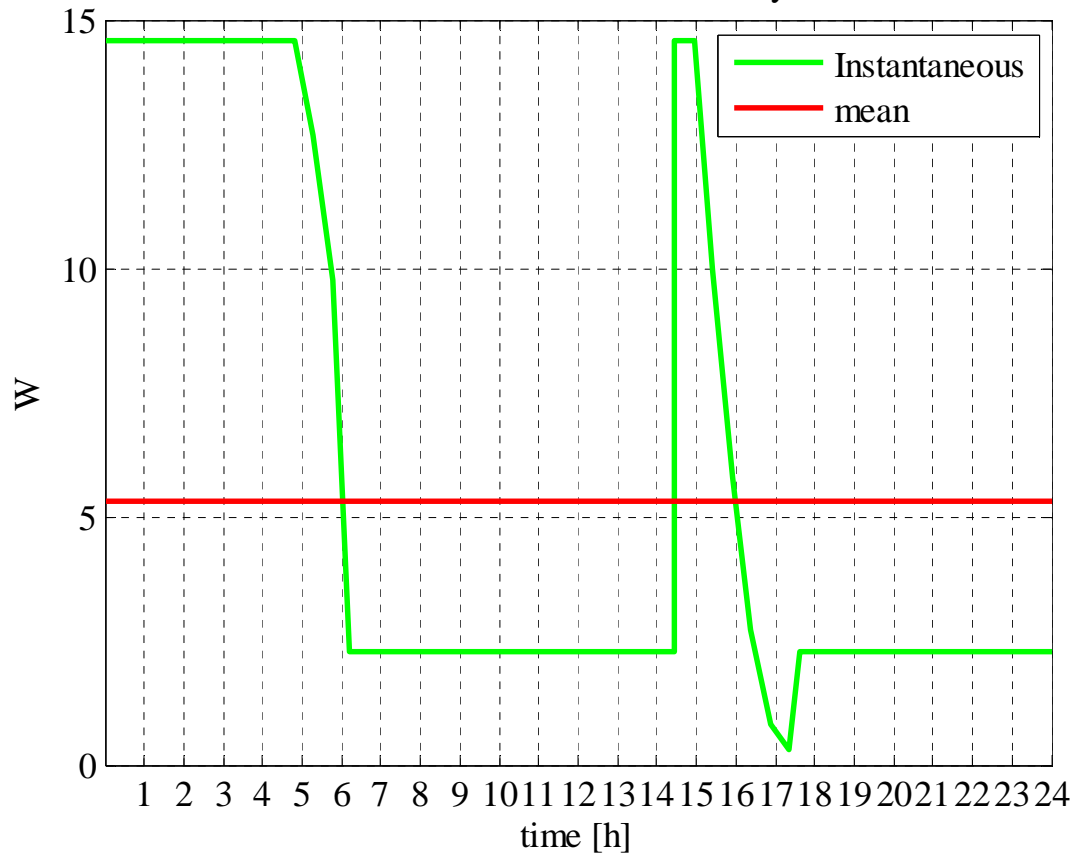
Daily energy profile of battery



Distribution loss without battery

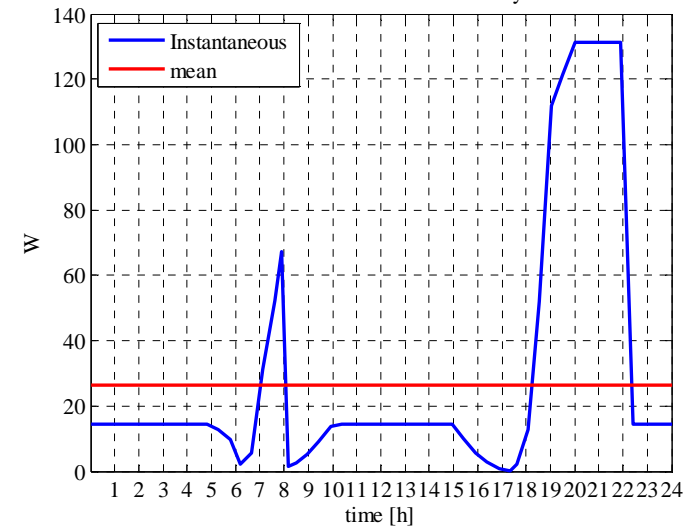


Distribution loss with battery

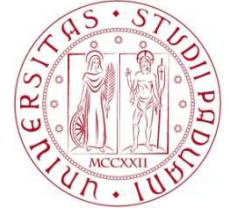


Distribution loss reduced by 85% (active power control only)

Distribution loss without battery



Distributed energy storage



Local functions (Energy Gateways)

- Regularization of power absorption
- Reduction of losses in the distribution feeder
- Peak power shaving
- Emergency supply in case of mains outage (UPS operation)
- Node voltage stabilization
- **Prosumer energy bill reduction**

Micro-grid functions (Utility Interface + Energy Gateways)

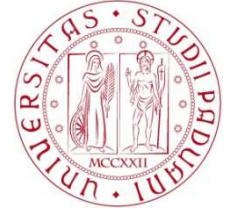
- Energy sharing & backup in case of islanded operation
- Smoothing of irregular power generation by renewable sources
- Programmable active and reactive power absorption
- Power delivery to the utility on demand
- **Cost-effective energy management and ROI planning**

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5. Control issues in smart micro-grids

Control objectives



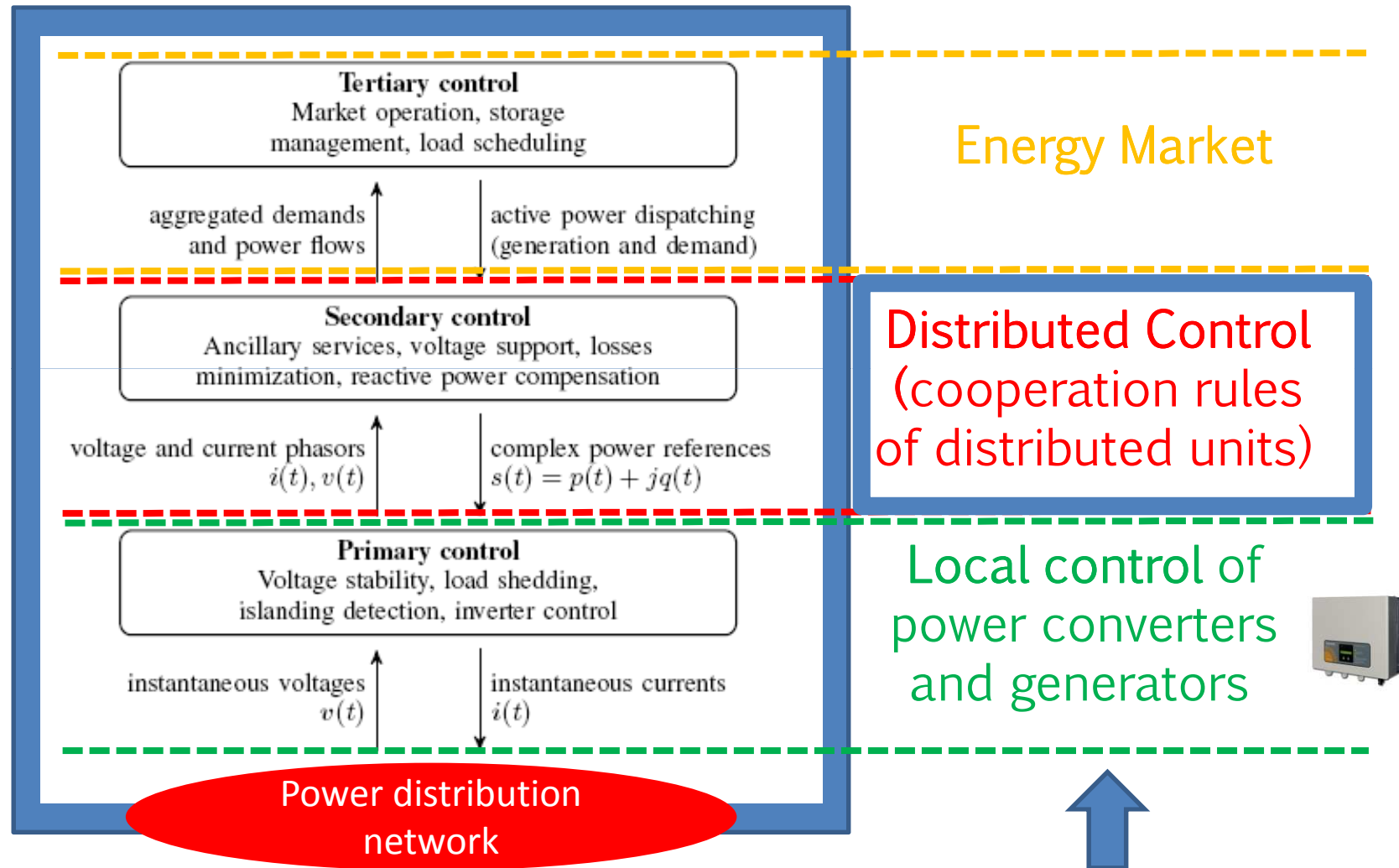
Local control functions (Energy Gateways)

- Exploitation of renewable energy sources (active power control)
- Management of energy storage (active power control)
- Voltage support (active & reactive power control)
- Reactive & harmonic compensation
- Load shedding & shifting

Micro-grid control functions (Utility Interface + Energy Gateways)

- Synergistic utilization of micro-grid resources
- Aggregate demand response and peak power shaving
- Load balancing by reactive current control (Steinmetz approach)
- Management of mains outages & grid dynamics
- Management of islanded operation
- Management of active and reactive power requests by the utility

Hierarchic control architecture



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6. Inverter modeling and control

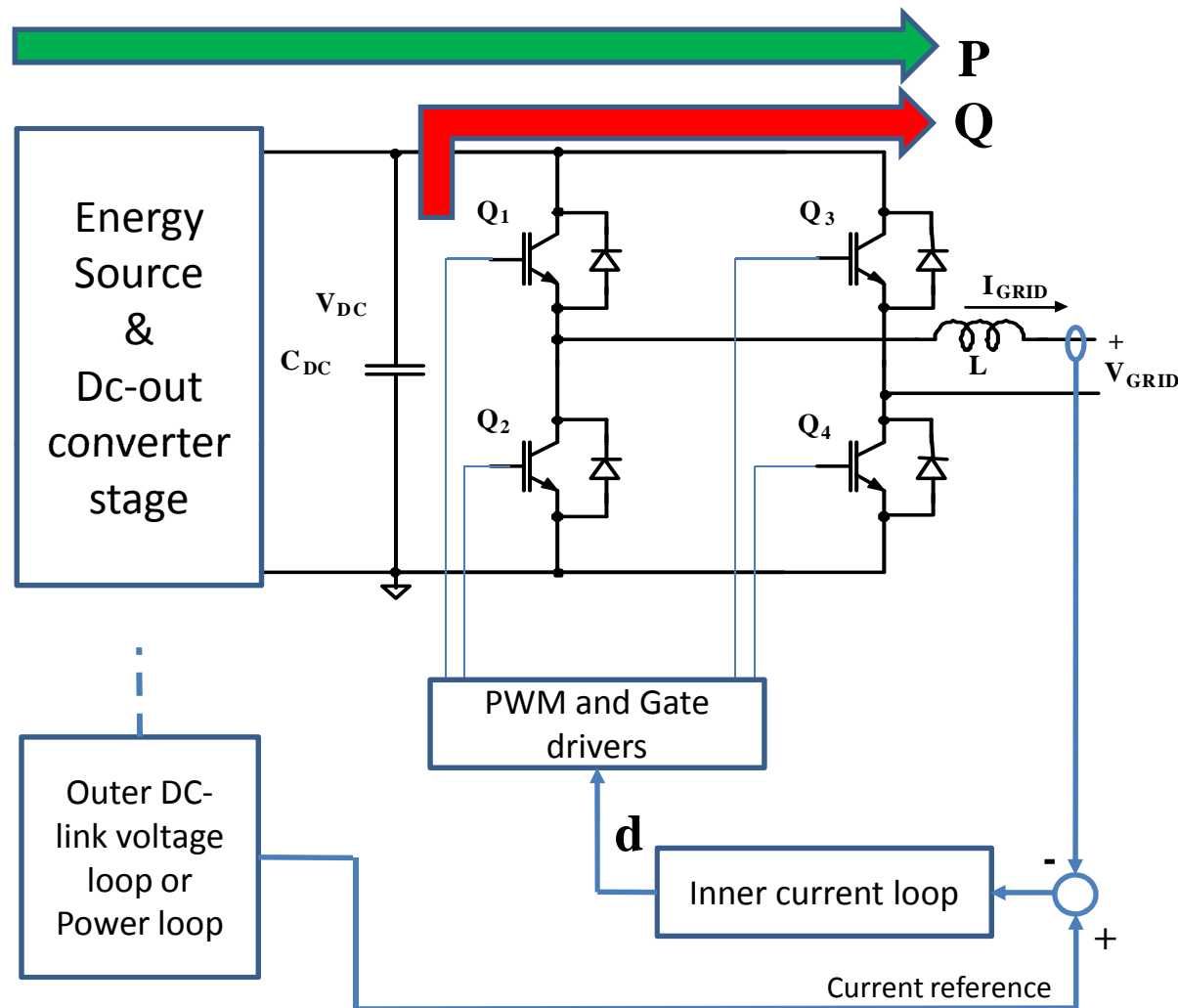
Inverter control modes



Single-phase voltage-fed full-bridge grid-connected inverters can be driven according to different control approaches:

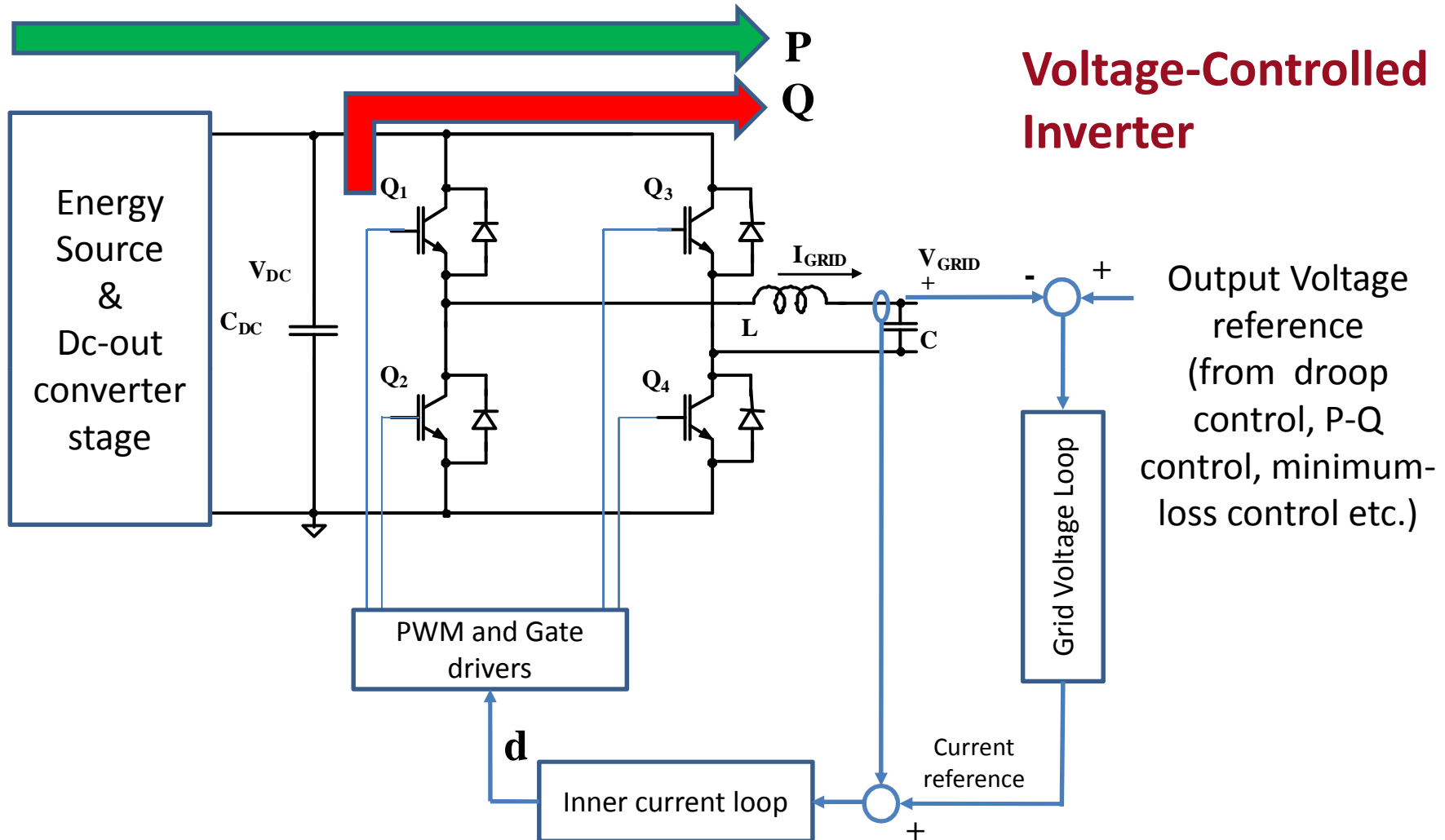
1. **Current-mode control:** the ac-side inductor current is controlled to track a current reference set by the DC link voltage controller (typical configuration of PV systems) or by an external power loop. The inverter appears as a **Controlled current (or Power) source**.
2. **Voltage-mode control:** the current loop is driven by an external voltage control loop that tracks a voltage reference (UPS applications or droop-controlled inverters, where the power flows are controlled by acting on module and phase of the inverter ac voltage). The inverter appears as a **Controlled voltage source**.

Inverter control: current-mode

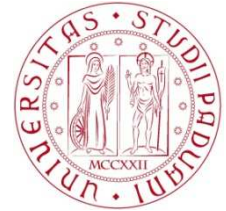


Current- Controlled / Power-Controlled Inverter

Inverter control: voltage-mode



Inverter control: current injection

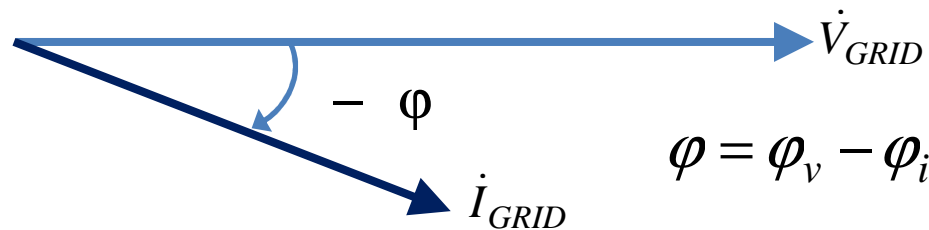


For current-controlled inverters the usual requirement in grid-connected operation (e.g., for PV inverters) is to **supply purely active power**, i.e., to inject a current in phase with the line voltage ($\cos\varphi=1$);

Assuming sinusoidal grid voltage \dot{V}_{GRID} and inverter current \dot{I}_{GRID} , the **phasorial representation** of this operating condition is:



In general, however, **the inverter can feed a current which can be leading or lagging** the grid voltage



RMS values

$$\dot{V}_{GRID} = V_{GRID} e^{j\varphi_v}$$

$$\dot{I}_{GRID} = I_{GRID} e^{j\varphi_i}$$

Inverter control: power injection

Complex Power supplied by the inverter:

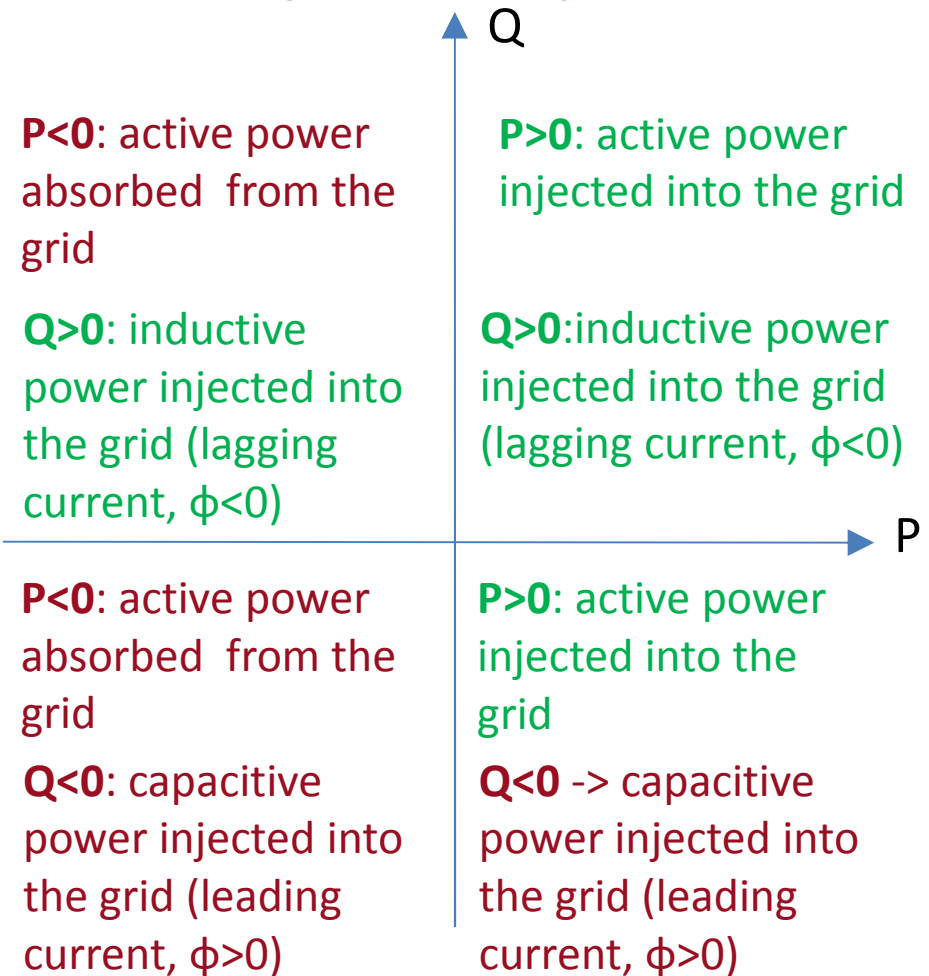
$$\dot{S} = \dot{V}_{GRID} \dot{I}_{GRID}^* = P + jQ$$

$$\begin{aligned} \dot{S} &= V_{GRID} \left(I_{GRID} e^{-j\phi} \right)^* = \\ &= V_{GRID} I_{GRID} e^{j\phi} = \\ &= V_{GRID} I_{GRID} \cos \phi + V_{GRID} I_{GRID} \sin \phi \end{aligned}$$

$$P = V_{GRID} I_{GRID} \cos \phi$$

$$Q = V_{GRID} I_{GRID} \sin \phi$$

Four quadrant operation

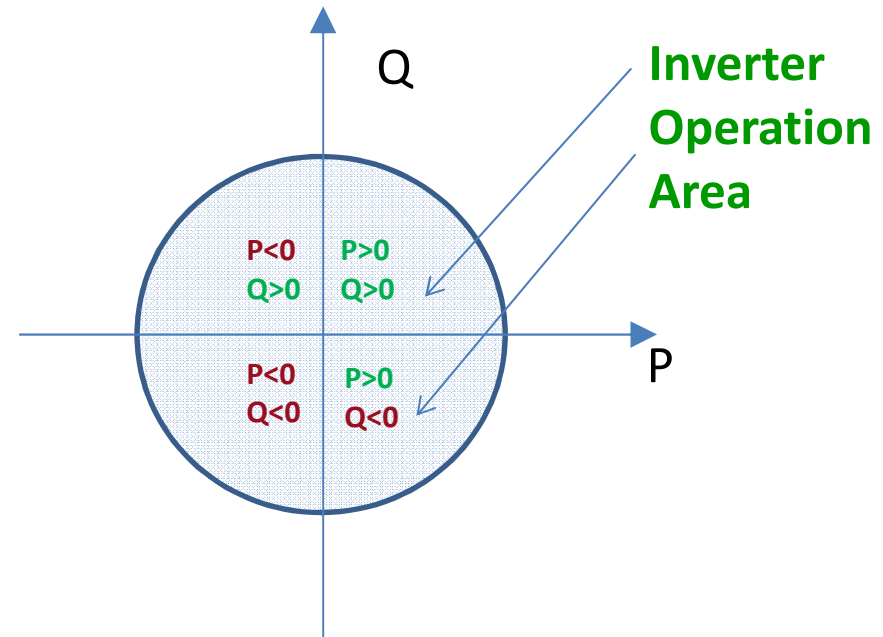


Inverter control: power capability

In distributed generation, the inverters operate in the **I and IV quadrants**, injecting **positive active power** and **either positive or negative reactive power**

POWER RATING:

The **complex power that can be injected by an inverter is limited by the current and voltage rating of the components** (V and I limits for the switches, I limits for the output inductors, V limit for the capacitors etc)



For a given grid voltage, these limits are represented by the apparent power

$$A = V_{GRID} I_{GRIDmax} = |\dot{S}_{max}| [VA]$$

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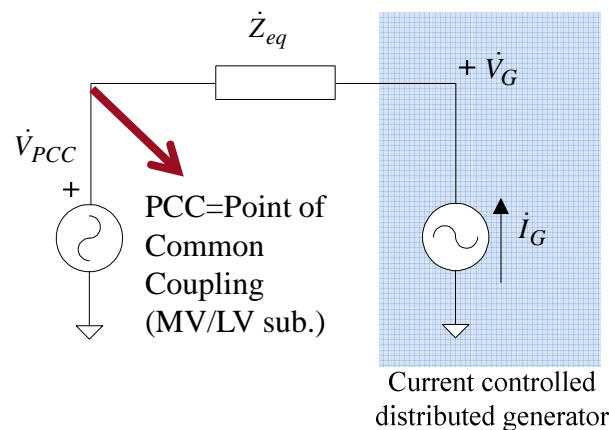
7. Micro-grid modeling and distribution loss analysis

Micro-grid modeling (1)

To analyze the micro-grid operation, a suitable **modelling** is required.

Power Systems approach: Network elements are represented as constant power loads / distributed generators, constant current loads, constant impedance loads. The grid is analyzed in terms of **Power Flow** relations, resulting in **nonlinear equation systems** which require numeric solvers (Newton-Raphson , etc).

LV distribution systems: the voltage is impressed at the Point of Common Coupling with the mains and its variation along the LV grid is within $\pm 5\%$ of rated value. Thus, under steady-state conditions, the constant-power loads can be represented as constant-current (or constant-impedance) elements. Similarly for the energy sources. Thus, the system model becomes linear and can be solved analytically by Kirchhoff's and load equations.



Moreover, LV distribution lines are usually made by **cables** with **constant section**, i.e. impedances with constant phase (modelled as R-L series). This further simplifies the analysis, making possible the analytical solution of radial and meshed grids as well.

Micro-grid modeling (2)

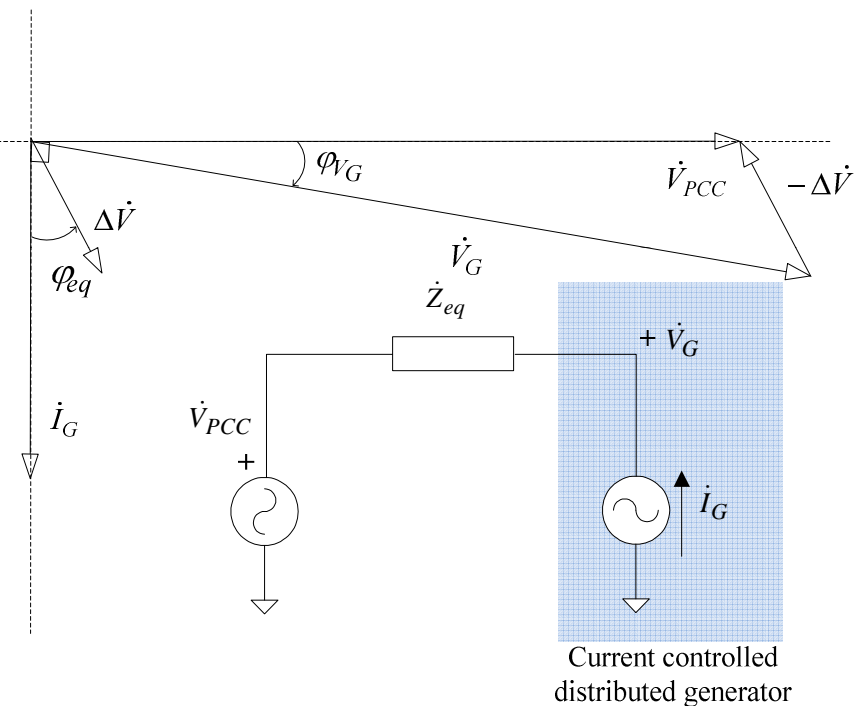
Assumption: the PCC voltage is taken as phase reference for the phasorial representation

$$\dot{V}_{PCC} = U_{rated} + j0 = 230 + j0 \text{ V}$$

Approximation: based on the assumption of negligible phase voltage differences between grid nodes, the **active and reactive** currents absorbed by the loads or injected by the generators nearly coincide with the **real and imaginary** components of such node currents referred to the PCC voltage.



- The **real part** of the node currents controls the **active power** absorbed/injected at the grid nodes
- The **imaginary part** of the node currents controls the **reactive power** absorbed/injected at the grid nodes



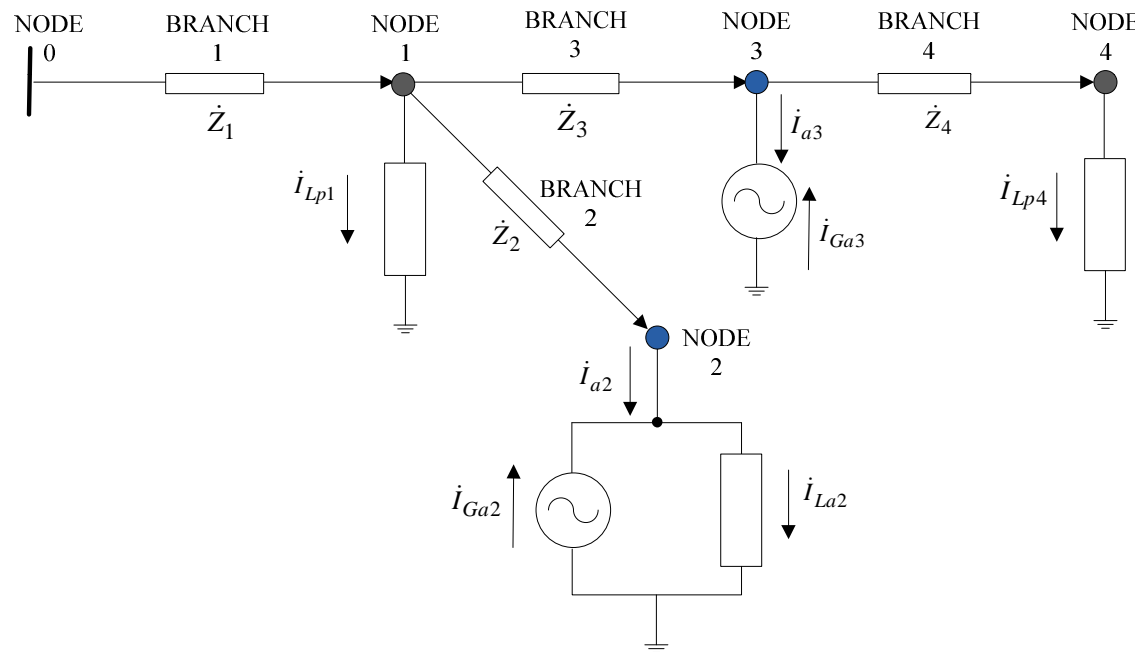
$$\Delta V \ll V_{PCC}$$

Incidence matrix of radial micro-grids (1)

Consider a **radial micro-grid** with $N+1$ nodes ($0 \dots N$) and N branches ($1 \dots N$), where the loads and the distributed generators are connected to the grid nodes.

The **complete incidence matrix** \underline{A}_c is defined as a $N \times (N+1)$ integer matrix whose elements are:

$$\underline{A}_c(\ell, n) = \begin{cases} -1 & \text{if branch } \ell \text{ leaves node } n \\ +1 & \text{if branch } \ell \text{ enters node } n \\ 0 & \text{otherwise} \end{cases}$$

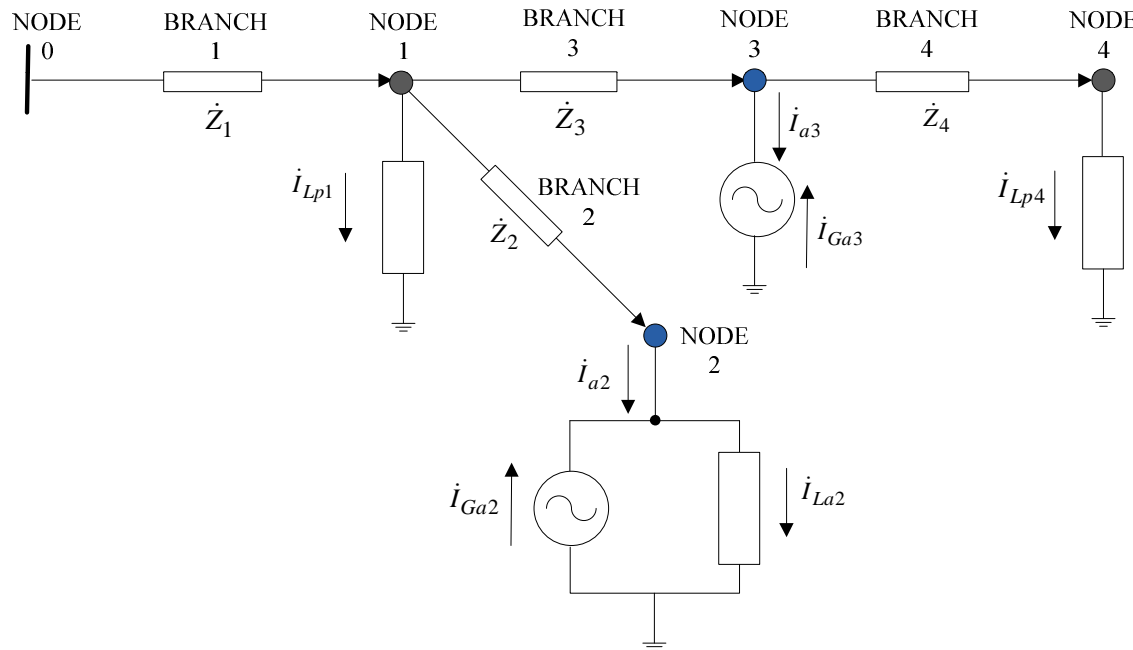


Complete Incidence Matrix \underline{A}_c

	Nodes					
	0	1	2	3	4	
$\underline{A}_c =$	-1	1	0	0	0	1
	0	-1	1	0	0	2
	0	-1	0	1	0	3
	0	0	0	-1	1	4
						Branches

Incidence matrix of radial micro-grids (2)

The reduced **incidence matrix \underline{A}** is defined as a $N \times N$ integer matrix obtained by eliminating the column of node 0 (**slack node**, i.e., the **Point of Common Coupling** with the utility, **PCC**)



Reduced Incidence Matrix \underline{A}

	Nodes				
	1	2	3	4	
$\underline{A}_c =$	1	0	0	0	1
	-1	1	0	0	2
	-1	0	1	0	3
	0	0	-1	1	4

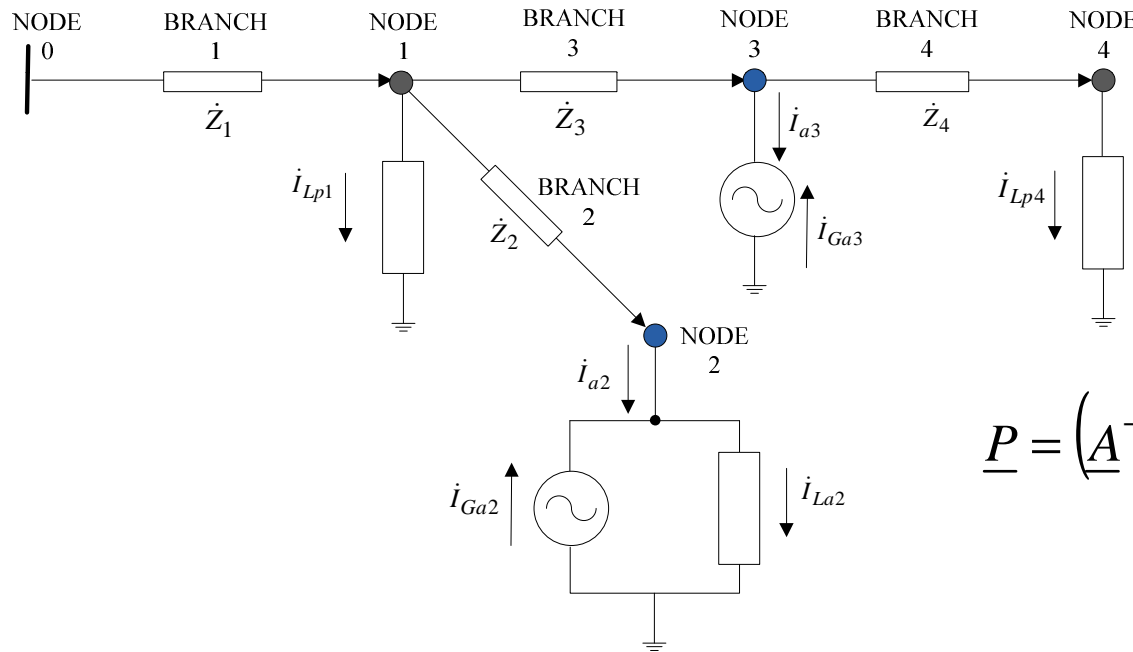
Branches

Note: The reduced Incidence Matrix \underline{A} is square and invertible

Path matrix of radial micro-grids



The transpose inverse of reduced incidence matrix \underline{A} is a $N \times N$ integer matrix called **path matrix \underline{P}** , whose n^{th} column gives the path from node 0 to node n .



Path Matrix \underline{P}

$$\underline{P} = (\underline{A}^{-1})^T = (\underline{A}^T)^{-1} = \begin{array}{c} \text{Nodes} \\ \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c|cccc|c} 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 0 & 1 & 0 & 0 & 2 \\ 3 & 0 & 0 & 1 & 1 & 3 \\ 4 & 0 & 0 & 0 & 1 & 4 \end{array} \\ \text{Branches} \end{array}$$

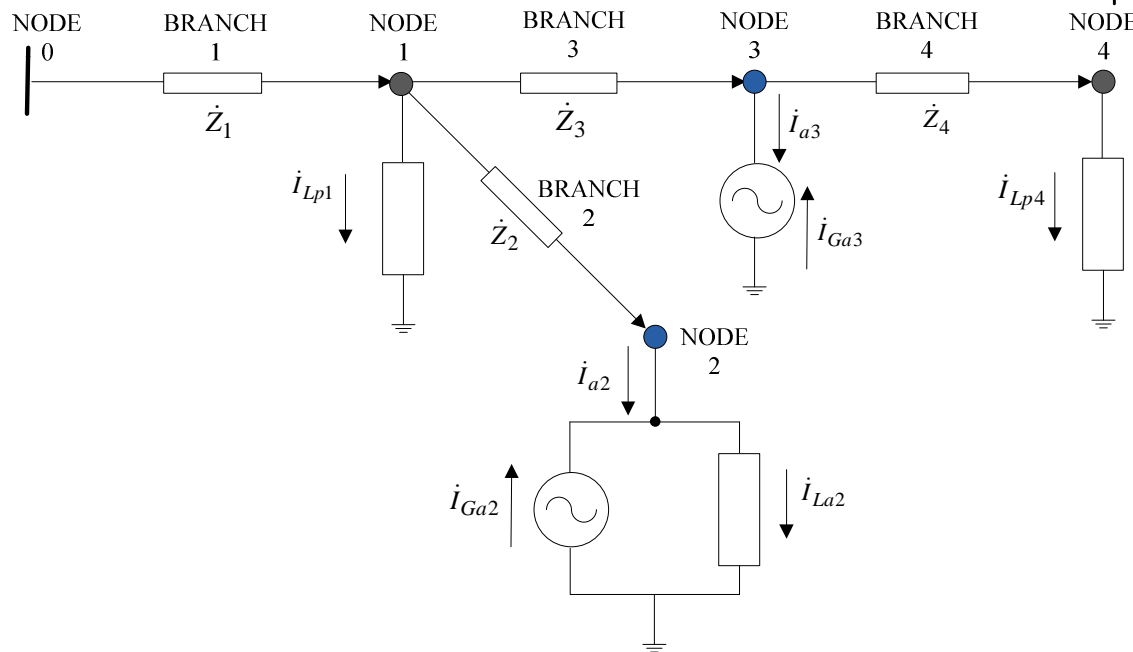
Kirchhoff's laws of radial micro-grids (1)

Let \underline{u}_c be the node voltages (including node 0) and \underline{v} the branch voltages, the **Kirchhoff's Law for voltages** (KLV) applied to voltage phasors gives:

$$\underline{v} = -\underline{A}_c \underline{u}_c \Rightarrow$$

$$\begin{bmatrix} \dot{V}_1 \\ \dot{V}_2 \\ \dot{V}_3 \\ \dot{V}_4 \end{bmatrix} = - \begin{bmatrix} -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix} \times \begin{bmatrix} \dot{U}_0 \\ \dot{U}_1 \\ \dot{U}_2 \\ \dot{U}_3 \\ \dot{U}_4 \end{bmatrix}$$

$\underbrace{\quad}_{\underline{a}_0} \quad \underbrace{\quad}_{\underline{A}}$



In a simplified form, let \underline{u} be the node voltages (excluding node 0), the Kirchhoff's Law for voltages (KLV) becomes:

$$\underline{\dot{V}} = -\underline{a}_0 \cdot \dot{U}_0 - \underline{A} \times \underline{\dot{U}}$$

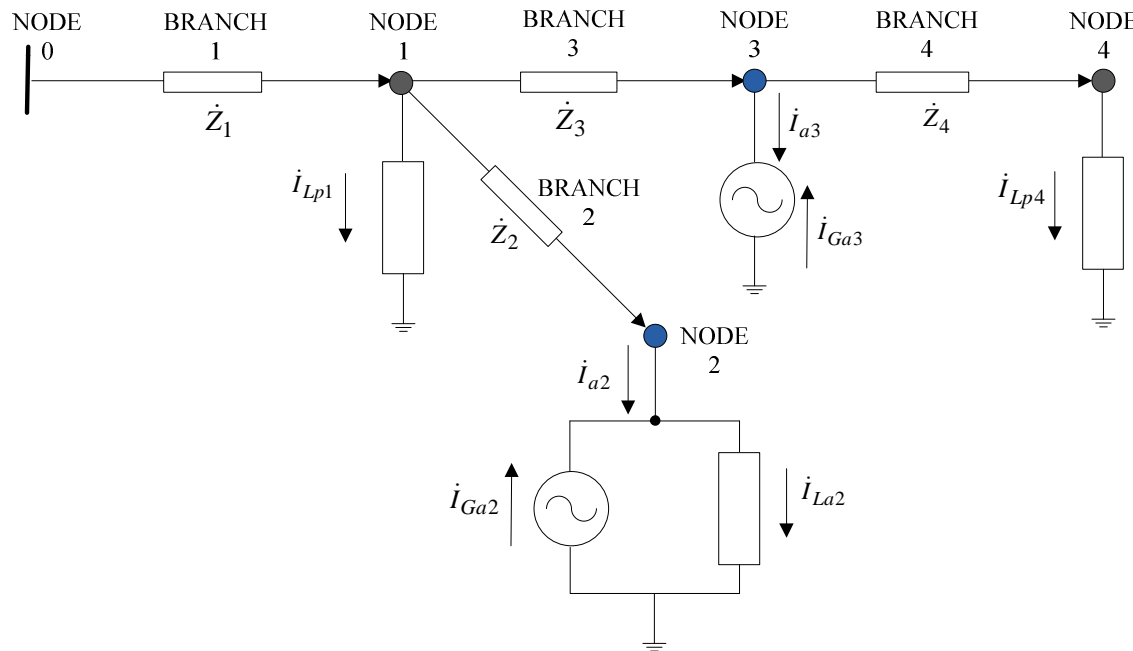
where \underline{a}_0 is the first column of complete incidence matrix \underline{A}_c

Kirchhoff's laws of radial micro-grids (2)

Let \underline{i}_c be the node currents (including node 0, with positive polarity if leaving the grid) and \underline{j} be the branch currents, the **Kirchoff's Law for currents** (KLC) applied to current phasors gives:

$$\underline{i}_c = \underline{A}_c^T \underline{j} \Rightarrow$$

$$\begin{bmatrix} \dot{I}_0 \\ \dot{I}_1 \\ \dot{I}_2 \\ \dot{I}_3 \\ \dot{I}_4 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 1 & -1 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \dot{J}_1 \\ \dot{J}_2 \\ \dot{J}_3 \\ \dot{J}_4 \end{bmatrix}$$



In a simplified form, let \underline{i} be the node currents (excluding node 0), the Kirchoff's Law for currents (KLC) becomes:

$$\begin{cases} \dot{I}_0 = \underline{a}_0^T \times \underline{j} \\ \underline{\dot{I}} = \underline{A}^T \times \underline{j} \end{cases}$$

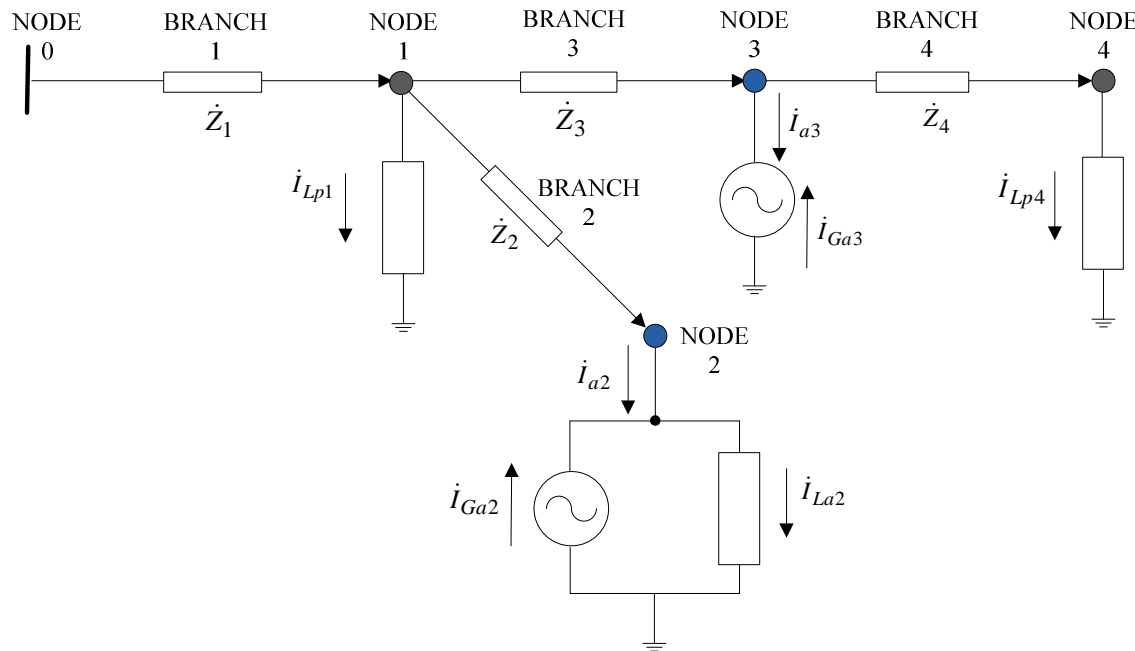
Kirchhoff's laws of radial micro-grids (3)

Note that:

$$i_0 = -\sum_{n=1}^N i_n \Rightarrow \underline{I}_0 = -\underline{1}_N^T \times \underline{I}$$

Thus:

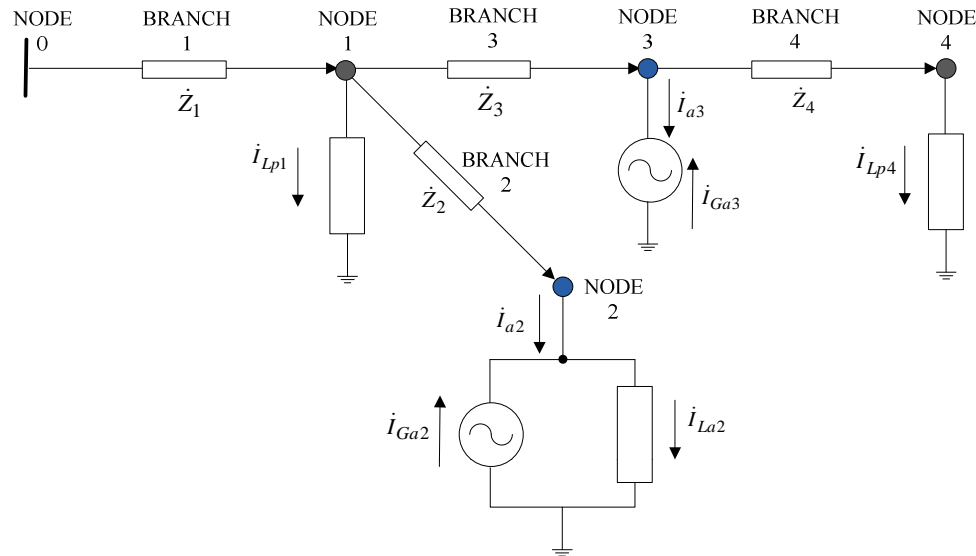
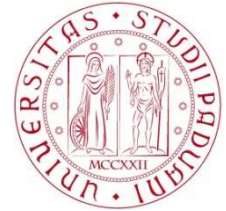
$$\underline{I}_0 = -\underline{1}_N^T \times \underline{I} = \underline{a}_0^T \times \underline{J} = \underline{a}_0^T \times (\underline{A}^T)^{-1} \underline{I} \Rightarrow \underline{a}_0^T \times (\underline{A}^T)^{-1} = \underline{a}_0^T \times \underline{P} = -\underline{1}_N^T$$



In a simplified form, let \underline{i} be the node currents (excluding node 0), the Kirchhoff's Law for currents (KLC) becomes:

$$\begin{cases} \underline{I}_0 = \underline{a}_0^T \times \underline{J} \\ \underline{I} = \underline{A}^T \times \underline{J} \end{cases}$$

Radial micro-grid equations (1)



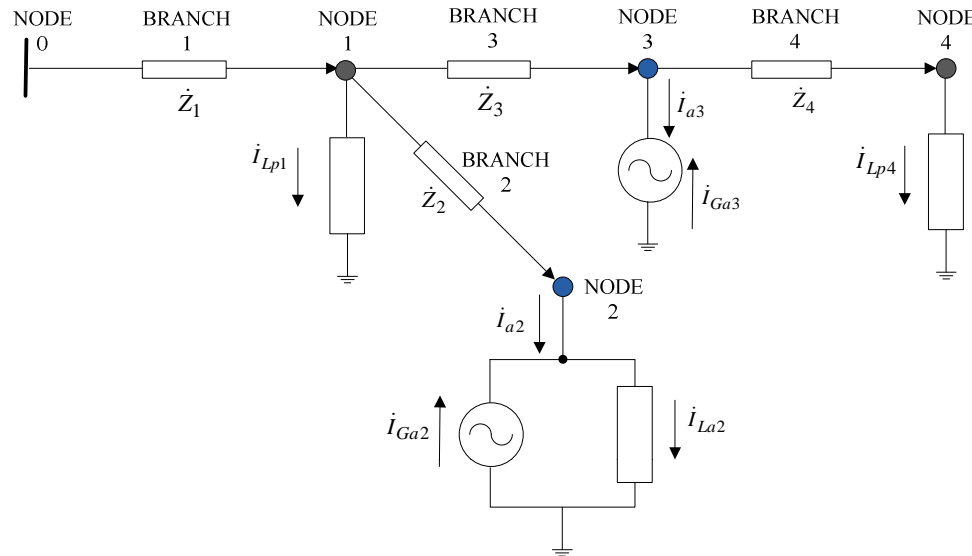
Branch impedances	
Z_1	$1+j1 \Omega$
Z_2	$2+j2 \Omega$
Z_3	$3+j3 \Omega$
Z_4	$4+j4 \Omega$
Load currents	
I_{Lp1}	$5+j5 \text{ A}$
I_{La2}	$10+j10 \text{ A}$
I_{Lp4}	$15+j15 \text{ A}$

For each branch of the distribution grid we can write: $\dot{V}_{ij} = \dot{U}_i - \dot{U}_j = \dot{Z}_{ij} \dot{J}_{i \rightarrow j}$

Let \underline{Z} be the diagonal matrix of the branch impedances, in vector form we get:

$$\underline{\dot{Z}} = \text{diag} \{ \dot{Z}_l \}_{l=1}^N = \begin{vmatrix} \dot{Z}_1 & 0 & 0 & 0 \\ 0 & \dot{Z}_2 & 0 & 0 \\ 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & \dot{Z}_N \end{vmatrix} \Rightarrow \underline{\dot{V}} = \underline{\dot{Z}} \times \underline{\dot{J}}$$

Radial micro-grid equations (2)



Branch impedances	
Z_1	$1+j1 \Omega$
Z_2	$2+j2 \Omega$
Z_3	$3+j3 \Omega$
Z_4	$4+j4 \Omega$
Load currents	
I_{Lp1}	$5+j5 \text{ A}$
I_{La2}	$10+j10 \text{ A}$
I_{Lp4}	$15+j15 \text{ A}$

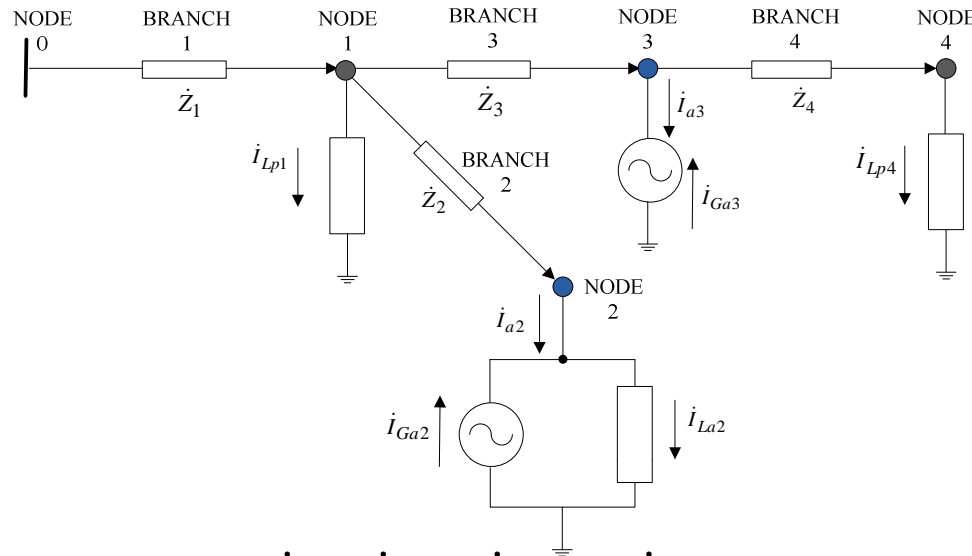
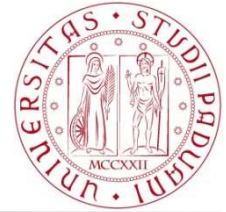
Recalling the previous definitions and results we get:

$$\underline{\dot{V}} = \underline{\dot{Z}} \times \underline{J} \Rightarrow \left\{ \begin{array}{l} \underline{\dot{V}} = -\underline{a}_0 \underline{\dot{U}}_0 - \underline{A} \times \underline{\dot{U}} \\ \underline{\dot{I}} = \underline{A}^T \times \underline{J} \end{array} \right\} \Rightarrow \underline{a}_0 \underline{\dot{U}}_0 + \underline{A} \times \underline{\dot{U}} = -\underline{\dot{Z}} \times \underbrace{\left(\underline{A}^T \right)^{-1}}_P \times \underline{\dot{I}}$$

The grid equations can therefore be expressed as a function of node currents and voltages in the form:

$$\underbrace{\underline{A}^{-1} \times \underline{a}_0}_{-\underline{1}_N} \underline{\dot{U}}_0 + \underline{\dot{U}} = -\underbrace{\underline{A}^{-1}}_{\underline{P}^T} \times \underline{\dot{Z}} \times \underline{P} \times \underline{\dot{I}} \Rightarrow \underline{\dot{U}} = \underbrace{\underline{\dot{U}}_0 \underline{1}_N}_{\underline{\dot{U}}_0} - \underbrace{\underline{P}^T \times \underline{\dot{Z}} \times \underline{P}}_{\underline{\dot{Z}}_{grid}} \times \underline{\dot{I}} = \underline{\dot{U}}_0 - \underline{\dot{Z}}_{grid} \times \underline{\dot{I}}$$

Radial micro-grid equations (3)



Branch impedances	
Z_1	$1+j1 \Omega$
Z_2	$2+j2 \Omega$
Z_3	$3+j3 \Omega$
Z_4	$4+j4 \Omega$
Load currents	
I_{Lp1}	$5+j5 \text{ A}$
I_{La2}	$10+j10 \text{ A}$
I_{Lp4}	$15+j15 \text{ A}$

Equations $\underline{\dot{U}} = \underline{\dot{U}}_0 - \underline{\dot{Z}}_{grid} \times \underline{\dot{I}}$ represent the Thevenin-equivalent for the grid

Equations $\underline{\dot{I}} = \underline{\dot{Z}}_{grid}^{-1} \times (\underline{\dot{U}}_0 - \underline{\dot{U}}) = \underline{\dot{I}}_{sc} - \underline{\dot{Y}}_{grid} \times \underline{\dot{U}}$ represent the Norton-equivalent

The impedance matrix $\underline{\dot{Z}}_{grid} = \underline{P}^T \times \underline{\dot{Z}} \times \underline{P}$ is symmetrical

It can be shown that the element (m,n) of \underline{Z}_{grid} represents the common impedance of the paths connecting node 0 with nodes m and n , respectively

Distribution loss in radial micro-grids

The distribution loss is defined as:

$$P_d = \sum_{\ell=1}^N R_{\ell} J_{\ell rms}^2 = \underline{\dot{J}}^T \times \underline{R} \times \underline{\dot{J}}^* \quad \underline{R} = \text{diagonal matrix of branch resistances}$$

Branch currents \underline{J} can be expressed as a function of node currents \underline{I} as:

$$\underline{\dot{I}} = \underline{A}^T \underline{\dot{J}} \Leftrightarrow \underline{\dot{J}} = \left(\underline{A}^T\right)^{-1} \times \underline{\dot{I}} = \underline{P} \times \underline{\dot{I}}$$

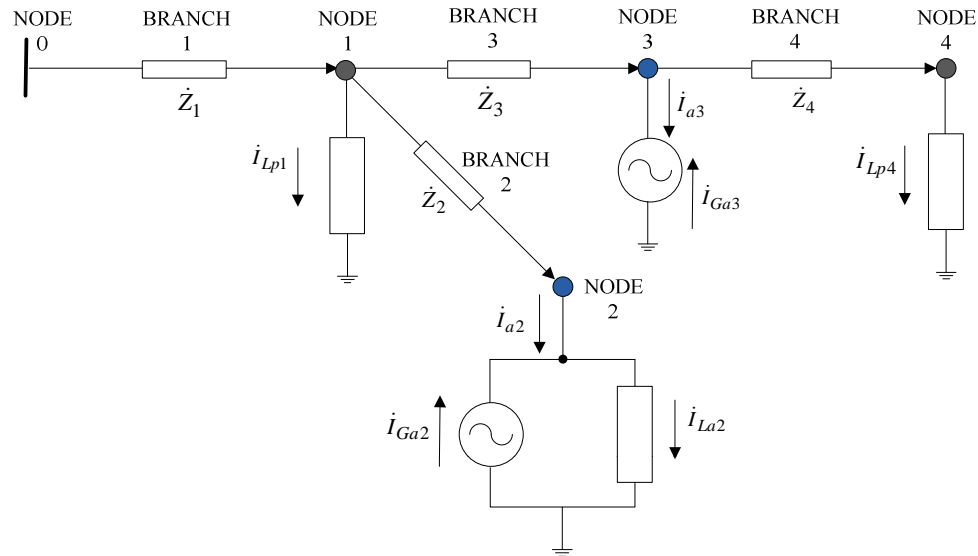
Thus:

$$P_d = \underline{\dot{I}}^T \times \underbrace{\underline{P}^T \times \underline{R} \times \underline{P}}_{\underline{R}_{grid}} \times \underline{\dot{I}}^* = \underline{\dot{I}}^T \times \underline{R}_{grid} \times \underline{\dot{I}}^*$$

Note: \underline{R}_{grid} is the real part of \underline{Z}_{grid} . In fact:

$$\underline{\dot{Z}}_{grid} = \underline{P}^T \times \underline{\dot{Z}} \times \underline{P} = \underline{P}^T \times (\underline{R} + j \underline{X}) \times \underline{P} = \underline{R}_{grid} + j \underline{X}_{grid}$$

Example (1)



Branch impedances	
Z_1	$1+j1 \Omega$
Z_2	$2+j2 \Omega$
Z_3	$3+j3 \Omega$
Z_4	$4+j4 \Omega$
Load currents	
I_{Lp1}	$5+j5 \text{ A}$
I_{La2}	$10+j10 \text{ A}$
I_{Lp4}	$15+j15 \text{ A}$

Simple microgrid, with 2 generators and 3 loads

Reduced incidence matrix

$$\underline{A} = \begin{matrix} \text{Nodes} \\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix} \\ \text{Branches} \end{matrix}$$

Remember that the slack node 0 , i.e. the PCC, is not considered in the matrix

Example (2)

$$\underline{R}_{grid} = \underline{A}^{-1} \times \underline{R} \times (\underline{A}^{-1})^T = \underline{P}^T \times \underline{R} \times \underline{P}$$

$$\underline{A}^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 \end{bmatrix} = \underline{P}^T$$

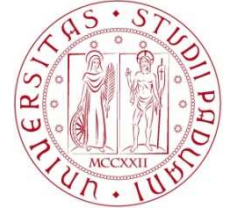
$$\underline{R}_{grid} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 3 & 1 & 1 \\ 1 & 1 & 4 & 4 \\ 1 & 1 & 4 & 8 \end{bmatrix} \Omega$$

Branch impedances	
Z_1	$1+j1 \Omega$
Z_2	$2+j2 \Omega$
Z_3	$3+j3 \Omega$
Z_4	$4+j4 \Omega$
Load currents	
I_{Lp1}	$5+j5 \text{ A}$
I_{La2}	$10+j10 \text{ A}$
I_{Lp4}	$15+j15 \text{ A}$

Matrix \underline{R} of branch resistances:

$$\underline{R} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix} \Omega$$

Example (3)



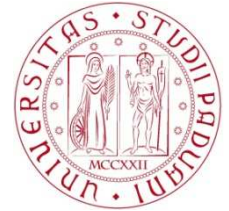
$$\underline{R}_{grid} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 3 & 1 & 1 \\ 1 & 1 & 4 & 4 \\ 1 & 1 & 4 & 8 \end{bmatrix} \Omega$$

Branch impedances	
Z_1	$1+j1 \Omega$
Z_2	$2+j2 \Omega$
Z_3	$3+j3 \Omega$
Z_4	$4+j4 \Omega$
Load currents	
I_{Lp1}	$5+j5 \text{ A}$
I_{La2}	$10+j10 \text{ A}$
I_{Lp4}	$15+j15 \text{ A}$

$$P_d = \underline{\dot{I}}^T \times \underline{R}_{grid} \times \underline{\dot{I}}^*$$

$$P_d = \begin{bmatrix} 5+j5 & 10+j10 & 0 & 15+j15 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 3 & 1 & 1 \\ 1 & 1 & 4 & 4 \\ 1 & 1 & 4 & 8 \end{bmatrix} \begin{bmatrix} 5-j5 \\ 10-j10 \\ 0 \\ 15-j15 \end{bmatrix} = 5350 \text{ W}$$

Loss analysis in meshed μG (1)



The analysis proposed for radial micro-grids can be applied to meshed micro-grids too, with a slightly different formulation.

In particular, the reduced incidence matrix is split in two sub-matrices: the tree sub-matrix \underline{A}_t and the co-tree sub-matrix \underline{A}_ℓ .

A *tree* is a generic subset of the micro-grid branches which connects all nodes and has a radial structure; the *co-tree* is the complementary subset of the micro-grid. The tree branches are called *twigs*, the co-tree branches are called *links*.

$$\underline{A} = \begin{bmatrix} \underline{A}_t \\ \underline{A}_\ell \end{bmatrix} \begin{array}{l} \leftarrow \text{Tree sub-matrix (includes all rows corresponding to twigs)} \\ \leftarrow \text{Co-tree sub-matrix (includes all rows corresponding to links)} \end{array}$$

The total distribution loss can be split in two terms, corresponding respectively to the twigs (tree) and the links (co-tree) giving:

$$P_d = \underline{j}^T \times \underline{R} \times \underline{j}^* = \begin{vmatrix} \underline{j}_t^T & \underline{j}_\ell^T \\ \underline{0} & \underline{R}_\ell \end{vmatrix} \times \begin{vmatrix} \underline{R}_t & \underline{0} \\ \underline{0} & \underline{R}_\ell \end{vmatrix} \times \begin{vmatrix} \underline{j}_t^* \\ \underline{j}_\ell^* \end{vmatrix} = \underline{j}_t^T \times \underline{R}_t \times \underline{j}_t^* + \underline{j}_\ell^T \times \underline{R}_\ell \times \underline{j}_\ell^*$$

Loss analysis in meshed μG (2)



In general, the circuit theory shows that the twig currents are depended variables, which can be expressed as a function of the node currents (absorbed by the loads or injected by the generators, which are independent variables) and the link currents (flowing in the co-tree, which are independent variables too).

Application of the superposition principle gives:

$$\underline{\dot{j}}_t^n = \left(\underline{A}_t^{-1}\right)^T \underline{\dot{i}} = \underline{P}_t \underline{\dot{i}} \quad \text{Twig currents due to node currents}$$

$$\underline{\dot{j}}_t^\ell = \left(\underline{A}_t^T\right)^{-1} \underline{A}_\ell^T \underline{\dot{j}}_\ell = \underline{P}_\ell \underline{\dot{j}}_\ell \quad \text{Twig currents due to link currents}$$

$$\underline{\dot{j}}_t = \underline{\dot{j}}_t^n + \underline{\dot{j}}_t^\ell = \underline{P}_t \underline{\dot{i}} + \underline{P}_\ell \underline{\dot{j}}_\ell \quad \text{Total twig currents}$$

Loss analysis in meshed μG (3)

Correspondingly, the distribution loss can be rewritten as:

$$\begin{aligned}
 P_d &= \underline{j}_t^T \underline{R}_t \underline{j}_t^* + \underline{j}_l^T \underline{R}_l \underline{j}_l^* = \underbrace{\left(\underline{i}_t^T \underline{P}_t^T + \underline{j}_l^T \underline{P}_l^T \right)}_{\underline{j}_t^T} \underline{R}_t \underbrace{\left(\underline{P}_t \underline{i}_t^* + \underline{P}_l \underline{j}_l^* \right)}_{\underline{j}_t^*} + \underline{j}_l^T \underline{R}_l \underline{j}_l^* = \\
 &= \underline{i}_t^T \underbrace{\left(\underline{P}_t^T \underline{R}_t \underline{P}_t \right)}_{\underline{\Omega}_t^t} \underline{i}_t^* + \underline{i}_t^T \underbrace{\left(\underline{P}_t^T \underline{R}_t \underline{P}_l \right)}_{\underline{\Omega}_t^\ell} \underline{j}_l^* + \underline{j}_l^T \underbrace{\left(\underline{P}_l^T \underline{R}_t \underline{P}_t \right)}_{\underline{\Omega}_\ell^t} \underline{i}_t^* + \underline{j}_l^T \left(\underbrace{\left(\underline{P}_l^T \underline{R}_t \underline{P}_l + \underline{R}_l \right)}_{\underline{\Omega}_\ell^\ell} \right) \underline{j}_l^*
 \end{aligned}$$

Since $\underline{\Omega}_t^\ell = \left(\underline{\Omega}_\ell^t \right)^T$, we can express the equation in the more synthetic form:

$$P_d = \underline{i}_t^T \underline{\Omega}_t^t \underline{i}_t^* + 2 \underline{j}_l^T \underline{\Omega}_\ell^t \underline{i}_t^* + \underline{j}_l^T \left(\underline{\Omega}_\ell^\ell + \underline{R}_l \right) \underline{j}_l^*$$

The distribution loss depends therefore on both **node currents** and **link currents** (twig currents have been removed from the equation).

In practice, also the link currents can be expressed as a function of the node currents, which distribute among twigs and links depending on their branch impedances.

Loss analysis in meshed μG (4)

To eliminate the dependence on the link currents it can be observed that, **if all distribution cables have the same section** (R/X constant), **the node currents distribute** among links and twigs depending on the branch resistances **in a way that necessarily minimizes the distribution losses**:

$$\frac{\partial P_d}{\partial \underline{j}_l} = 0 \Rightarrow 2 \underline{\Omega}_l^t \underline{i}^* + 2 (\underline{\Omega}_l^l + \underline{R}_l) \underline{j}_l^* = 0 \Rightarrow \underline{j}_l = -(\underline{\Omega}_l^l + \underline{R}_l)^{-1} \underline{\Omega}_l^t \underline{i}$$

$$P_d = \underline{i}^T \underline{\Omega}_t^t \underline{i}^* + 2 \underline{j}_l^T \underline{\Omega}_l^t \underline{i}^* + \underline{j}_l^T (\underline{\Omega}_l^l + \underline{R}_l) \underline{j}_l^*$$

$$P_d = \underline{i}^T \underline{\Omega}_t^t \underline{i}^* - \underline{i}^T \underline{\Omega}_t^l \left[(\underline{\Omega}_l^l + \underline{R}_l)^{-1} \right]^T \underline{\Omega}_l^t \underline{i}^* = \underline{i}^T \underline{R}_{grid}^{mesh} \underline{i}^*$$

This latter expression is formally equivalent to that applicable for radial micro-grids

Smart micro-grids

Properties, trends and local control of energy sources

8. Optimum control of smart micro-grids

Optimization goals



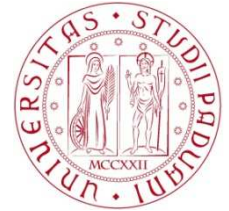
In the basic optimization process, the **distribution loss in the micro-grid** is taken as the quantity to be minimized (cost function). The motivations are:

- This is an optimum choice in terms of *energy efficiency*
- The *power consumption* of the micro-grid is minimized
- The *currents flowing in the distribution grid are minimized*; this implies that:
- The *loads are fed by the nearest sources*, which corresponds to the most *effective load power sharing among distributed generators*
- The voltage drops across the branch impedances are minimized, resulting in a *voltage stabilization* effect at all nodes of the micro-grid

The optimization will be firstly done in the assumption that a **central controller** drives all energy gateways of the micro-grid and has a *complete knowledge of grid topology and impedances*

The results of such optimization are unrealistic, since several other aspects (mentioned later) should be considered. However, this sets a **benchmark** to compare the performances of any other control technique.

Distribution Loss Minimization (1)



Ideal optimization:

- Linear micro-grid modelling
- Grid topology (matrix \underline{A}) and path impedances (matrix \underline{Z}) known to the controller
- **Unconstrained active and reactive current injection** by distributed EPPs

Let $\underline{\dot{I}}_a = \underline{K}_a \underline{\dot{I}}$ currents injected at the N_a active nodes (**energy gateways**)
 $\underline{\dot{I}}_p = \underline{K}_p \underline{\dot{I}}$ currents absorbed at the N_p passive nodes (**loads**)

Distribution loss:

$$P_d = \underline{\dot{I}}^T \underline{R}_{grid} \underline{\dot{I}}^* \rightarrow P_d = \underline{\dot{I}}_a^T \underline{R}_{a,a} \underline{\dot{I}}_a^* - 2\Re \left(\underline{\dot{I}}_a^T \underline{R}_{a,p} \underline{\dot{I}}_p^* \right) + \underline{\dot{I}}_p^T \underline{R}_{p,p} \underline{\dot{I}}_p^*$$

where:

$$\underline{R}_{a,a} = \underline{K}_a \underline{R}_{grid} \underline{K}_a^T, \quad \underline{R}_{a,p} = \underline{K}_a \underline{R}_{grid} \underline{K}_p^T, \quad \underline{R}_{p,a} = \underline{K}_p \underline{R}_{grid} \underline{K}_a^T, \quad \underline{R}_{p,p} = \underline{K}_p \underline{R}_{grid} \underline{K}_p^T, \quad \underline{R}_{a,p} = \underline{R}_{p,a}^T$$

Optimization goal

Find active node currents $\underline{\dot{I}}_a$ that minimize P_d for a given set of load currents $\underline{\dot{I}}_p$

Distribution Loss Minimization (2)

Let: $\underline{\dot{I}}_a = \underline{x} + j\underline{y}$ $\underline{\dot{I}}_p = \underline{a} + j\underline{b}$

$$\left(\frac{\partial P_d}{\partial \underline{\dot{I}}_a} = 0 \right) \Rightarrow \begin{cases} \frac{\partial P_d}{\partial \underline{x}} = 0 \Rightarrow 2\underline{R}_{a,a}\underline{x} - 2\underline{R}_{a,p}\underline{a} = 0 \\ \frac{\partial P_d}{\partial \underline{y}} = 0 \Rightarrow 2\underline{R}_{a,a}\underline{y} - 2\underline{R}_{a,p}\underline{b} = 0 \end{cases} \Rightarrow \underline{R}_{a,a}\underline{\dot{I}}_a - \underline{R}_{a,p}\underline{\dot{I}}_p = 0$$

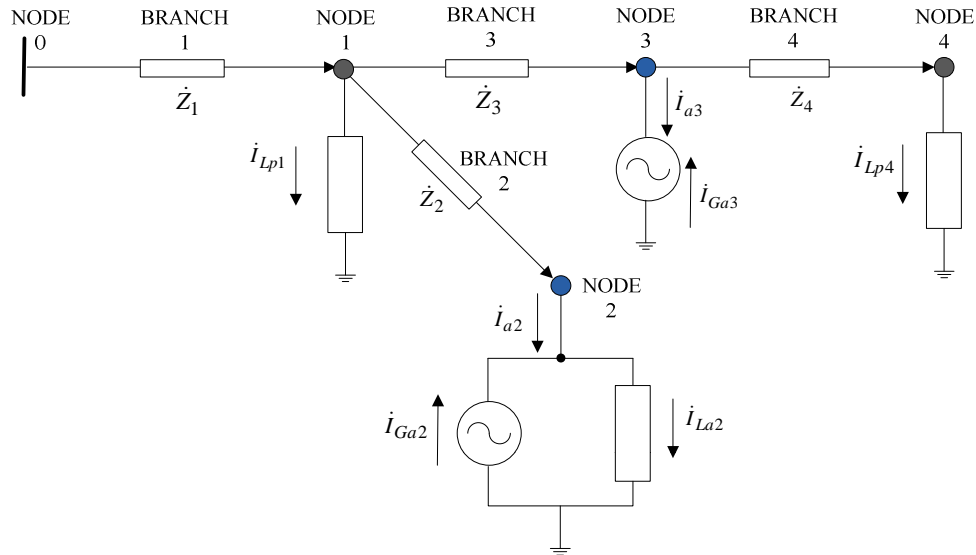
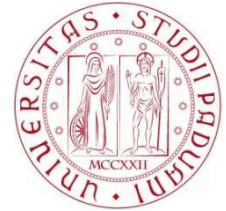
↓

$$\underline{\dot{I}}_{a,opt} = \underline{R}_{a,a}^{-1} \underline{R}_{a,p} \underline{\dot{I}}_p$$

Observe that:

- A **centralized controller** which knows topology and impedances of the micro-grid, given the load currents, can directly drive the active nodes currents (both active and reactive terms) so as to target the minimum distribution loss condition
- The distribution loss minimization can be done separately for the real (active) and imaginary (reactive) part of the injected currents. This may be **important in those cases when only reactive currents can be used for distribution loss minimization**, the active currents being constrained by power or energy limitations of the distributed energy resources (renewable sources, batteries, etc.).

Application example (1)



Branch impedances	
Z_1	$1+j1 \Omega$
Z_2	$2+j2 \Omega$
Z_3	$3+j3 \Omega$
Z_4	$4+j4 \Omega$
Load currents	
I_{Lp1}	$5+j5 \text{ A}$
I_{La2}	$10+j10 \text{ A}$
I_{Lp4}	$15+j15 \text{ A}$

Simple microgrid, with 2 generators and 3 loads

STEP 1: Reduced incidence matrix

$$\underline{A} = \begin{matrix} \text{Nodes} \\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix} \\ \text{Branches} \end{matrix}$$

Application example (2)



STEP 2: Inverse of incidence matrix

$$\underline{A}^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

STEP 3: Matrix of branch resistances:

$$\underline{R} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix} \Omega$$

STEP 4: Matrix $\underline{R}_{grid} = \underline{A}^{-1} \underline{R} (\underline{A}^{-1})^T$

$$\underline{R}_{grid} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 3 & 1 & 1 \\ 1 & 1 & 4 & 4 \\ 1 & 1 & 4 & 8 \end{bmatrix} \Omega$$

Application example (3)



Given the above matrices, the inherent distribution loss (with all inverters switched off) can be derived as a function of load currents \underline{I}_L :

$$P_{do} = \underline{\dot{I}}_L^T \underline{R}_{grid} \underline{\dot{I}}_L^* = [5 + j5 \quad 10 + j10 \quad 0 \quad 15 + j15] \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 3 & 1 & 1 \\ 1 & 1 & 4 & 4 \\ 1 & 1 & 4 & 8 \end{bmatrix} \begin{bmatrix} 5 - j5 \\ 10 - j10 \\ 0 \\ 15 - j15 \end{bmatrix} = 5350 \text{ W}$$

STEP 5: Matrices \underline{K}_a and \underline{K}_p (identify active and passive nodes)

$$\underline{K}_a = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad \underline{K}_p = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

STEP 6: Sub-matrices of \underline{R}_{grid}

$$\underline{R}_{a,a} = \underline{K}_a \underline{R}_{grid} \underline{K}_a^T, \quad \underline{R}_{a,p} = \underline{K}_a \underline{R}_{grid} \underline{K}_p^T$$

$$\underline{R}_{a,a} = \begin{bmatrix} 3 & 1 \\ 1 & 4 \end{bmatrix}, \quad \underline{R}_{a,p} = \begin{bmatrix} 1 & 1 \\ 1 & 4 \end{bmatrix}$$

$$\underline{R}_{p,a} = \underline{K}_p \underline{R}_{grid} \underline{K}_a^T, \quad \underline{R}_{p,p} = \underline{K}_p \underline{R}_{grid} \underline{K}_p^T$$

$$\underline{R}_{p,a} = \begin{bmatrix} 1 & 1 \\ 1 & 4 \end{bmatrix}, \quad \underline{R}_{p,p} = \begin{bmatrix} 1 & 1 \\ 1 & 8 \end{bmatrix}$$

Application example (4)



STEP 7: Calculation of the optimum currents to be injected at the active nodes given the currents I_p absorbed by the loads at the passive grid nodes. Let:

$$\underline{\dot{I}}_p = \begin{bmatrix} \dot{I}_{Lp1} \\ \dot{I}_{Lp4} \end{bmatrix} = \begin{bmatrix} 5 + j5 \\ 15 + j15 \end{bmatrix}$$

The optimum currents are:

$$\underline{\dot{I}}_{a,opt} = \underline{R}_{a,a}^{-1} \underline{R}_{a,p} \underline{\dot{I}}_p = \begin{bmatrix} 3 & 1 \\ 1 & 4 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 1 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} 5 + j5 \\ 15 + j15 \end{bmatrix} = \begin{bmatrix} -1.3636 - j1.3626 \\ -15.9091 - j15.9091 \end{bmatrix}$$

In practice, currents \underline{I}_a can be expressed as the difference between the currents absorbed by the loads connected at the active grid nodes (\underline{I}_{La}) and the currents injected by the distributed energy resources at the same nodes (\underline{I}_{Ga}). Thus:

$$\underline{\dot{I}}_a = \begin{bmatrix} \dot{I}_{La_2} - \dot{I}_{Ga_2} \\ \dot{I}_{La_3} - \dot{I}_{Ga_3} \end{bmatrix} \Rightarrow \begin{bmatrix} \dot{I}_{Ga_2} \\ \dot{I}_{Ga_3} \end{bmatrix} = \begin{bmatrix} 10 + j10 \\ 0 \end{bmatrix} - \begin{bmatrix} -1.3636 - j1.3626 \\ -15.9091 - j15.9091 \end{bmatrix} = \begin{bmatrix} 11.3636 + j11.3626 \\ 15.9091 + j15.9091 \end{bmatrix}$$

STEP 8: Calculation of the distribution loss in the optimum condition

$$P_{d,opt} = \underline{\dot{I}}_a^T \underline{R}_{a,a} \underline{\dot{I}}_a^* + 2 \Re \left(\underline{\dot{I}}_a^T \underline{R}_{a,p} \underline{\dot{I}}_p^* \right) + \underline{\dot{I}}_p^T \underline{R}_{p,p} \underline{\dot{I}}_p^* = 1827.3 \text{ W}$$

Remarks



The above loss minimization approach represents a first step towards optimum control. In practice, the optimization procedure can be extended to consider also:

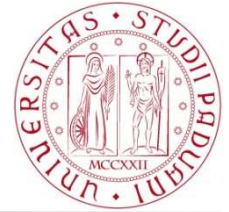
- **islanded operation**, when the micro-grid is disconnected by the utility ($i_0 = 0$)
- **inverter losses**, which affect the distribution efficiency since the inverters manage the full power generated by the distributed energy resources
- **current capability of the inverters**, which actually limits the active and reactive power deliverable at the active grid nodes
- **actual power capability of distributed generators and energy capability of distributed energy sources**, which constraint the active power deliverable at the active grid nodes
- other aspects, like *intermittent power generation* of renewable sources, *lifetime optimization of storage batteries*, *daily cost of energy* and *revenues from power trading* that might influence the optimization process in a wider perspective, both technical and economic

Smart micro-grids

Properties, trends and local control of energy sources

9. On-line Identification of micro-grid parameters

Identification goals



The previous optimization has been done in the assumption that a **central controller** has a *complete knowledge of grid topology and impedances*.

In this section we analyze some techniques which allow **on-line evaluation of node-to-node distances and identification of micro-grid topology**.

These techniques take advantage of the capabilities of modern *powerline communication* (PLC) technologies, which are particularly suited for micro-grid applications.

In fact, in low-voltage residential micro-grids, *the same power lines connecting the users can be used to convey data*. The small distances between users and the absence of transformers make possible a direct powerline communication among grid nodes, *without requiring any additional communication infrastructure*.

The on-line identification approach can also be extended to *estimate the line impedances*.

However, in a residential micro-grid the size of the distribution cables is usually the same, thus the *knowledge of node-to-node distances is sufficient* to run the optimum control algorithm, as well as the distributed quasi-optimum control techniques which will be discussed in the next sections.

Node-to-node communication

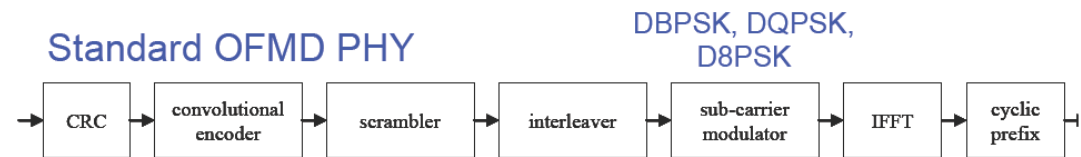
- Node-to-node **communication architecture**
- Node-to-node **distance measurement**



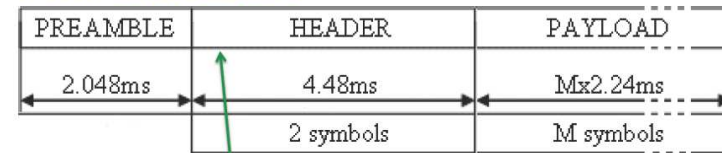
Standard PRIME
 (PowerLine Intelligent
 Metering Evolution)

PRIME overview:

- Designed for outdoor applications
- OFDM physical layer
- Maximum bit rate **128kbps**
- Transmission over CENELEC A band, in the range 45kHz-92kHz with 97 equally spaced sub-carriers
- MAC layer needs to be “customized” to fit peer-to-peer communication (PRIME is originally master-slave)



PHY Packet Structure



Symbols=OFDM 288bits
 symbols – **M<64**

Node-to-node **distance measurement**: PLC enables the use of **TOA (Time Of Arrival)** techniques, currently under testing over $\approx 1\text{km}$ of real distribution cables in the Smart Micro-Grid Facility at DEI



Node-to node distance measurement

The knowledge of **grid map (incidence matrix)**, node-to-node **distances** and **branch impedances** is generally required to implement loss minimization techniques.

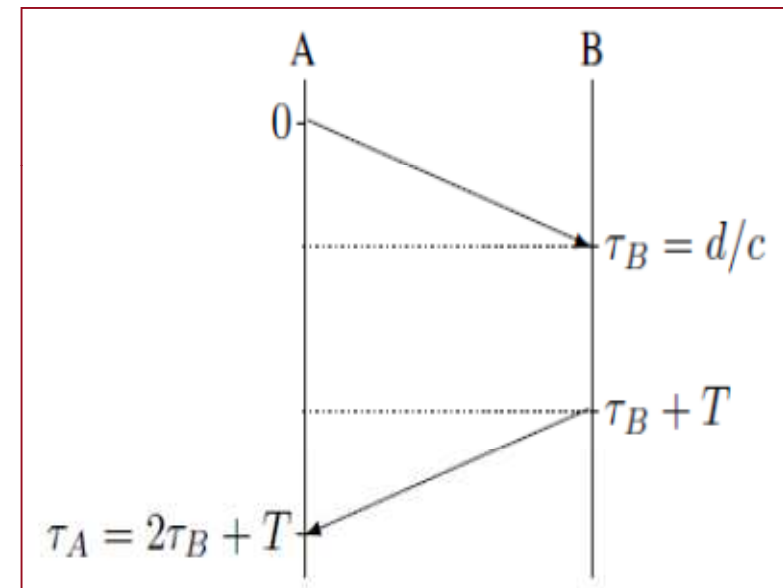
In practice, the knowledge of branch impedances is not required if the distribution lines have **constant section**. In this case, node-to-node distances are sufficient.

PLC-based distance measurement by Time of Arrival (TOA) ranging technique

- Node A broadcasts a data packet, which is received by node B at time τ_B
- Node B waits a fixed time T and then replies to A with another data packet.
- A receives the packet at time $\tau_A = 2\tau_B + T$
- Time τ_B depends on the distance d_{AB} between nodes A and B by the relation $\tau_B = d_{AB}/c$, with c the speed of light.

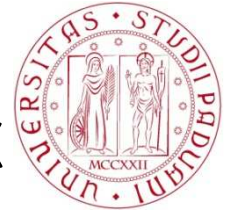
Thus:

$$d_{AB} = \frac{c(\tau_A - T)}{2}$$



Distance measurement accuracy
1.5-10 m

Neighbours map/Incidence matrix



Grid mapping algorithm

- If the ranging procedure is repeated for each pair of nodes in the micro-grid, the **distance matrix \underline{D}** can be determined, whose generic element d_{mn} gives the distance between nodes m and n .
- We say that **two nodes n and m are neighbors** if their distance is the minimum among the lengths of all paths connecting them, i.e.:

$$d_{nm} < d_{nk} + d_{km}, \quad k = 1 \dots N$$

- Neighbor nodes are directly connected by a branch of the distribution grid, thus **each pair of neighbor nodes identifies a row of the complete incidence matrix \underline{A}_c** .
- The reduced incidence matrix \underline{A} is then obtained by suppressing the column corresponding to node 0 (slack node).
- Finally, the tree and co-tree sub-matrices \underline{A}_t and \underline{A}_l are derived by partitioning \underline{A} into a full-rank (*tree*) sub-matrix and the residual (*co-tree*) sub-matrix.

Smart micro-grids

Properties, trends and local control of energy sources

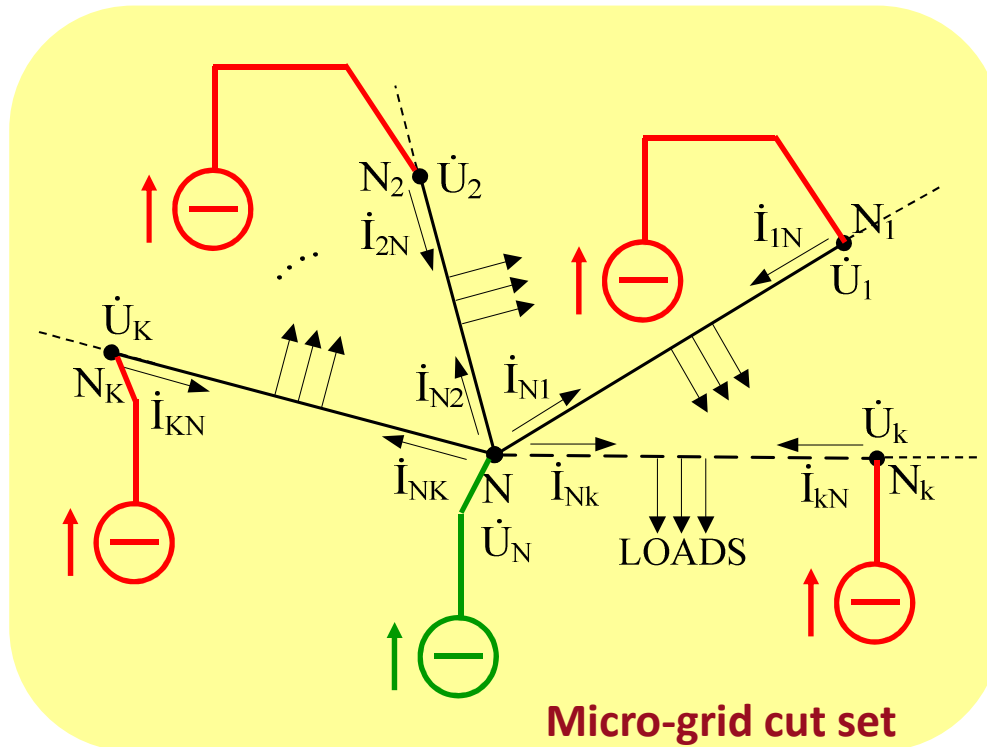
10. Distributed surround control of smart micro-grids

Introduction



- In this section we analyze a **distributed plug & play control** technique, called **surround control**, which provides *local minimization of the distribution losses*, resulting in a quasi-optimum operation of the entire micro-grid.
- The technique requires that every grid node, both active and passive, is equipped with a *smart meter*, i.e., a local measurement unit capable of data processing and powerline communication.
- This allows identification of both the incidence matrix (*network topology*) and the distance matrix (*node-to-node distances*), extended to active and passive nodes.
- Given the incidence matrix, each active node identifies the *neighbor nodes*, i.e., the active nodes connected by a direct link and the passive nodes fed by such links.
- Then, a local optimum control algorithm is applied, which only requires *data exchange among neighbor nodes*.
- The proposed control technique ensures **flexibility and scalability**, i.e., it can be applied irrespective of micro-grid architecture, and automatically adapts when a new node is implemented in the micro-grid.

Token ring sequential control



The distributed EPPs (grid-connected inverters) operate as current sources (to stabilize the grid impedances)

- The distributed grid-connected inverters cyclically update their ac current references (**control phase**).
- Outside the control phase, the inverters keep constant their ac current references (**hold phase**).
- When an inverter is in the control phase, the neighbors keep the hold phase. This prevents possible detrimental control interactions.

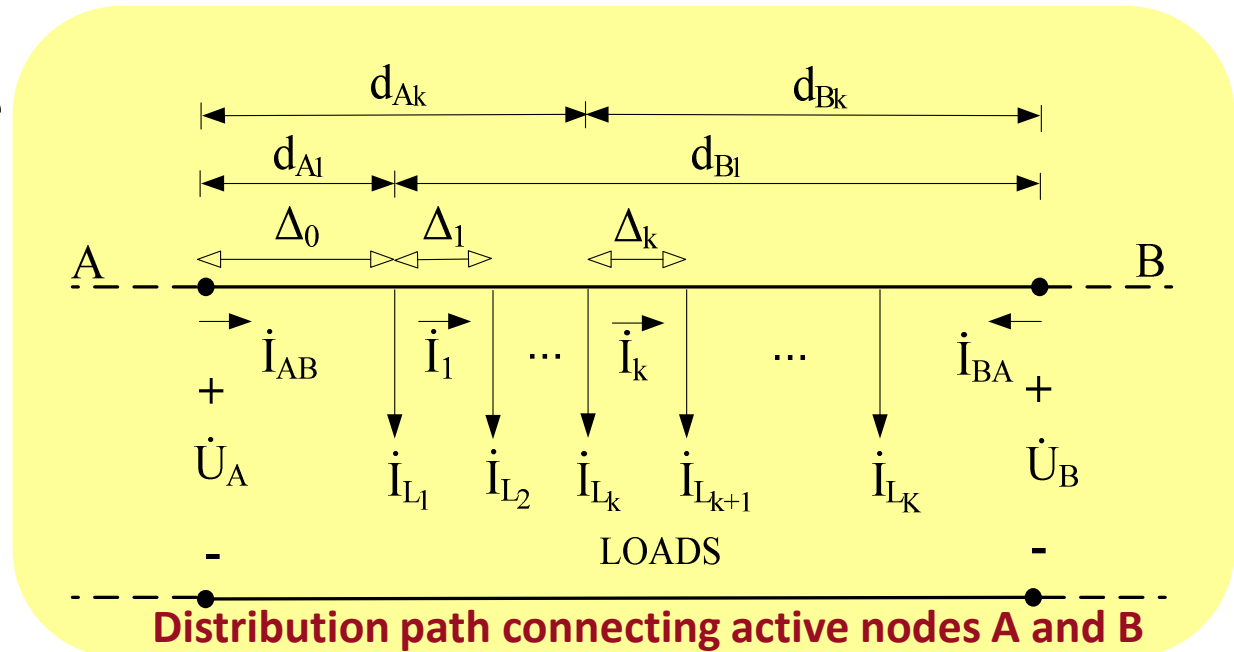
Conduction loss in distribution lines

Let r_{AB} be the resistance per unit of length of the line, the distribution loss in the line between active nodes A and B is given by:

$$\begin{aligned}
 P_{LOSS} &= \sum_{k=0}^K r_{AB} \Delta_k |\dot{i}_k|^2 = \\
 &= \sum_{k=0}^K r_{AB} \Delta_k \dot{i}_k \dot{i}_k^*
 \end{aligned}$$

where:

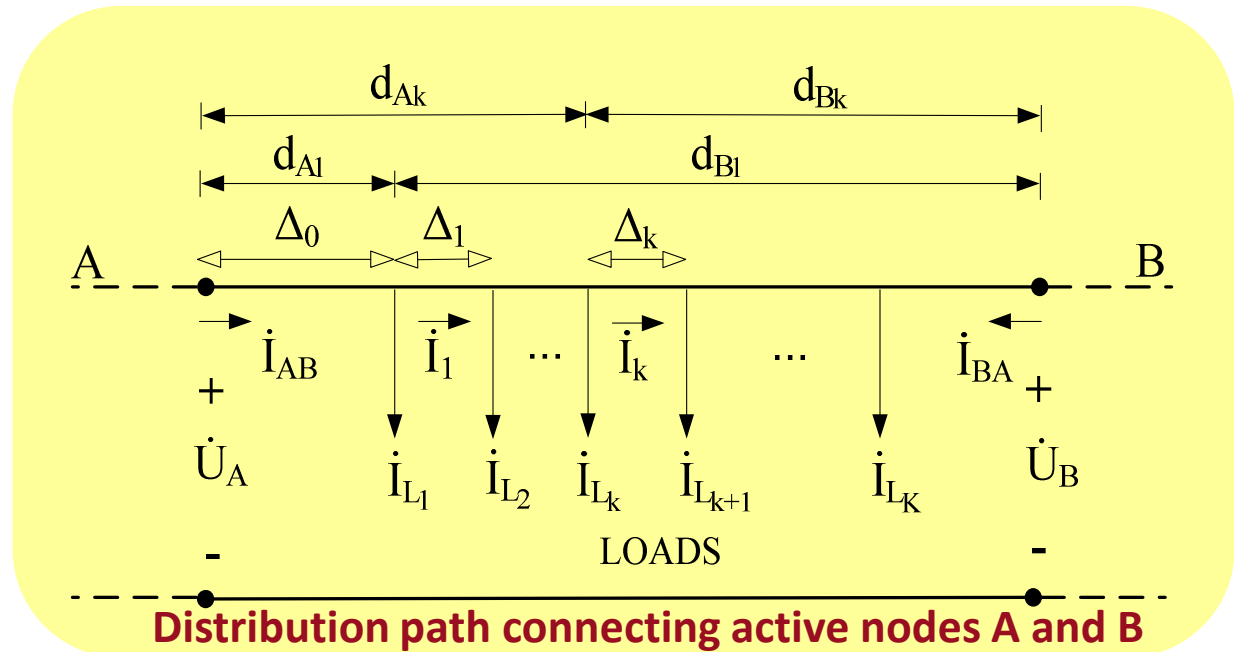
$$\begin{cases}
 \dot{i}_k = \dot{i}_{AB} - \sum_{\ell=1}^k \dot{i}_{L_\ell} \\
 \dot{i}_k = - \left(\dot{i}_{BA} - \sum_{\ell=k+1}^K \dot{i}_{L_\ell} \right)
 \end{cases}$$



Given the currents absorbed by the passive loads fed along path A-B, the distribution loss in path A-B can therefore be expressed as a function of active node current I_{AB} (or I_{BA}).

Optimization goal: find the values of I_{AB} and I_{BA} that minimize the conduction losses in path A-B

$$\frac{\partial P_{LOSS}}{\partial \dot{I}_{AB}} = 0 \quad \frac{\partial P_{LOSS}}{\partial \dot{I}_{BA}} = 0$$



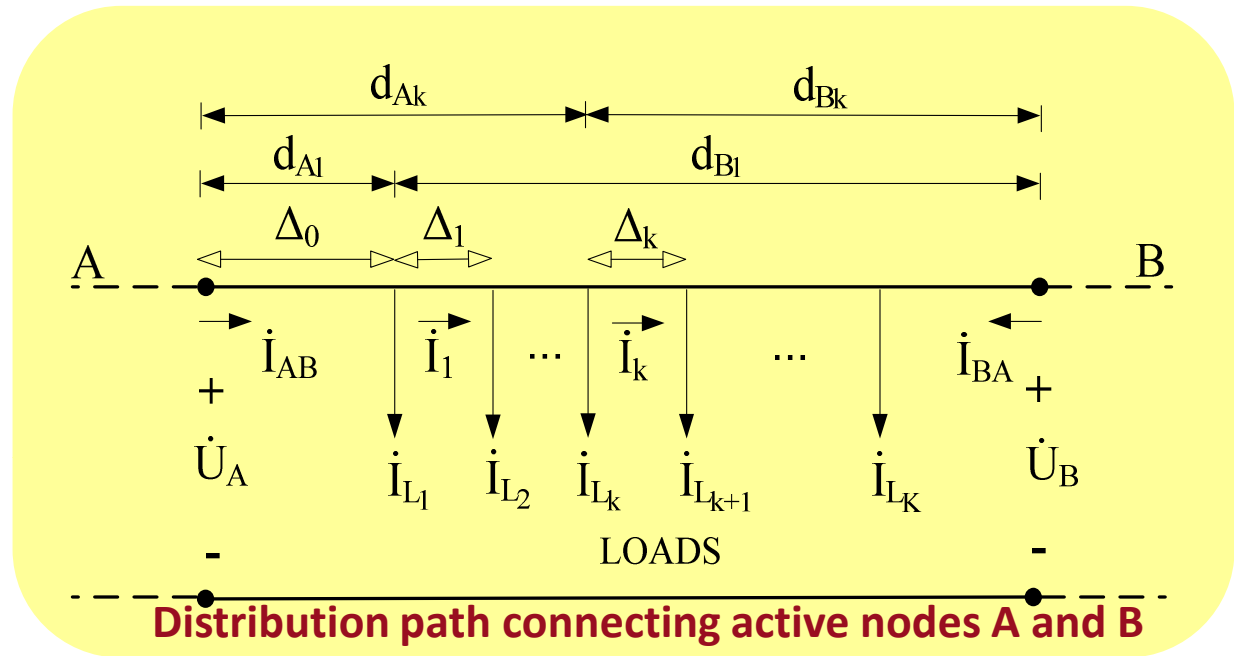
$$\begin{cases} \dot{I}_{AB}^{opt} = \frac{1}{d_{AB}} \sum_{k=1}^K \dot{I}_{Lk} d_{Bk} \\ \dot{I}_{BA}^{opt} = \frac{1}{d_{AB}} \sum_{k=1}^K \dot{I}_{Lk} d_{Ak} \end{cases}$$

The optimum node currents depend only on the loads and their distribution along path A-B

Moreover:

$$\begin{cases} \dot{I}_{AB} = \dot{I}_{AB}^{opt} \\ \dot{I}_{BA} = \dot{I}_{BA}^{opt} \end{cases} \Leftrightarrow \dot{U}_A = \dot{U}_B$$

Loss minimization in distribution lines (2)



In general, nodes A and B are not equipotential, thus:

$$\dot{i}_{AB} = \dot{i}_{AB}^{opt} + \frac{\dot{U}_A - \dot{U}_B}{\dot{z} d_{AB}} = \dot{i}_{AB}^{opt} + \dot{i}_{AB}^{circ}$$

$$\dot{i}_{BA} = \dot{i}_{BA}^{opt} + \frac{\dot{U}_B - \dot{U}_A}{\dot{z} d_{AB}} = \dot{i}_{BA}^{opt} + \dot{i}_{BA}^{circ}$$

Impedance per unit of length of distribution line

Circulation current

Optimum current

Loss minimization in cut sets (2)

Consider a cut-set of the micro-grid

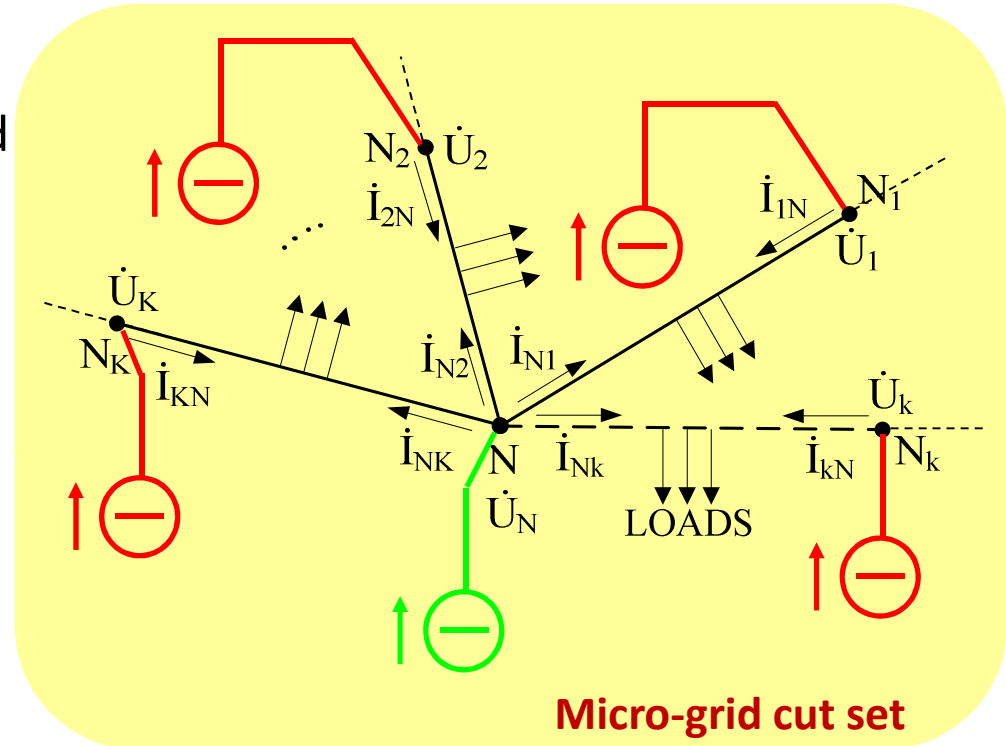
The current at node N can be expressed

as:

$$\dot{I}_N = \sum_{k=1}^K \dot{I}_{Nk} = \underbrace{\sum_{k=1}^K \dot{I}_{Nk}^{opt}}_{\dot{I}_N^{opt}} + \underbrace{\sum_{k=1}^K \frac{\dot{U}_N - \dot{U}_k}{\dot{Z}_k}}_{\dot{I}_N^{circ}}$$



Depends on loads connected to paths $L_1 - L_K$

Depends on voltage differences



Minimum distribution loss condition

$$\dot{I}_N^{circ} = 0 \Rightarrow \begin{cases} \dot{I}_N = \dot{I}_N^{opt} \\ \dot{U}_N = \dot{U}_N^{opt} = \frac{\sum_{k=1}^K \frac{\dot{U}_k}{\dot{Z}_k}}{\sum_{k=1}^K \frac{1}{\dot{Z}_k}} \end{cases}$$

-  EPP in control phase
-  EPPs in hold phase

Node current/voltage optimization



Node current optimization

$$\dot{I}_N = \dot{I}_N^{opt} = \sum_{k=1}^K \dot{I}_{Nk}^{opt} = \sum_{k=1}^K \frac{1}{d_{Nk}} \sum_{m=1}^{M^{Nk}} \dot{I}_{L_m}^{Nk} d_m^{Nk}$$

This equation holds separately for active and reactive terms, thus optimization can be done by acting on active currents, reactive currents, or both

Node voltage optimization

$$\dot{U}_N = \dot{U}_N^{opt} = \frac{\sum_{k=1}^K \frac{\dot{U}_k}{\dot{Z}_k}}{\sum_{k=1}^K \frac{1}{\dot{Z}_k}} \approx \frac{\sum_{k=1}^K \frac{\dot{U}_k}{d_k}}{\sum_{k=1}^K \frac{1}{d_k}}$$

This method is very sensitive to voltage measurement errors

The computation of optimum node current (EPP reference current) requires *distance estimation* (ranging), *local grid mapping*, and *current measurement* at surrounding passive nodes)

The computation of optimum node voltage (EPP reference voltage) requires *local grid mapping*, knowledge of *path impedances* (or *node-to-node distances*), and *voltage measurement* at surrounding active nodes

Node current/voltage optimization



Node current optimization

$$\dot{I}_N = \dot{I}_N^{opt} = \sum_{k=1}^K \dot{i}_{Nk}^{opt} = \sum_{k=1}^K \frac{1}{d_{Nk}} \sum_{m=1}^{M^{Nk}} \dot{i}_{L_m}^{Nk} d_m^{Nk}$$

Optimum current control does not excite network dynamics !

In fact injecting currents at the grid nodes affects marginally the node voltages, thus grid operation is not influenced.

Node voltage optimization

$$\dot{U}_N = \dot{U}_N^{opt} = \frac{\sum_{k=1}^K \frac{\dot{U}_k}{\dot{Z}_k}}{\sum_{k=1}^K \frac{1}{\dot{Z}_k}} \approx \frac{\sum_{k=1}^K \frac{\dot{U}_k}{d_k}}{\sum_{k=1}^K \frac{1}{d_k}}$$

Optimum voltage control does excite network dynamics !

In fact, changing the voltage at node N may cause significant variations of the line currents, thus affecting also the voltages of the other nodes in the micro-grid.

Current/voltage relation at node N



1. Given the optimum node voltage and current, assuming the same impedance z per unit of length for all distribution paths, from the measured voltage and current at node N we estimate this impedance as:

$$\dot{z} = \frac{\dot{U}_N - \dot{U}_N^{opt}}{\dot{I}_N - \dot{I}_N^{opt}} \sum_{k=1}^K \frac{1}{d_{Nk}}$$

2. The Thevenin equivalent circuit at node N (characterized by internal impedance and no-load voltage) can be determined as:

$$\dot{Z}_N^{eq} = \dot{z} \left(\sum_{k=1}^K \frac{1}{d_{Nk}} \right)^{-1}$$

$$\dot{U}_N^o = \dot{U}_N^{opt} - \dot{Z}_N^{eq} \dot{I}_N^{opt}$$

3. The general relation between voltage and current at node N is expressed by:

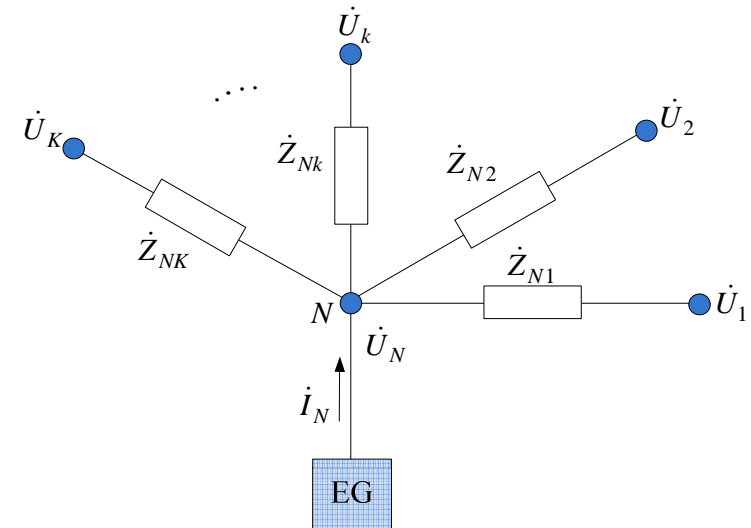
$$\dot{U}_N = \dot{U}_N^o + \dot{Z}_N^{eq} \dot{I}_N$$

This latter equation allows conversion of voltage references into current references and vice versa (current-mode \Leftrightarrow voltage-mode control)

Surround control implementation

Token ring control

- A token moves along the micro-grid, and only the active node (N) keeping the token is enabled to modify its current reference according to the minimum distribution loss criterion
- When an active node receives the token, it:
 1. collects voltage phasors from neighbour nodes
 2. measures (or recalls) the distances from neighbour nodes
 3. computes the optimum voltage reference
 4. computes the current reference variation needed to reach the optimum voltage
 5. sends the token to the next active node



$$\dot{U}_N^{opt} \approx \frac{\sum_{k=1}^K \dot{U}_k}{\sum_{k=1}^K \frac{1}{d_{Nk}}}$$

$$\Delta \dot{I}_N = \frac{\dot{U}_N^{opt} - \dot{U}_N}{\dot{Z}_N^{eq}}$$

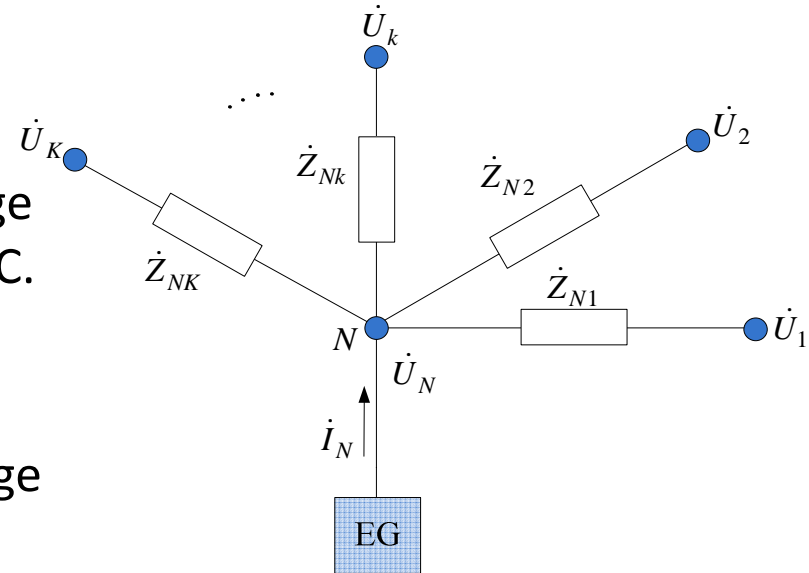
Convergence of surround control



Convergence of control algorithm

- **Note:** The minimum loss condition is reached when **all nodes are equipotential**, their voltage being equal to the voltage impressed at the PCC.
- The control theory shows that this condition is progressively approached if the nodes which sequentially receive the token drive their voltage toward the value:

$$\dot{U}_N^{ref} = \sum_{k \in [1, K]} b_{Nk} \dot{U}_k$$



- The choice of coefficients b_{Nk} defines **how fast the algorithm converges to the steady state optimum condition**.
- The convergence condition for the control algorithm is:
- This condition is satisfied with surround control since we assume:

$$\sum_{k \in [1, K]} b_{kN} = 1, b_{kN} \geq 0$$

$$b_{Nk} = \frac{1}{d_{Nk}} \bigg/ \sum_{k=1}^K \frac{1}{d_{Nk}}$$

Smart micro-grids

Properties, trends and local control of energy sources

11. Distributed cooperative control of smart micro-grids

Surround Control ensures minimum distribution loss, but requires a full knowledge of the micro-grid topology, which requires data exchange among all grid nodes.

Moreover, it has **strict requirements** in terms of node-to-node communication and synchronization (PMU, Phasor Measurement Unit), which are not easily satisfied with cheap commercial technology.

Question: there is a different distributed control technique which has easier implementation and still keeps good performances?
(**Sub-optimum** solution)

Remark: Beyond the mathematical analysis, an intuitive interpretation of distribution loss minimization is that **“the distribution loss reduces if the loads are supplied by the generators nearby”**

Principle of cooperative control



Cooperative control approach

1. Each load m splits its active and reactive power demand P_m and Q_m among the active nodes n in inverse proportion to their distances:

$$P_m^n = \frac{P_m}{d_m^n \underbrace{\left(\sum_{n=0}^N \frac{1}{d_m^n} \right)^{-1}}_{d_m^{eq}}} = P_m \frac{d_m^{eq}}{d_m^n} \Rightarrow \sum_{n=1}^N P_m^n = P_m$$

$$Q_m^n = \frac{Q_m}{d_m^n \left(\sum_{n=0}^N \frac{1}{d_m^n} \right)^{-1}} = Q_m \frac{d_m^{eq}}{d_m^n} \Rightarrow \sum_{n=1}^N Q_m^n = Q_m$$

2. Each active node n , within its current capability, supplies the total power requested by the passive loads:

$$P_n = \sum_{m=1}^M P_m^n = \sum_{m=1}^M P_m \frac{d_m^{eq}}{d_m^n} \quad Q_n = \sum_{m=1}^M Q_m^n = \sum_{m=1}^M Q_m \frac{d_m^{eq}}{d_m^n}$$

Advantages of cooperative control

- *Use of PMUs* (phasor measurement units) *can be avoided*, since the loads address their requests in terms of active and reactive power, which are conservative quantities and do not depend on the phase of the node voltages.
- There is *no need for micro-grid topology identification*, since only the node-to-node distances are requested to implement the control algorithm.

Disadvantage of cooperative control

- The solution can diverge from the optimum condition in case of saturation of the current capability of the inverters.

Upgrade of cooperative control

- The saturation conditions must be properly managed by shifting the power requests from the saturated active nodes to the non-saturated nodes.

Managing saturation

The splitting algorithm of the load power is modified as follows:

$$P_m^n = P_m \frac{\beta_{nP}}{d_m^n} \bigg/ \sum_{n=0}^N \frac{\beta_{nP}}{d_m^n} \quad Q_m^m = Q_m \frac{\beta_{nQ}}{d_m^n} \bigg/ \sum_{n=0}^N \frac{\beta_{nQ}}{d_m^n}$$

where:

$$\begin{aligned}
 \beta_{nP}(k) &= \beta_{nP}(k-1) \cdot \alpha_{nP}(k) & \beta_{nP_{min}} \leq \beta_{nP}(k) \leq 1 & & \alpha_{nP}(k) = P_{n,MAX} / P_n(k-1) \\
 \beta_{nQ}(k) &= \beta_{nQ}(k-1) \cdot \alpha_{nQ}(k) & \beta_{nQ_{min}} \leq \beta_{nQ}(k) \leq 1 & \text{and} & \alpha_{nQ}(k) = Q_{n,MAX} / Q_n(k-1)
 \end{aligned}$$

k = sampling interval (sampling frequency = 10 Hz)

- Coefficients α express the residual power capability of active nodes ($\alpha < 1$ means saturated current capability).
- Coefficients β represent the corrective terms applied to the ideal power distribution criterion (inverse of distance). $\beta < 1$ means limited contribution due to saturation, $\beta = 1$ means full contribution.
- At every sampling interval coefficients β are updated: they can be further reduced if saturation still holds, while can be increased (up to 1) if saturation disappears (e.g., due to a reduction of load power request).

Managing saturation

The splitting algorithm of the load power is modified as follows:

$$P_m^n = P_m \frac{\beta_{nP}}{d_m^n} \bigg/ \sum_{n=0}^N \frac{\beta_{nP}}{d_m^n} \quad Q_n^m = Q_m \frac{\beta_{nQ}}{d_m^n} \bigg/ \sum_{n=0}^N \frac{\beta_{nQ}}{d_m^n}$$

where:

$$\begin{aligned} \beta_{nP}(k) &= \beta_{nP}(k-1) \cdot \alpha_{nP}(k) & \beta_{nP_{min}} &\leq \beta_{nP}(k) \leq 1 & \alpha_{nP}(k) &= P_{n,MAX} / P_n(k-1) \\ \beta_{nQ}(k) &= \beta_{nQ}(k-1) \cdot \alpha_{nQ}(k) & \beta_{nQ_{min}} &\leq \beta_{nQ}(k) \leq 1 & \alpha_{nQ}(k) &= Q_{n,MAX} / Q_n(k-1) \end{aligned} \quad \text{and}$$

Advantages

- The power limits of the active nodes are automatically met
- Recovery from saturation happens quickly
- Load power requests are met precisely
- The power splitting criterion approaches the “minimum distance” criterion as close as possible, within the power limits of the active nodes
- Control is inherently stable

Smart micro-grids

Properties, trends and local control of energy sources

12. Simulation results

Simulation approach



Assumptions

- The proposed control techniques have been validated by simulation in the Matlab – Simulink environment.
- To minimize the complexity of simulation and to reduce the simulation times a **phasorial simulation** tool has been developed.
- The graphs showing the time behaviour of the system represent must be interpreted as **sequences of steady states** (quasi-stationary behavior), where fast dynamics are neglected.

Simulation Example (1)

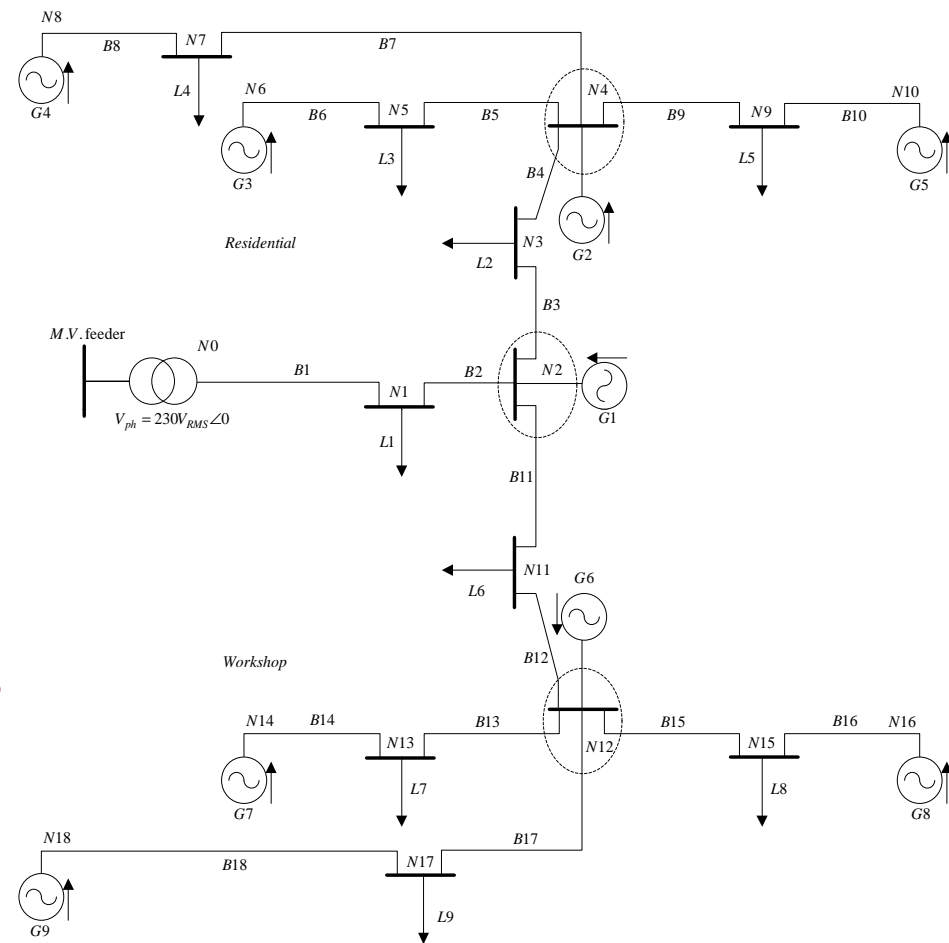
DG	P_{MAX} kW	S_{MAX} kVA	Load $Z=R+j\omega L$	Power @ $230V_{RMS}$
G1	1	2	L1	5kW $\cos\phi=0.91$
G2	1	2	L2	5kW $\cos\phi=0.91$
G3	3	5	L3	2.5kW $\cos\phi=0.96$
G4	3	5	L4	2.5kW $\cos\phi=0.96$
G5	3	5	L5	2.5kW $\cos\phi=0.96$
G6	1	2	L6	5kW $\cos\phi=0.91$
G7	10	15	L7	10kW $\cos\phi=0.80$
G8	10	15	L8	10kW $\cos\phi=0.80$
G9	10	15	L9	10kW $\cos\phi=0.80$

$P_{RL} = 52.5kW$ $\cos\phi_{RL} = 0.857$ **Total Loads**

$P_{RG} = 55kW$ $S_{RG} = 85kVA$ **Total DERs / EPPs**

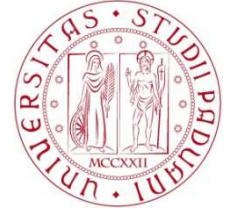
$r = 0.08\Omega/km$
 $S = 240mm^2$ $l = 255\mu H/km$
 $\Phi = \pi/4$ rad

18-bus LV network



Total length of distribution line **1.8km**

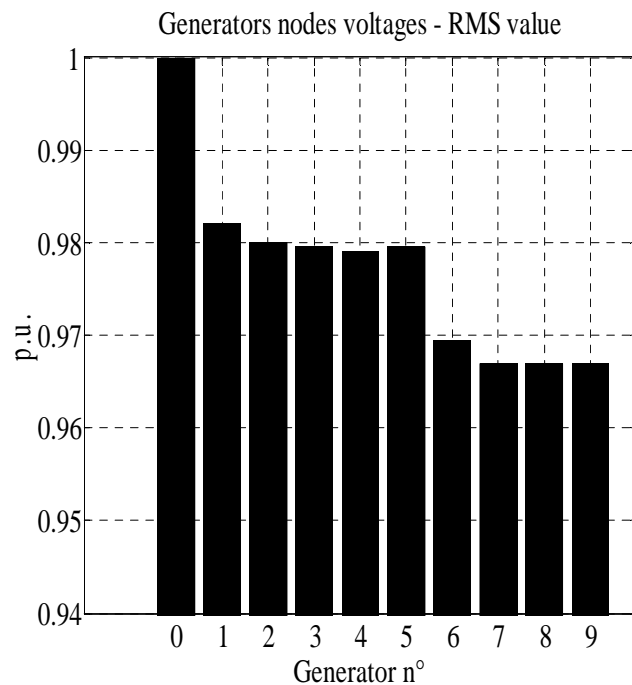
Simulation Example (2)



Initial situation: Inverters OFF

Loss: $P_{R_{\max}} = 1.2\text{kW}$

Voltages:



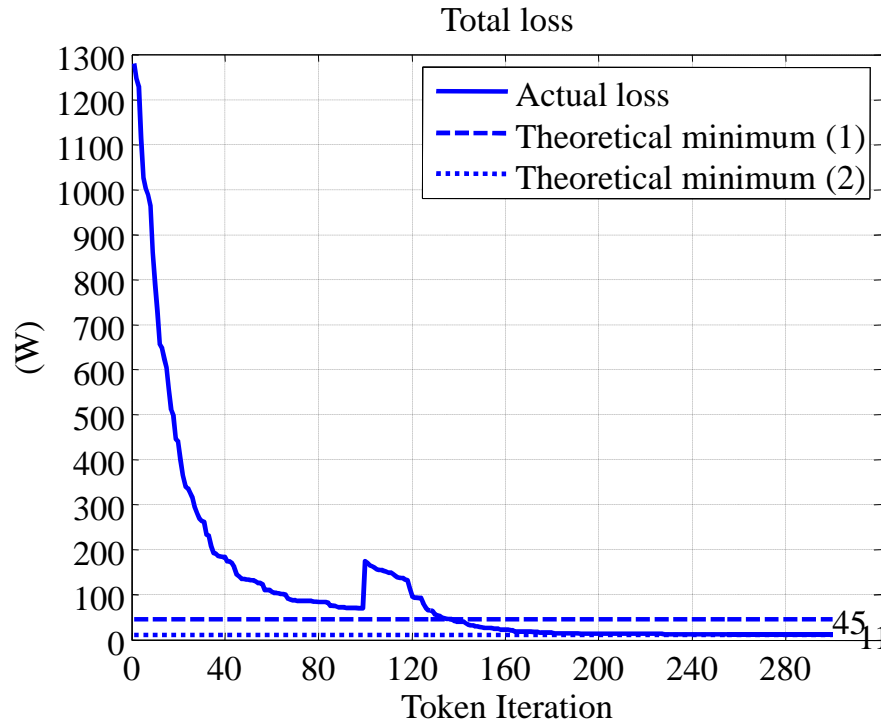
The distributed control techniques are analyzed in specific operating conditions, their performance being compared with those of optimum control.

Two cases are considered:

- **Active and Reactive** current control constrained only by **converters saturation** (to show the achievable performances in a real system with power generation & energy storage)
 - In practice, actual active power capability is determined by energy storage & generated power constraints (sun, wind, batteries, etc), while reactive power can be regulated within the current capability of the inverters.
- **Purely reactive current control** constrained by converters saturation (to show the achievable improvement without power generation & energy storage)

The actual micro-grid performances are intermediate between these two cases

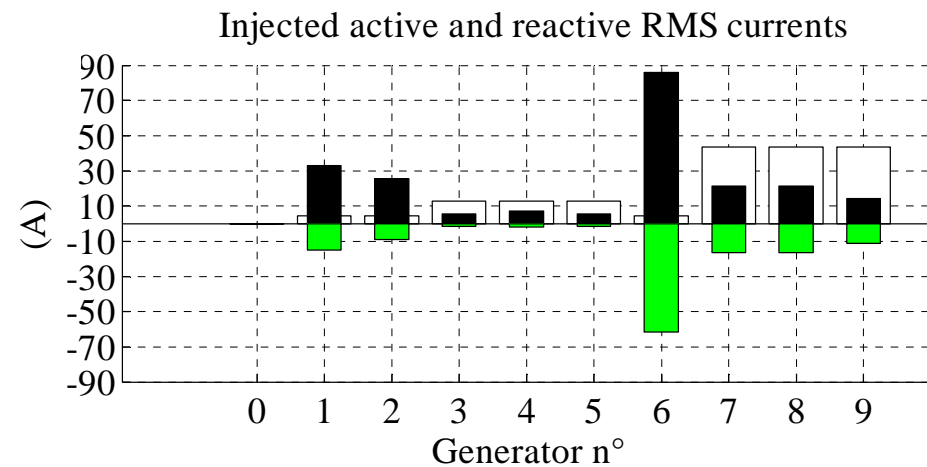
Surround control (1)



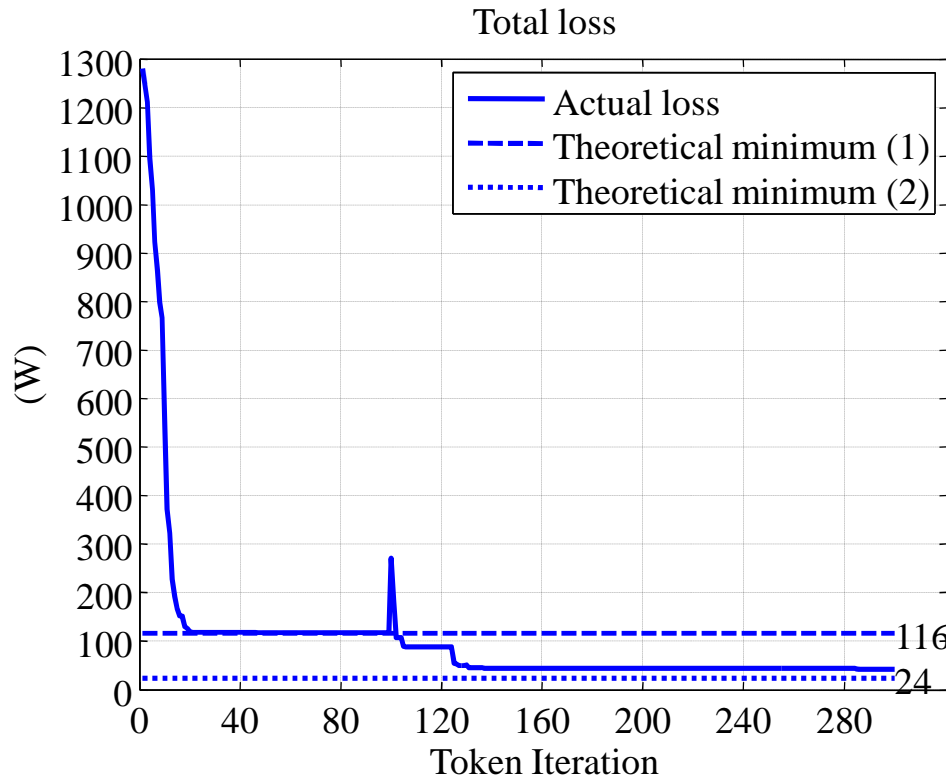
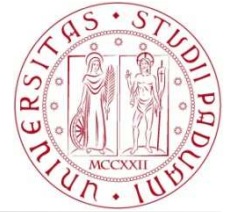
**After 100 iterations all loads reduce
 their power absorption to 50% of the
 nominal ratings**

Surround control neglecting saturation Active and reactive current control

A Token Ring approach is adopted, where
 the 9 generators are activated and updated
 in sequence. Every 9 token jumps, the loads
 update their current demands.



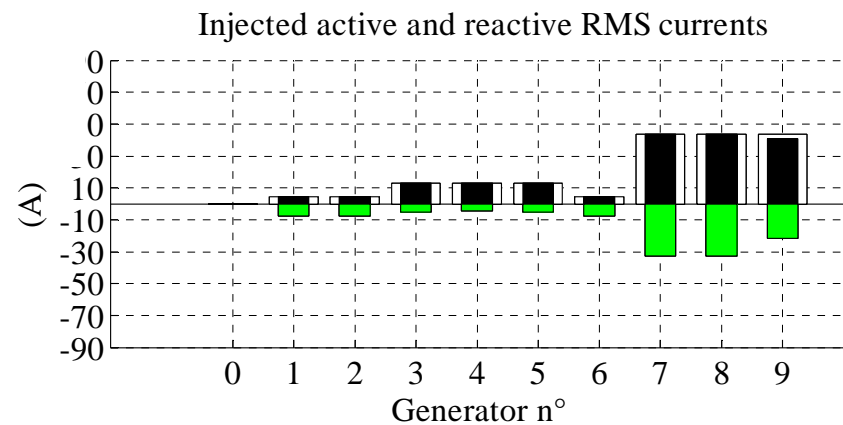
Surround control (2)



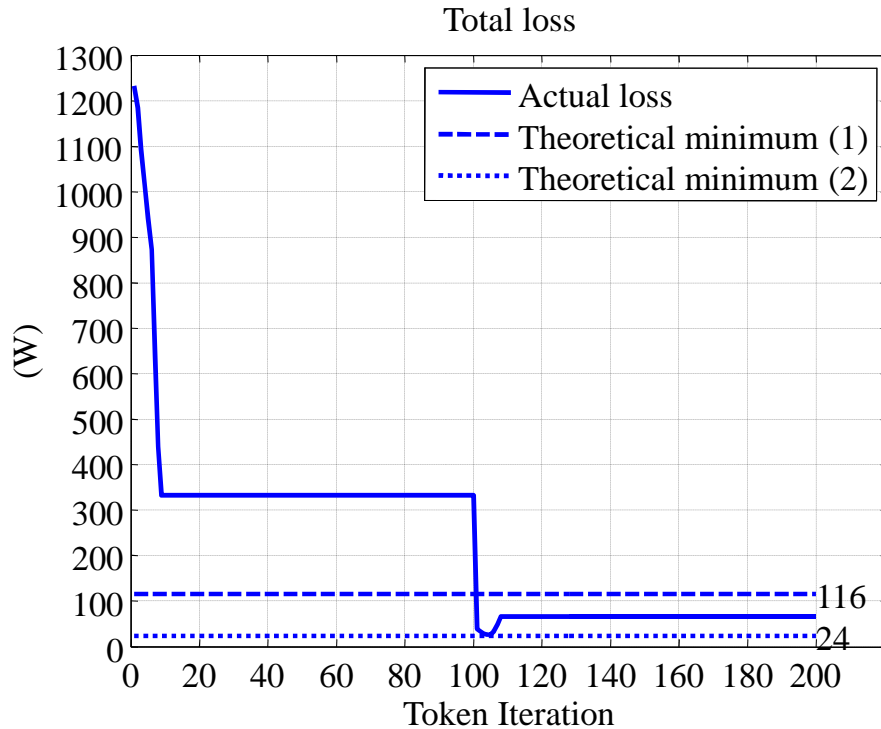
**After 100 iterations all loads reduce
 their power absorption to 50% of the
 nominal ratings**

Surround control considering saturation Active and reactive current control

A Token Ring approach is adopted, where
 the 9 generators are activated and updated
 in sequence. Every 9 token jumps, the loads
 update their current demands.



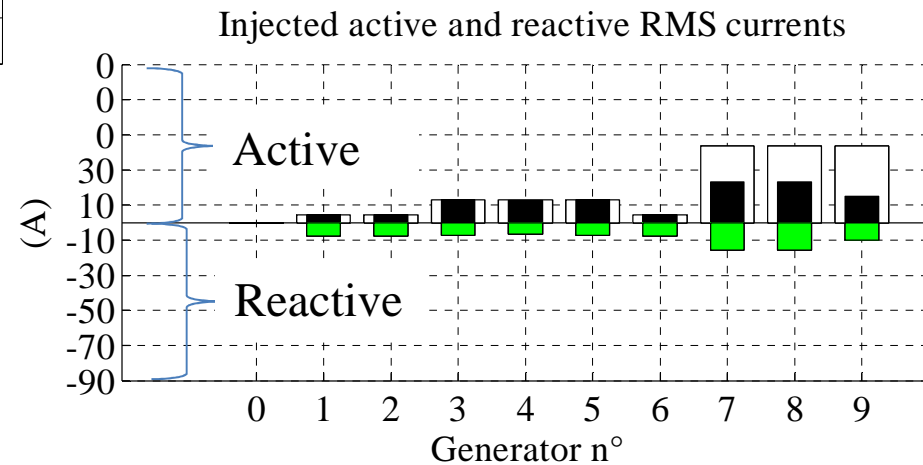
Cooperative Control (1)



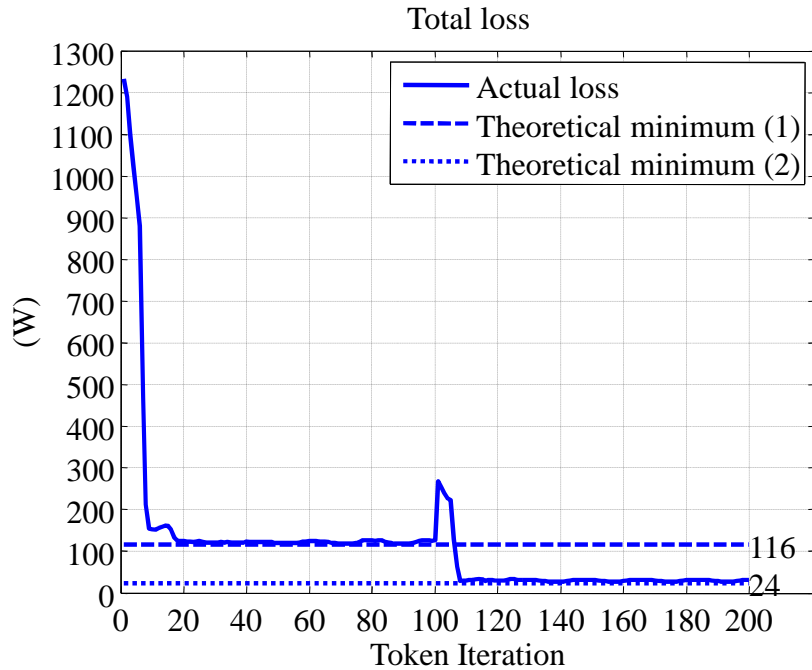
**After 100 iterations all loads reduce
 their power absorption to 50% of the
 nominal ratings**

Cooperative control without saturation management Active and reactive current control

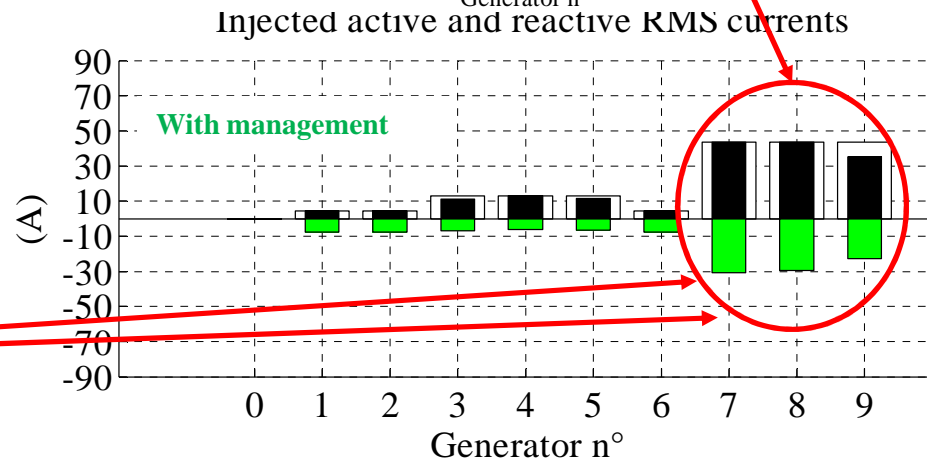
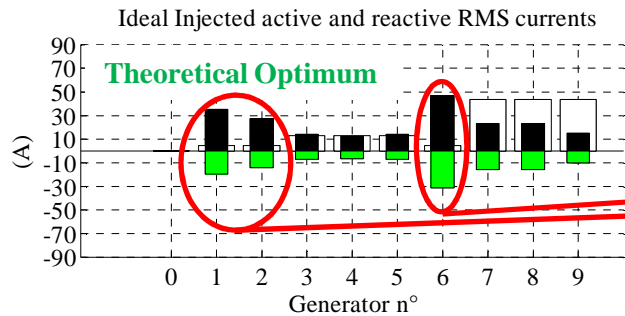
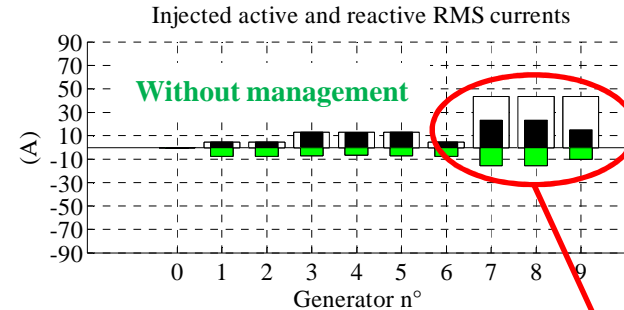
A Token Ring approach is adopted, where the 9 generators are activated and updated in sequence. Every 9 token jumps, the loads update their current demands.



Cooperative control (2)



Cooperative control with Saturation Management Active and reactive current control



Purely reactive current control



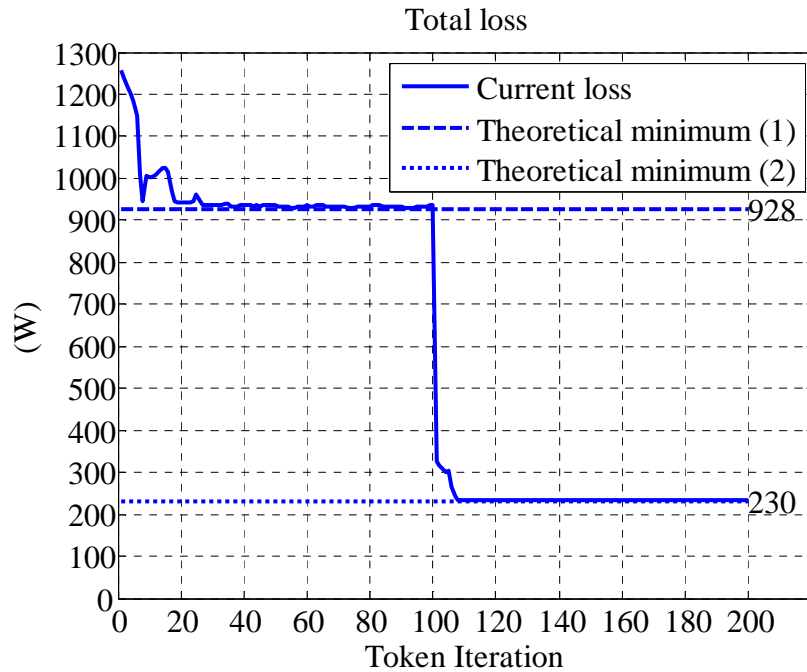
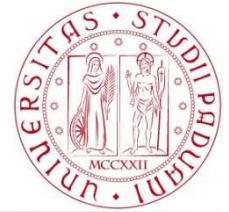
Assuming that **only reactive currents** are injected in the grid by the distributed grid-connected inverters, the distribution losses become:

- **Surround Control** $P_{\text{LOSS}}=928\text{W}$ (23% loss reduction)
- **Cooperative Control** $P_{\text{LOSS}}=935\text{W}$ (22% loss reduction)

This represents the worst case condition, i.e., the case of a micro-grid without energy storage capability and distributed power generation.

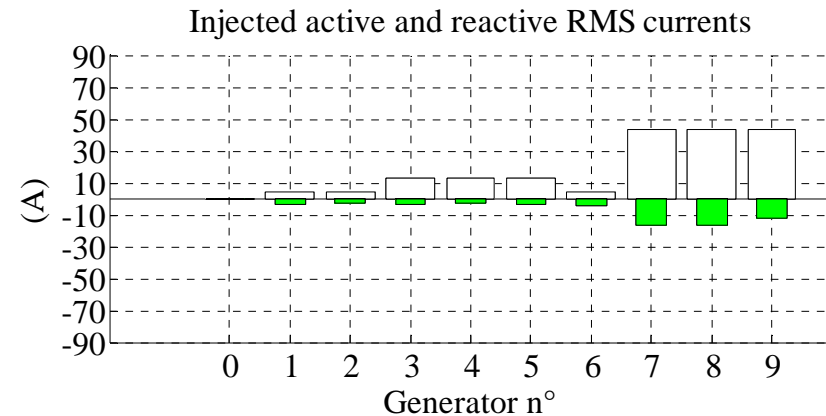
- The presence of distributed power generators allows a first level of improvement, since their active power can partially compensate for the active power demand of local loads.
- The situation is further improved if the grid-connected inverters can manage the energy of storage devices too, because this allows a local compensation for the entire active and reactive power demand by the loads, resulting in minimum distribution losses.

Purely reactive current control



Cooperative control with Saturation Management (pure reactive current control)

After 100 iterations all loads reduce their power absorption to 50% of the nominal ratings



Conclusions

1. Smart micro-grids represent a fast-growing and challenging arena for ICT, power electronics and power systems research and applications
2. The bottom-up revolution made possible by an extensive implementation of the micro-grid paradigm can have a dramatic impact on the entire value chain of the electrical market
3. A structured multi-layer reorganization of the electrical grid can provide huge benefits in terms of energy savings, quality of service and flexibility of operation, without altering the physical infrastructure of the grid
4. The development of suitable distributed control & communication techniques can provide flexibility, scalability, power quality, integration and exploitation of any kind of energy resources, energy efficiency and stability of operation
5. The successful Internet paradigm can possibly be replicated in the domain of distributed energy generation, distribution and utilization

Smart micro-grids

Properties, trends and local control of energy sources

Research activities at DEI/UniPD

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