UNICAMP – UNESP Sorocaba
August 2012

Conservative Power Theory

A theoretical background to understand energy issues of electrical networks under non-sinusoidal conditions

and

to approach measurement, accountability and control problems in smart grids

Paolo Tenti
Department of Information Engineering
University of Padova, Italy

Seminar Outline

1. Motivation of work
2. Mathematical and physical foundations of the theory
   - Mathematical operators and their properties
   - Instantaneous and average power & energy terms in poly-phase networks
3. Definition of current and power terms in single-phase networks under non-sinusoidal conditions
4. Extension to poly-phase domain: 3-wires / 4-wires
5. Sequence components under non-sinusoidal conditions
6. Measurement & accountability issues

1. Motivation of work

Why do we need to define power terms

- **Describe physical phenomena**
  - energy transfer,
  - energy storage,
  - rate of utilization of power sources and distribution infrastructure,
  - unwanted voltage and current terms, ….
- **Allow unambiguous measurement of quantities**
  - load and source characterization,
  - revenue metering, …
- **Compensation**
  - identify provisions which make the equipment or the plant compliant with standards & regulations in terms of symmetry, purity of waveforms, power factor …

1. Motivation of work

Few basic questions

While the definition and meaning of instantaneous power and its average value (active power) are universally agreed, the situation is less clear with other popular power terms

- What is/means reactive power ?
- What is/means distortion power ?
- What is/means apparent power ?

These power terms are unambiguously defined when at least the voltage supply is sinusoidal, but are matter of controversial discussions (since nearly one century) in case of distorted voltages and currents.

1. Motivation of work

Milestones of power theory history

- ✓ **In the frequency domain**
  - ➢ **Budeanu (1927)**
  - ➢ **Sheperd & Zakikhani (1971)**
  - ➢ **Czarnecki (1984 …)**
- ✓ **In the time domain**
  - ➢ **Fryze (1931)**
  - ➢ **Kusters & Moore (1975)**
  - ➢ **Depenbrock (1993)**
  - ➢ **Akagi & Nabae (1983)**

- **No one of these theories was able to target all goals (characterization of physical phenomena, load & line identification, compensation).**
- **The time-domain theory presented here tries to target all goals at the same time.**
- **It represents an outcome of a long-standing cooperation between UNIPD, UNICAMP and UNESP.**

1. Motivation of work

Need for a revision of power terms

- In modern scenarios (e.g., micro-grids) where:
  - the grid is weak,
  - frequency may change,
  - voltages may be asymmetrical,
  - distortion may affect voltages and currents,

  are the usual definitions of reactive, unbalance and distortion power still valid ?
- Which is the physical meaning of such terms ?
- Are they useful for compensation ?
- To which extent are power measurements affected by source non-ideality ?
- It is possible to identify supply and load responsibility on voltage distortion and asymmetry at a given network port ?

Conservative Power Theory

DEPARTMENT OF INFORMATION ENGINEERING
UNIVERSITY OF PADOVA

2. Mathematical and physical foundations

- **Definition of mathematical operators and their properties**
- **Definition of instantaneous power and energy terms**
- **Conservative quantities**
- **Selection of voltage reference**
- **Definition of average power terms and their physical meaning in real networks**

Mathematical operators for periodic scalar quantities

Let *T* be the period of variables *x* and y, we define:

- **Average value** $\bar{x} = \langle x \rangle = \frac{1}{T}\int_0^T x(t)\,dt$
- **Time derivative** $\breve{x} = \frac{dx}{dt}$
- **Time integral** $x_\int = \int_0^t x(\tau)\,d\tau$
- **Unbiased time integral** $\hat{x} = x_\int - \bar{x}_\int$
- **Internal product** $\langle x, y \rangle = \frac{1}{T}\int_0^T x \cdot y\,dt$
- **Norm (rms value)** $X = \|x\| = \sqrt{\langle x, x \rangle}$
- **Orthogonality** $\langle x, y \rangle = 0$

Mathematical operators for periodic vector quantities

Let $\underline{x}$ and $\underline{y}$ be vector quantities of size N, we define:

- **Scalar product** $$\underline{x} \circ \underline{y} = \sum_{n=1}^{N} x_n \, y_n$$
- **Magnitude** $$|\underline{x}| = \sqrt{\underline{x} \circ \underline{x}} = \sqrt{\sum_{n=1}^{N} x_n^2}$$
- **Internal product** $$\langle \underline{x}, \underline{y} \rangle = \langle \underline{x} \circ \underline{y} \rangle = \sum_{n=1}^{N} \langle x_n, y_n \rangle$$
- **Norm** $$\boldsymbol{X} = \|\underline{x}\| = \sqrt{\sum_{n=1}^{N} \langle x_n, x_n \rangle} = \sqrt{\sum_{n=1}^{N} X_n^2}$$
- **Orthogonality** $$\langle \underline{x}, \underline{y} \rangle = 0$$

- **The vector norm is also called *collective rms* value**

Properties of mathematical operators
(valid for scalar and vector quantities)

The above operators have the following properties:

- **Orthogonality**

$$\langle x, \breve{x} \rangle = 0 \qquad \langle x, \widehat{x} \rangle = 0 \quad \Rightarrow \quad \langle \underline{x}, \underline{\breve{x}} \rangle = 0 \qquad \langle \underline{x}, \underline{\widehat{x}} \rangle = 0$$

- **Equivalences**

$$\langle x, \breve{y} \rangle = -\langle \breve{x}, y \rangle$$
$$\langle x, \widehat{y} \rangle = -\langle \widehat{x}, y \rangle$$
$$\langle x, y \rangle = -\langle \breve{x}, \widehat{y} \rangle = -\langle \widehat{x}, \breve{y} \rangle$$

$$\Rightarrow$$

$$\langle \underline{x}, \underline{\breve{y}} \rangle = -\langle \underline{\breve{x}}, \underline{y} \rangle$$
$$\langle \underline{x}, \underline{\widehat{y}} \rangle = -\langle \underline{\widehat{x}}, \underline{y} \rangle$$
$$\langle \underline{x}, \underline{y} \rangle = -\langle \underline{\breve{x}}, \underline{\widehat{y}} \rangle = -\langle \underline{\widehat{x}}, \underline{\breve{y}} \rangle$$

- **For sinusoidal quantities**

$$X = \|x\| = \omega \|\widehat{x}\| = \frac{1}{\omega} \|\breve{x}\| \qquad x^2 + \omega^2 \widehat{x}^2 = x^2 + \frac{\breve{x}^2}{\omega^2} = 2X^2$$

$$\langle x, y \rangle = XY \cos\varphi \qquad \langle \widehat{x}, y \rangle = \frac{1}{\omega} XY \, sen\varphi$$

Instantaneous power definitions
(for periodic variables)

Given the vectors of the *N* phase currents i_n and voltages u_n measured at a generic network port we define:

Instantaneous (active) power:

$$p = \underline{u} \cdot \underline{i} = \sum_{n=1}^{N} u_n \, i_n = \sum_{n=1}^{N} p_n$$

Instantaneous reactive energy (new definition):

$$w = \underline{\hat{u}} \cdot \underline{i} = \sum_{n=1}^{N} \hat{u}_n \, i_n = \sum_{n=1}^{N} w_n$$

- **Both quantities do not depend on the voltage reference**
- **Both quantities are conservative in every real network**

Conservation of instantaneous power and reactive energy

For every real network *π*, let *u̲* and *i̲* be the vectors of the *L* branch voltages and currents, we claim that:

- ✓ **Branch voltages, *their time derivative and unbiased integral* are consistent with network *π*, i.e. they comply with KLV (Kirchhoff's law for voltages)**
- ✓ **Branch currents, *their time derivative and unbiased integral* are consistent with network π, i.e. they comply with KLC (Kirchhoff's law for currents)**

Thus, according to Tellegen's Theorem all quantities shown here are conservative

$$\underline{u}\cdot\underline{i}=\underline{\widehat{u}}\cdot\underline{\breve{i}}=\underline{\breve{u}}\cdot\underline{\widehat{i}}=0$$

$$\underline{\widehat{u}}\cdot\underline{i}=\underline{u}\cdot\underline{\widehat{i}}=0$$

$$\underline{\breve{u}}\cdot\underline{i}=\underline{u}\cdot\underline{\breve{i}}=0$$

Average power definitions
(valid for periodic quantities)

Active power:	$P = \overline{p} = \langle \underline{u}, \underline{i} \rangle$
Reactive energy:	$W = \overline{w} = \langle \widehat{\underline{u}}, \underline{i} \rangle = -\langle \underline{u}, \widehat{\underline{i}} \rangle$
Apparent power:	$A = \lVert \underline{u} \rVert \lVert \underline{i} \rVert = \mathbf{U}\,\mathbf{I}$
Power factor:	$\lambda = \dfrac{P}{A}$

- **All quantities are defined in the time domain.**
- **Reactive energy is a new definition, whose properties will be analyzed in the following.**
- **Active power and reactive energy are conservative quantities which do not depend on the voltage reference.**
- **Unlike *P* and *W*, apparent power *A* is non-conservative and depends on the voltage reference.**

Skip voltage reference

Selection of voltage reference (1)

Cauchy-Schwartz inequality:

$$\left|\langle \underline{u}, \underline{i} \rangle\right| \le \|\underline{u}\| \|\underline{i}\| \quad \Rightarrow |\lambda| = \frac{|P|}{A} \le 1$$

The equal sign is possible if:

$$\|\underline{u}\| \propto \|\underline{i}\| \quad \Rightarrow \quad \left|\langle \underline{u}, \underline{i} \rangle\right| = \|\underline{u}\| \|\underline{i}\| \quad \Rightarrow \quad |\lambda| = 1$$

We select the voltage reference so as to ensure unity power factor in case of symmetrical resistive load. This gives a physical meaning to the apparent power, which is the maximum active power that a supply line rated for V_{rms} Volts and I_{rms} Amps can deliver to a (purely resistive and symmetrical) load.

Selection of voltage reference (2)
N-phase systems without neutral wire

The proportionality condition between phase voltages and currents for symmetrical resistive load determines voltage reference

$$\underline{u} = R\,\underline{i}$$

$$\sum_{n=1}^{N} i_n = 0 \quad \Rightarrow \quad \sum_{n=1}^{N} u_n = 0$$

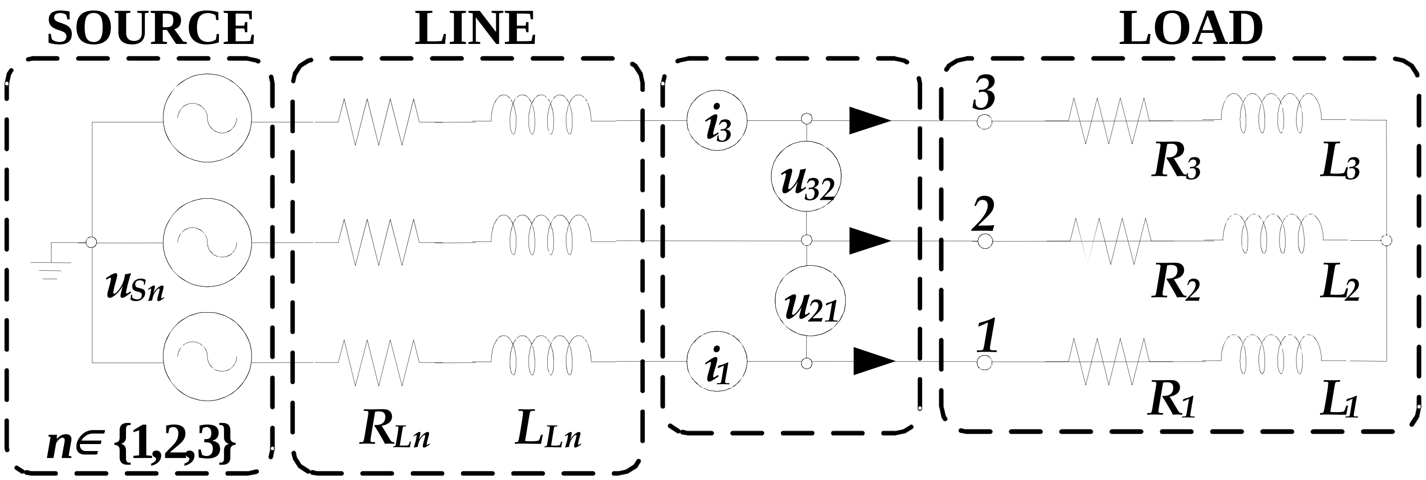
Thus, the voltage reference must be selected to comply with the zero-sum condition:

$$\sum_{n=1}^{N} u_n = 0 \quad \Rightarrow \quad \sum_{n=1}^{N} \underbrace{\left(u_{n_{measure}} - u_{ref}\right)}_{u_n} = 0 \quad \Rightarrow \quad u_{ref} = \frac{1}{N} \sum_{n=1}^{N} u_{n_{measure}}$$

This choice minimizes the norm of the voltage vector

Selection of voltage reference (3)
N-phase systems without neutral wire

Measurement of voltages and currents



Derivation of phase voltages

$$u_n = \frac{1}{N} \sum_{j=1,\, j \neq n}^{N} u_{nj}$$

$$|\underline{u}|^2 = \frac{1}{2N} \sum_{n=1}^{N} \sum_{j=1,\, j \neq n}^{N} u_{nj}^2$$

$$u_n^2 = \frac{1}{N} \left(\sum_{j=1,\, j \neq n}^{N} u_{nj}^2 - |\underline{u}|^2 \right), \quad n = 1 \div N$$

Selection of voltage reference (4)
N-phase systems with neutral wire

In case of symmetrical resistive load the proportionality condition between phase voltages and currents holds only if the voltage reference is set to the neural wire.

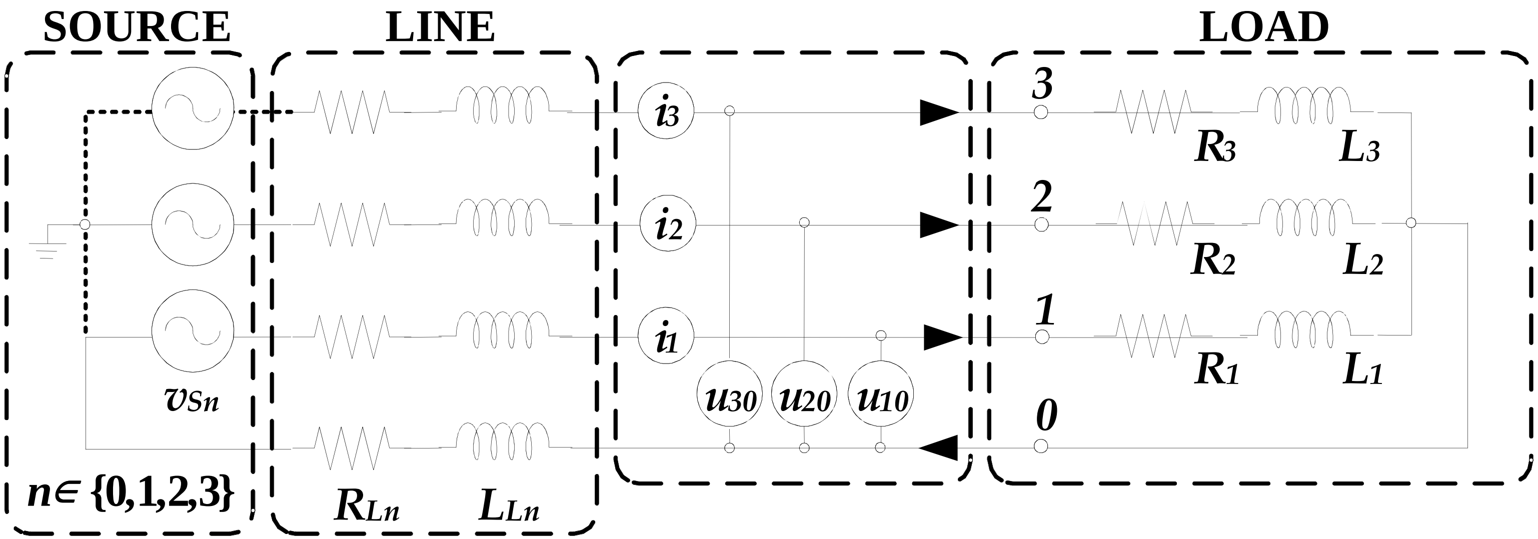
$$u_{ref} = u_o = 0 \quad \Rightarrow \quad u_n = R\, i_n, n = 0 \div N$$

Unity power factor may occur only if the neutral current is disregarded for apparent power computation (only phase currents are considered). Thus:

$$A = P = \boldsymbol{U}\,\boldsymbol{I}, \qquad \boldsymbol{U} = \sqrt{\sum_{n=1}^{N} U_n^2} \left(= \sqrt{\sum_{n=0}^{N} U_n^2} \right) \quad \boldsymbol{I} = \sqrt{\sum_{n=1}^{N} I_n^2} \left(\neq \sqrt{\sum_{n=0}^{N} I_n^2} \right)$$

Selection of voltage reference (5)
N-phase systems with neutral wire

Measurement of voltages and currents



Collective rms voltage and current

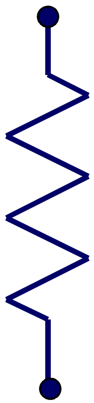
$$U = \sqrt{\sum_{n=1}^{N} U_n^2} \qquad I = \sqrt{\sum_{n=1}^{N} I_n^2}$$

Homopolar voltage and current

$$u^z = \frac{1}{N}\sum_{n=1}^{N} u_n \qquad i^z = \frac{1}{N}\sum_{n=1}^{N} i_n = -\frac{i_o}{N}$$

Power terms in passive networks

Resistor



$$u = R\,i$$

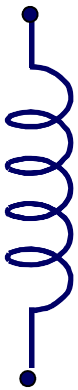
$$i = G\,u$$

$$P_R = \langle u, i \rangle = G\,\|u\|^2 = R\,\|i\|^2$$

$$W_R = \langle \widehat{u}, i \rangle = R\langle \widehat{i}, i \rangle = 0$$

Power terms in passive networks

Inductor



$$u = L\frac{di}{dt} = L\,\breve{i}$$

$$i = \frac{\overset{\frown}{u}}{L}$$

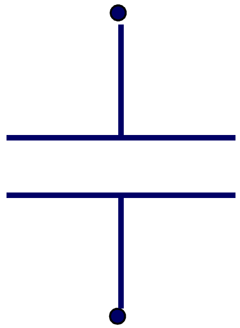
$$P_L = \langle u, i\rangle = \left\langle u, \frac{\overset{\frown}{u}}{L}\right\rangle = 0 \qquad W_L = \langle \overset{\frown}{u}, i\rangle = \langle L\,i, i\rangle = L\|i\|^2$$

Inductor energy

$$\varepsilon_L = \frac{1}{2}L\,i^2 \;\Rightarrow\; \bar{\varepsilon}_L = \mathrm{E}_L = \frac{1}{2}L\|i\|^2 = \frac{W_L}{2}$$

Power terms in passive networks

Capacitor



$$i = C\frac{du}{dt} = C\,\overset{\smile}{u}$$

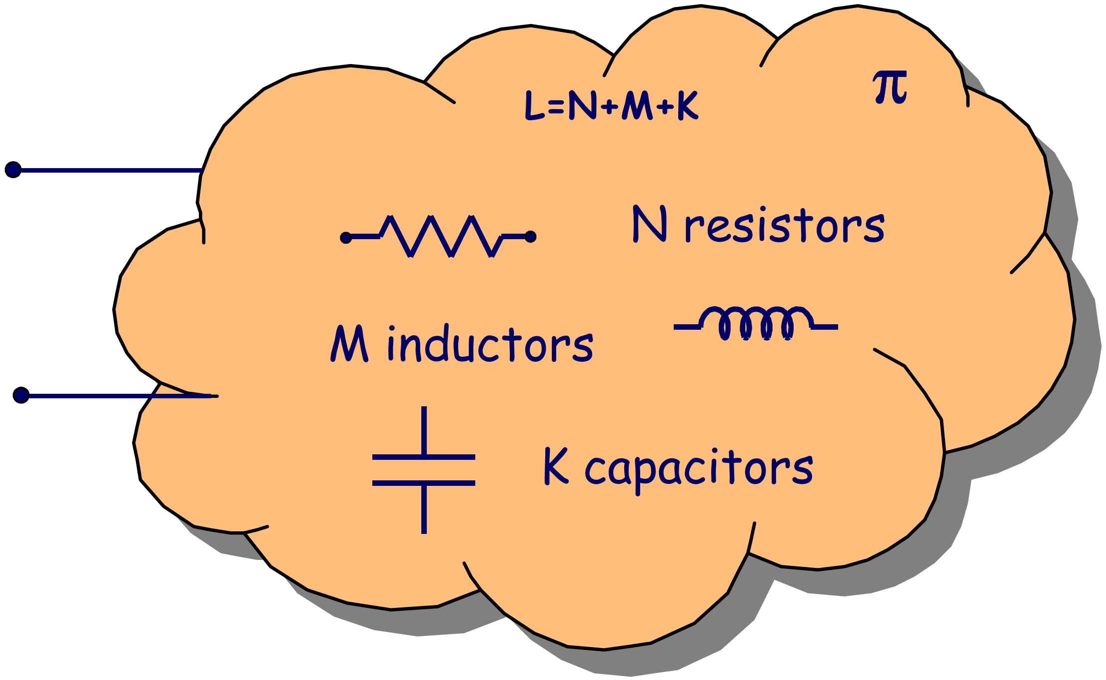
$$u = \frac{\overset{\frown}{i}}{C}$$

$$P_C = \langle u, i \rangle = \left\langle \frac{\overset{\frown}{i}}{C}, i \right\rangle = 0 \qquad W_C = -\langle u, \overset{\frown}{i} \rangle = -\langle u, C\,u \rangle = -C\|u\|^2$$

Capacitor energy

$$\varepsilon_C = \frac{1}{2}Cu^2 \quad \Rightarrow \quad \bar{\varepsilon}_C = \mathrm{E}_C = \frac{1}{2}C\|u\|^2 = -\frac{W_C}{2}$$

Active and reactive power absorption of a linear passive network π



Remark: Whichever is the origin of reactive energy, including active and nonlinear loads, it can be compensated by reactive elements with proper energy storage capability

Total active power and reactive energy

$$P = \sum_{l=1}^{L} \langle u_l, i_l \rangle = \sum_{n=1}^{N} P_{R_n} = P_{R_{tot}}$$

$$W = \sum_{l=1}^{L} \langle \hat{u}_l, i_l \rangle = \sum_{m=1}^{M} W_{L_m} + \sum_{k=1}^{K} W_{C_k} = 2\left(\sum_{m=1}^{M} \mathrm{E}_{L_m} - \sum_{k=1}^{K} \mathrm{E}_{C_k}\right) = 2\left(\mathrm{E}_{L_{tot}} - \mathrm{E}_{C_{tot}}\right)$$

Seminar Outline

1. Motivation of work
2. Mathematical and physical foundations of the theory
   - Mathematical operators and their properties
   - Instantaneous and average power & energy terms in poly-phase networks
3. **Definition of current and power terms in single-phase networks under non-sinusoidal conditions**
4. **Extension to poly-phase domain: 3-wires / 4-wires**
5. **Sequence components under non-sinusoidal conditions**
6. **Measurement & accountability issues**

Conservative Power Theory

DEPARTMENT OF INFORMATION ENGINEERING
UNIVERSITY OF PADOVA

3. Definition of current and power terms in single-phase networks under non-sinusoidal conditions

- Orthogonal current decomposition into active, reactive and void terms
- Physical meaning of current terms
- Apparent power decomposition into active, reactive and void terms
- Physical meaning of power terms
- Application examples

Orthogonal current decomposition in single-phase networks

(voltage and current measured at a generic network port)

✓ **Current terms**

$$i = i_a + i_r + i_v = i_a + i_r + \underbrace{i_{sa} + i_{sr} + i_g}_{i_v}$$

- i_a ***active current***
- i_r ***reactive current***
- i_v ***void current***
- i_{sa} ***scattered active current***
- i_{sr} ***scattered reactive current***
- i_g ***generated current***

✓ **Orthogonality: all terms in the above equations are orthogonal**

$$\|i\|^2 = \|i_a\|^2 + \|i_r\|^2 + \|i_v\|^2 = \|i_a\|^2 + \|i_r\|^2 + \|i_{sa}\|^2 + \|i_{sr}\|^2 + \|i_g\|^2$$

Orthogonal current decomposition in single-phase networks

(voltage and current measured at a generic network port)

✓ **Active current: the minimum current (i.e., with minimum rms value) needed to convey the active power *P* flowing through the port**

$$i_a = \frac{\langle u, i \rangle}{\|u\|^2} u = \frac{P}{U^2} u = G_e u$$

u **= port voltage**

U **= rms value of port voltage**

G_e **= equivalent conductance**

$$P_a = \langle u, i_a \rangle = G_e \langle u, u \rangle = G_e U^2 = P$$
$$W_a = \langle \hat{u}, i_a \rangle = G_e \langle \hat{u}, u \rangle = 0$$

Active current conveys full active power and zero reactive energy

Orthogonal current decomposition in single-phase networks

(voltage and current measured at a generic network port)

✓ **Reactive current: the minimum current needed to convey the reactive energy *W* flowing through the port**

$$i_r = \frac{\langle \hat{u}, i \rangle}{\|\hat{u}\|^2}\hat{u} = \frac{W}{\hat{U}^2}\hat{u} = B_e \hat{u}$$

B_e = equivalent reactivity

$$P_r = \langle u, i_r \rangle = B_e \langle u, \hat{u} \rangle = 0$$

$$W_r = \langle \hat{u}, i_r \rangle = B_e \langle \hat{u}, \hat{u} \rangle = B_e \hat{U}^2 = W$$

Reactive current conveys full reactive energy and no active power

$$\langle i_a, i_r \rangle = G_e B_e \langle u, \hat{u} \rangle = 0$$

Active and reactive current are orthogonal

Orthogonal current decomposition in single-phase networks

(voltage and current measured at a generic network port)

✓ **Void current: is the remaining current component**

$$i_v = i - i_a - i_r$$

Void current is not conveying active power or reactive energy

$$P_v = \langle u, i_v \rangle = \langle u, i \rangle - \langle u, i_a \rangle - \langle u, i_r \rangle = P - P_a - P_r = 0$$
$$W_v = \langle \hat{u}, i_v \rangle = \langle \hat{u}, i \rangle - \langle \hat{u}, i_a \rangle - \langle \hat{u}, i_r \rangle = W - W_a - W_r = 0$$

Void current is orthogonal to active and reactive terms

$$\langle i_v, i_a \rangle = G_e \langle i_v, u \rangle = G_e P_v = 0$$
$$\langle i_v, i_r \rangle = B_e \langle i_v, \hat{u} \rangle = B_e W_v = 0$$

Orthogonal current decomposition in single-phase networks

(voltage and current measured at a generic network port)

The void current reflects the presence of scattered active, scattered reactive and load-generated harmonic terms

$$i_v = i_{sa} + i_{sr} + i_g$$

Scattered current terms: Account for different values of equivalent admittance at different harmonics

Load-generated current harmonics: Harmonic terms that exist in currents only, not in voltages

$$\langle i_{sa}, i_{sr} \rangle = \langle i_{sa}, i_g \rangle = \langle i_{sr}, i_g \rangle = 0$$

Scattered and load-generated harmonic currents are orthogonal

Skip void current components

Orthogonal current decomposition in single-phase networks

Scattered active current

For each co-existing harmonic components of voltage and current we define:

✓ **Harmonic active current terms**

$$i_{ak} = \frac{\langle u_k, i_k \rangle}{\|u_k\|^2} u_k = \frac{P_k}{U_k^2} u_k = \frac{I_k \cos\varphi_k}{U_k} u_k = G_k u_k$$

✓ **Total harmonic active current**

$$i_{ha} = \sum_{k\in K} i_{ak}$$

$$P_{ha} = \sum_{k\in K} P_k = P_a = P, \quad W_{ha} = 0$$

✓ **Scattered active current**

$$i_{sa} = i_{ha} - i_a = \sum_{k\in K} (G_k - G_e) u_k$$

$$P_{sa} = P_{ha} - P_a = 0, \quad W_{sa} = 0$$

Orthogonal current decomposition in single-phase networks

Scattered reactive current

For each co-existing harmonic components of voltage and current we define:

- ✓ **Harmonic reactive current terms**

$$i_{rk} = \frac{\langle \hat{u}_k , i_k \rangle}{\|\hat{u}_k\|^2} \hat{u}_k = \frac{W_k}{\hat{U}_k^2} \hat{u}_k = \frac{\omega k \, I_k \sin \varphi_k}{U_k} \hat{u}_k = B_k \hat{u}_k$$

- ✓ **Total harmonic reactive current**

$$i_{hr} = \sum_{k \in K} i_{rk}$$

$$W_{hr} = \sum_{k \in K} W_k = W_r = W, \quad P_{hr} = 0$$

- ✓ **Scattered reactive current**

$$i_{sr} = i_{hr} - i_r = \sum_{k \in K} (B_k - B_e) \hat{u}_k$$

$$W_{sr} = W_{hr} - W = 0, \quad P_{sr} = 0$$

Apparent power decomposition in single-phase networks

$$A = \|u\|\|i\| = U\,I = \sqrt{P^2 + Q^2 + V^2}$$

- ✓ **Active power:** $$P = \|u\|\,\|i_a\| = U\,I_a$$
- ✓ **Reactive power:** $$Q = \|u\|\,\|i_r\| = U\,I_r$$
- ✓ **Void power:** $$V = \|u\|\,\|i_v\| = U\,I_v = \sqrt{S_a^2 + S_r^2 + V_g^2}$$
  - ✓ **Scattered active power:** $$S_a = \|u\|\,\|i_{sa}\| = U\,I_{sa}$$
  - ✓ **Scattered reactive power:** $$S_r = \|u\|\,\|i_{sr}\| = U\,I_{sr}$$
  - ✓ **Load-generated harmonic power:** $$V_g = \|u\|\,\|i_g\| = U\,I_g$$

Reactive Power

U **and *Û* can be decomposed in fundamental and harmonic components**
(THD means total harmonic distortion)

$$U = \sqrt{U_f^2 + U_h^2} = U_f \sqrt{1 + [THD(u)]^2}$$

$$\widehat{U} = \sqrt{\widehat{U}_f^2 + \widehat{U}_h^2} = \widehat{U}_f \sqrt{1 + [THD(\widehat{u})]^2}$$

Recalling that: $U_f / \widehat{U}_f = \omega$

We have:

$$Q = U\, I_r = \frac{U}{\widehat{U}} W = \omega W \frac{\sqrt{1 + [THD(u)]^2}}{\sqrt{1 + [THD(\widehat{u})]^2}}$$

Note that, unlike reactive energy W, REACTIVE POWER Q IS NOT CONSERVATIVE. In fact, it depends on line frequency and (local) voltage distortion.

Under sinusoidal conditions, the definition of Q coincides with the conventional one

Void Power Terms

Void Power:

$$V = U\,I_v = \sqrt{S_a^2 + S_r^2 + V_g^2}$$

✓ **Scattered active power:**

$$S_a = U\,I_{sa} = \sqrt{U^2 \sum_{k\in\{K\}} \left(\frac{P_k}{U_k^2} - \frac{P}{U^2} \right)^2 U_k^2}$$

✓ **Scattered reactive power:**

$$S_r = U\,I_{sr} = \omega \frac{\sqrt{1+THD_u^2}}{\sqrt{1+THD_{\hat{u}}^2}} \sqrt{\hat{U}^2 \sum_{k\in\{K\}} \left(\frac{W_k}{\hat{U}_k^2} - \frac{W}{\hat{U}^2} \right)^2 \hat{U}_k^2}$$

✓ **Load-generated harmonic power:**

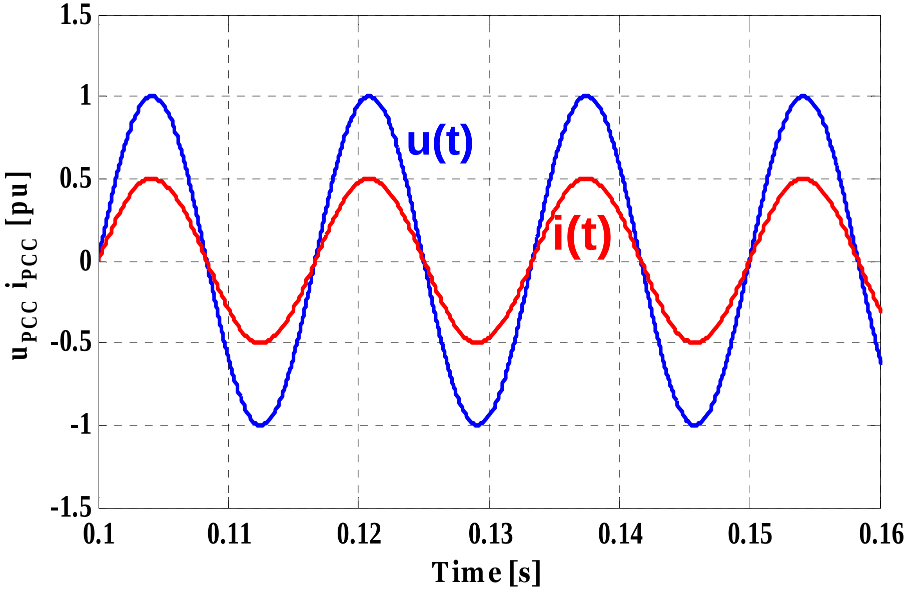
$$V_g = U\,I_g$$

Skip examples

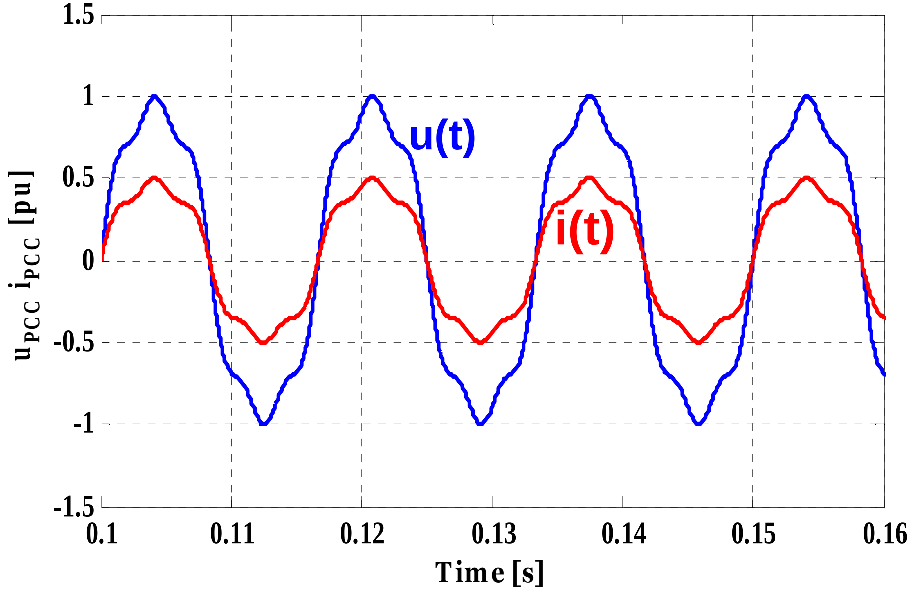
Application Examples

Example # 1

Voltage and Current : Resistive Load



Sinusoidal voltage



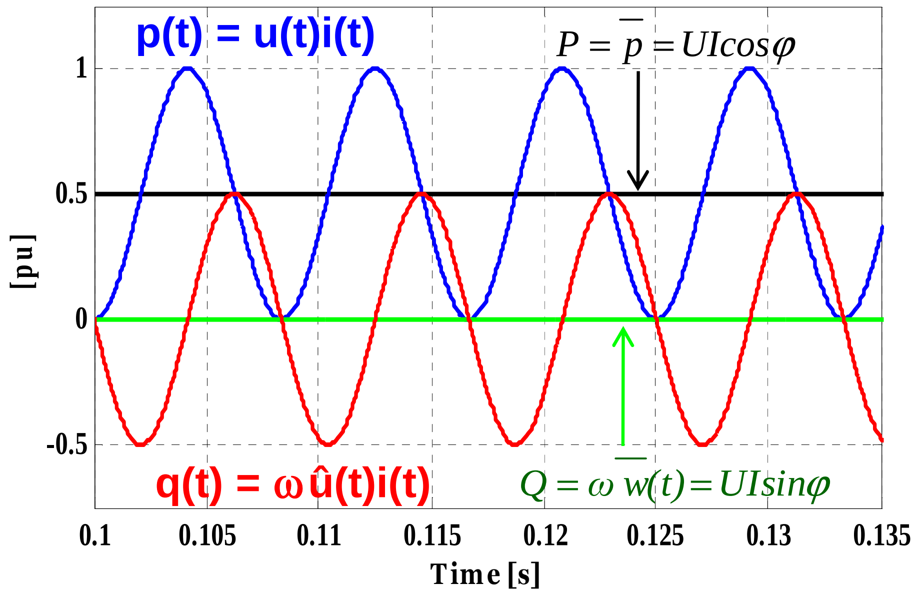
Non – sinusoidal voltage

Current = $i_{pu}(t)/2$

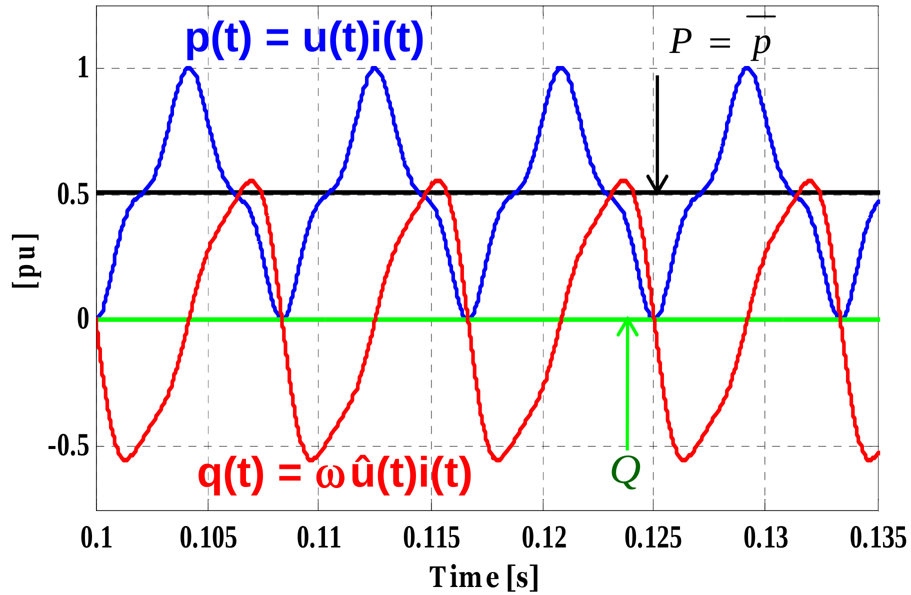
Application Examples

Example # 1

Conservative Power Terms: Resistive Load



Sinusoidal voltage



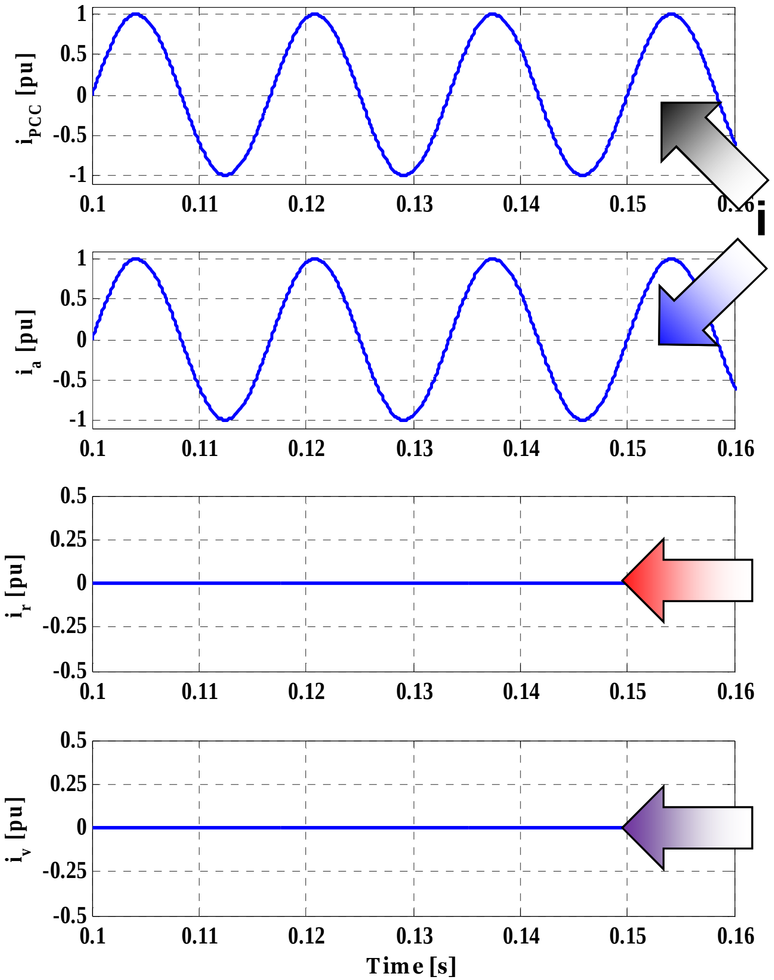
Non – sinusoidal voltage

This example shows the correspondences between the CPT theory and conventional theory

Application Examples

Example # 1 – Single-phase

Current Terms: Resistive Load



PCC current

$i_{PCC}(t) = i_a(t)$

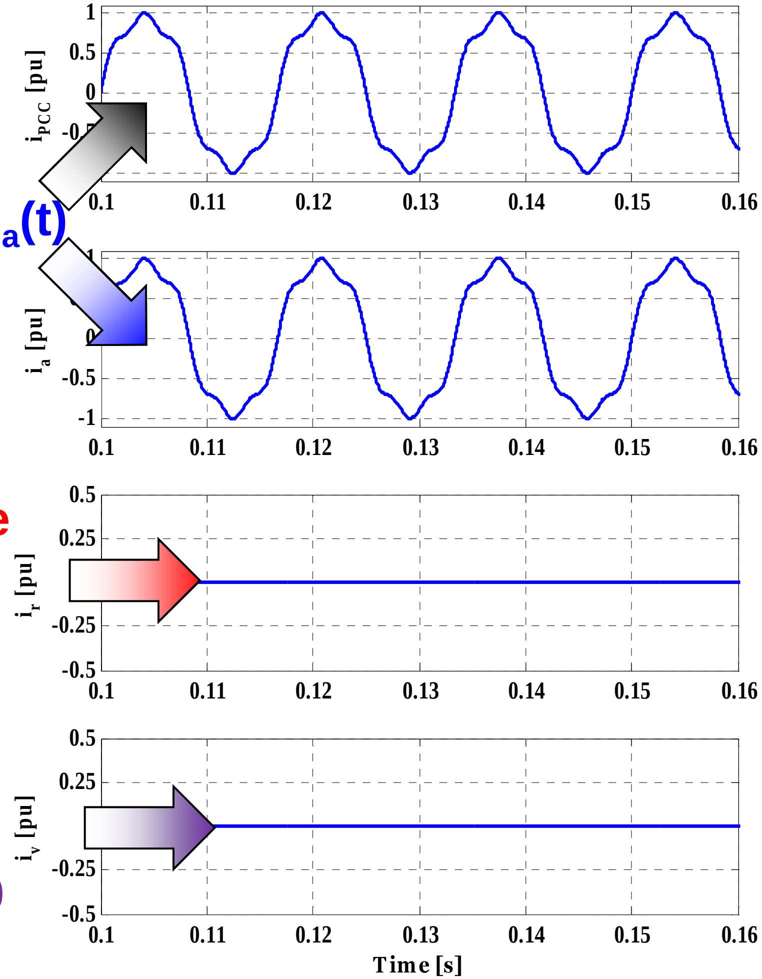
Active current

Reactive current

$i_r(t) = 0$

Void current

$i_v(t) = 0$



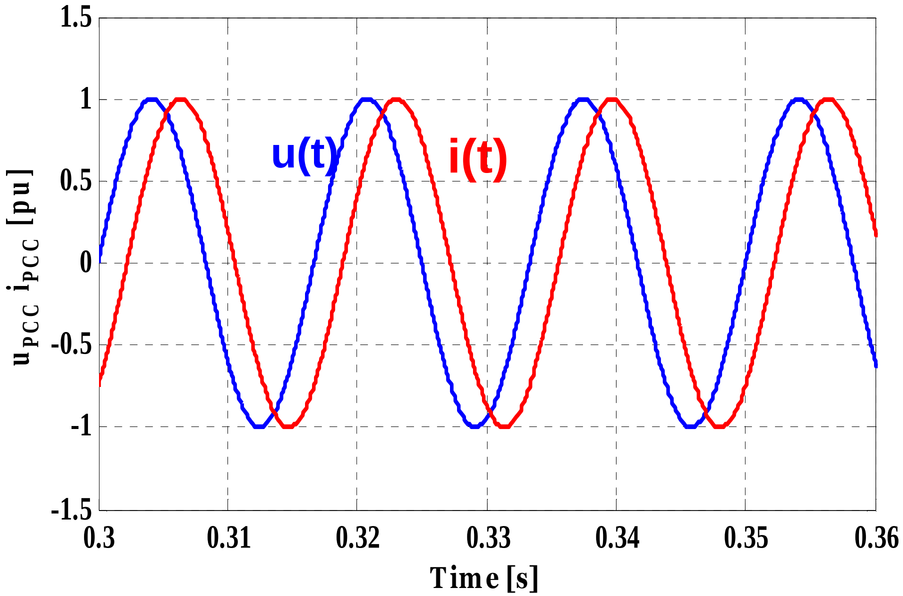
Sinusoidal voltage

Non – sinusoidal voltage

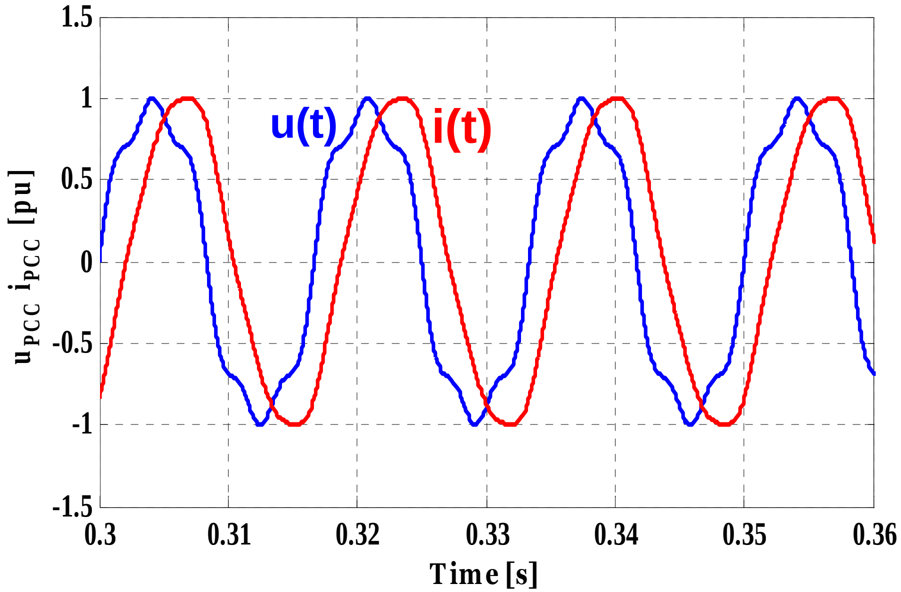
Application Examples

Example # 2

Voltage and Current : Ohmic-inductive Load



Sinusoidal voltage

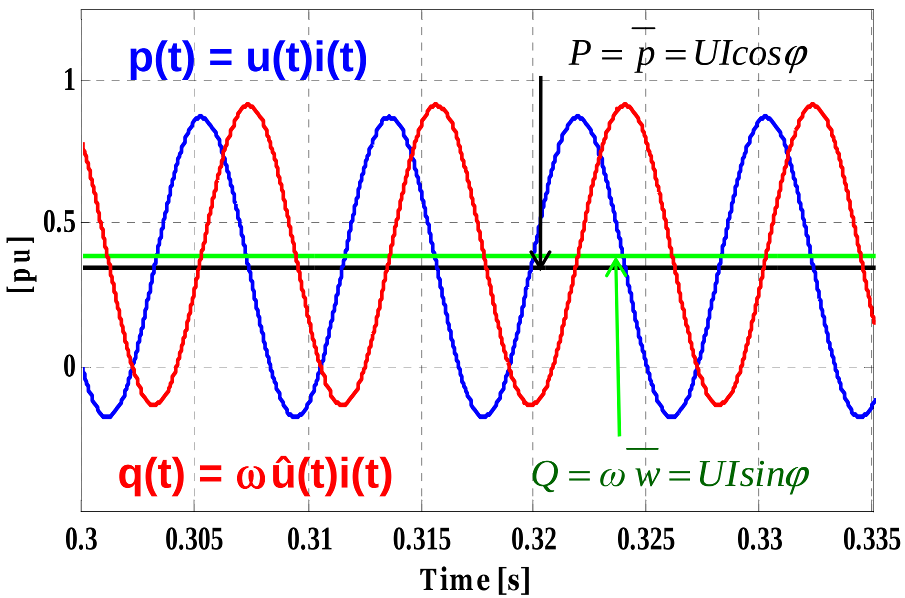


Non – sinusoidal voltage

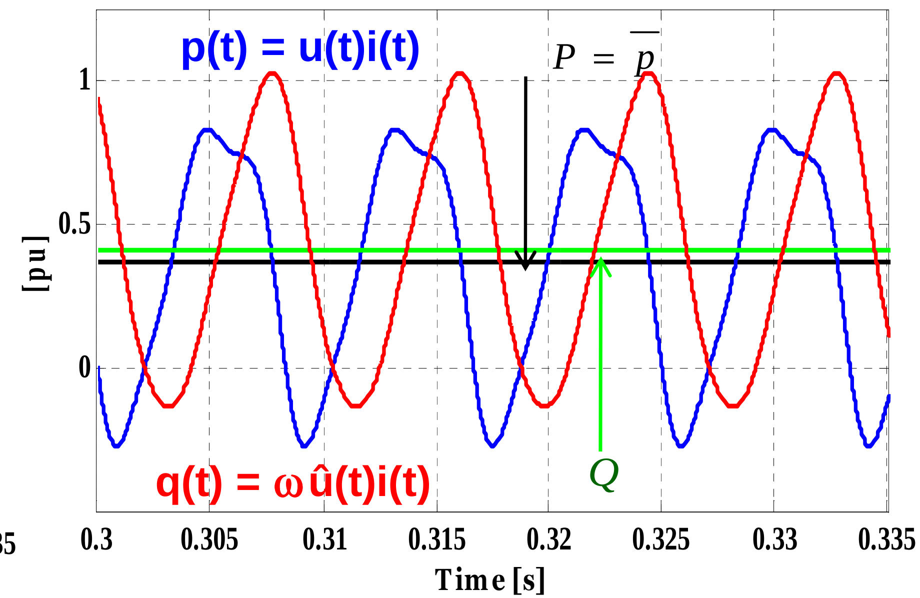
Application Examples

Example # 2

Conservative Power Terms: Ohmic-inductive Load



Sinusoidal voltage



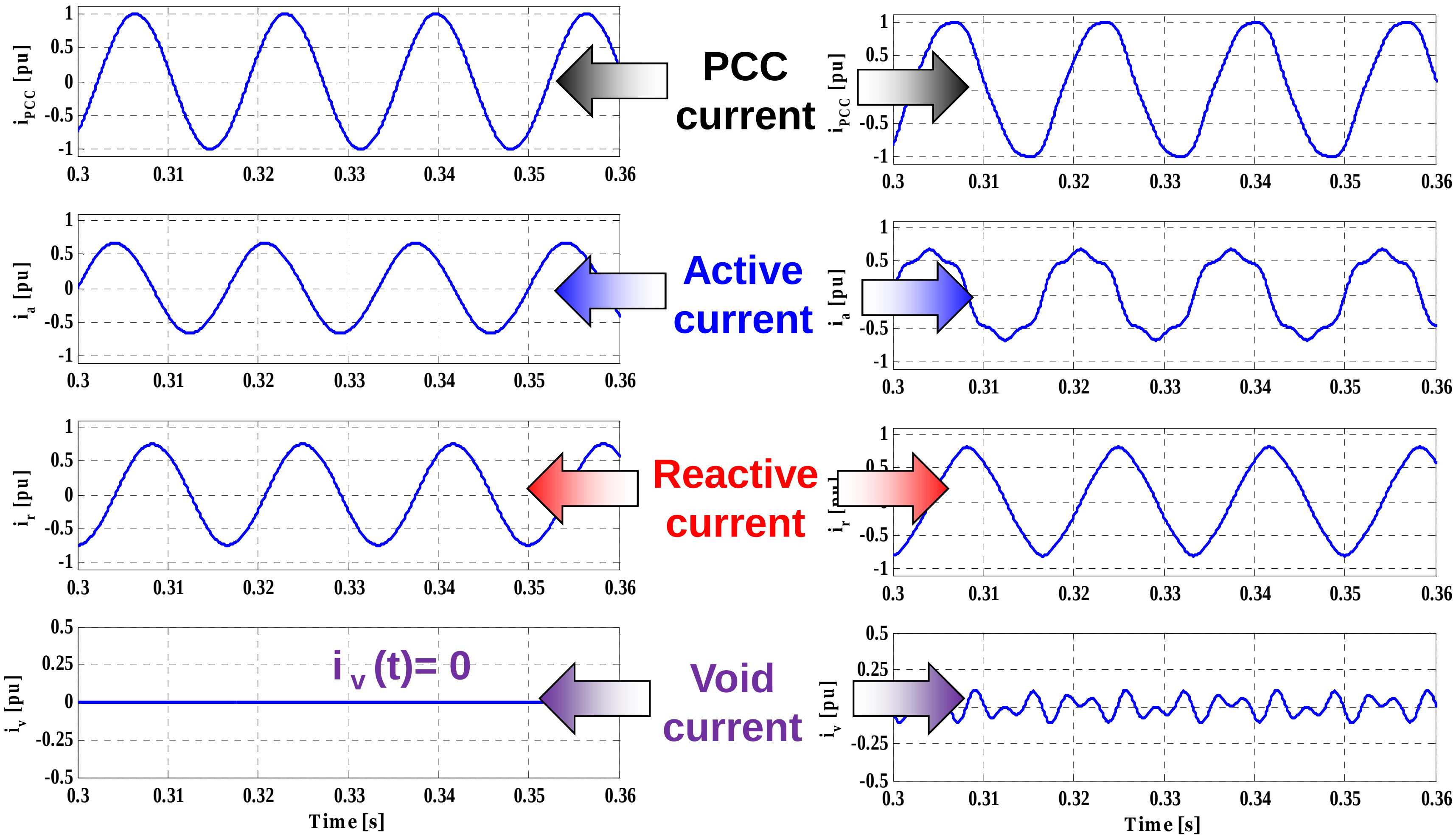
Non – sinusoidal voltage

This example shows the correspondence between CPT and conventional theory under sinusoidal conditions

Application Examples

Example # 2

Current Terms: Ohmic-inductive Load



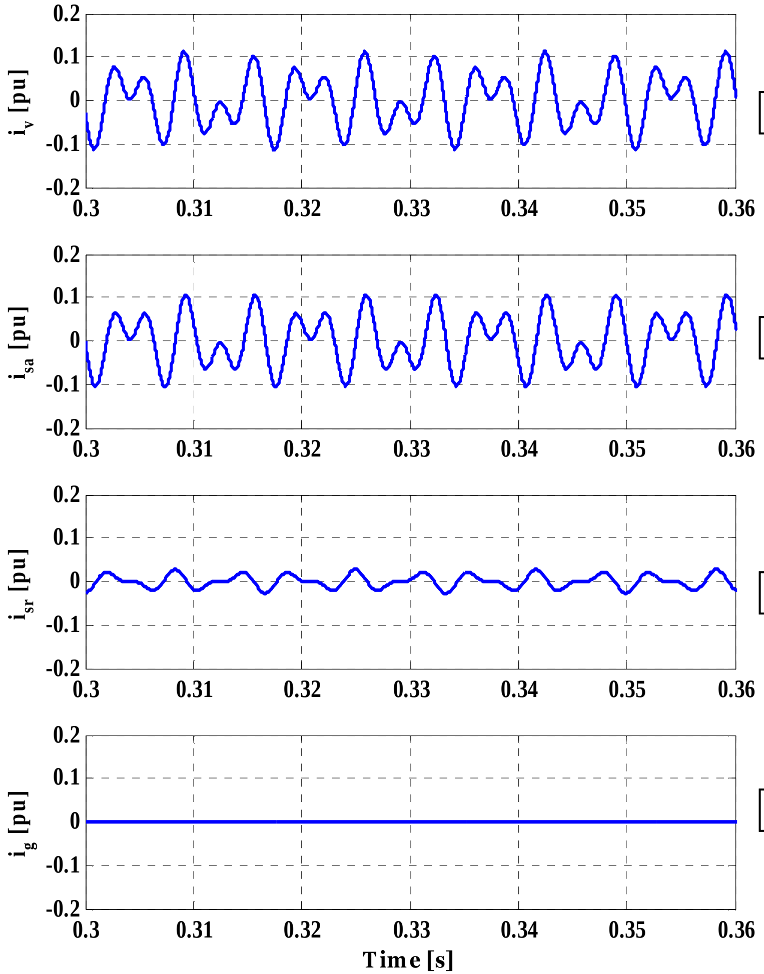
Sinusoidal voltage

Non – sinusoidal voltage

Physical meaning of void current

Example # 2

Void Current Terms: Ohmic-inductive Load



Void current

Scattered active current

Scattered reactive current

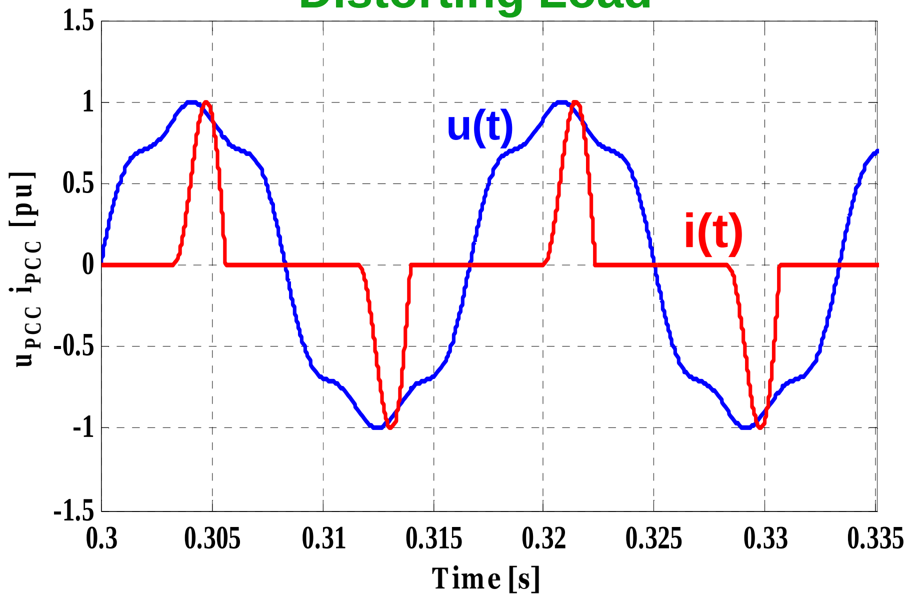
Load-generated harmonic current

Non – sinusoidal voltage

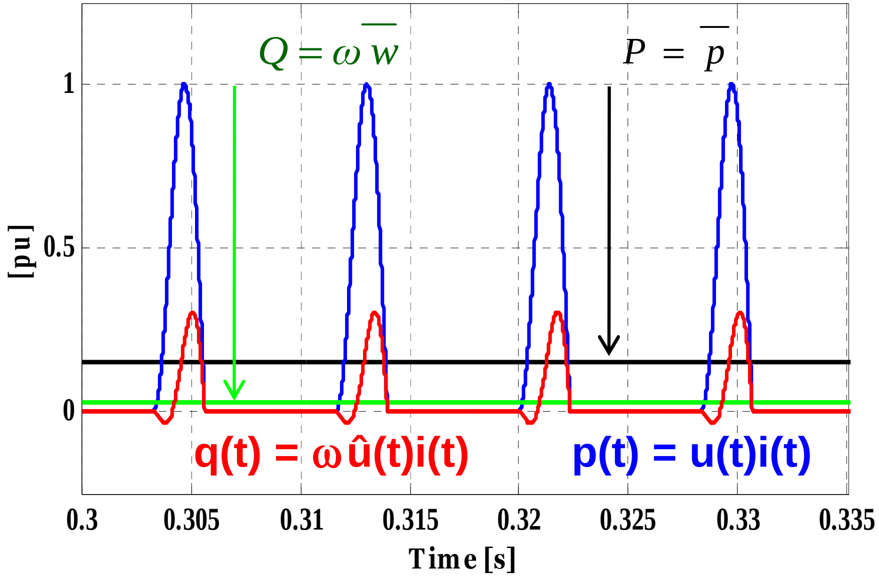
Application Examples

Example # 3

Voltage and Current
Distorting Load



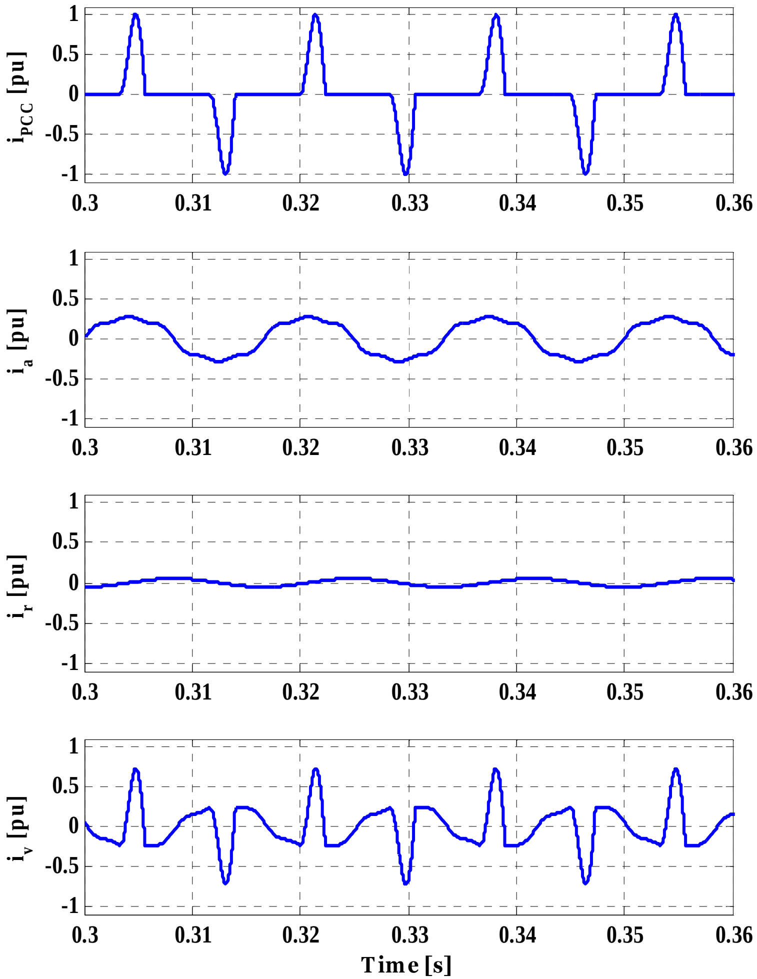
Conservative Power Terms
Distorting Load



Non – sinusoidal voltage

Physical meaning of void current

Void Current Terms: Ohmic-inductive Load



PCC current

Active current

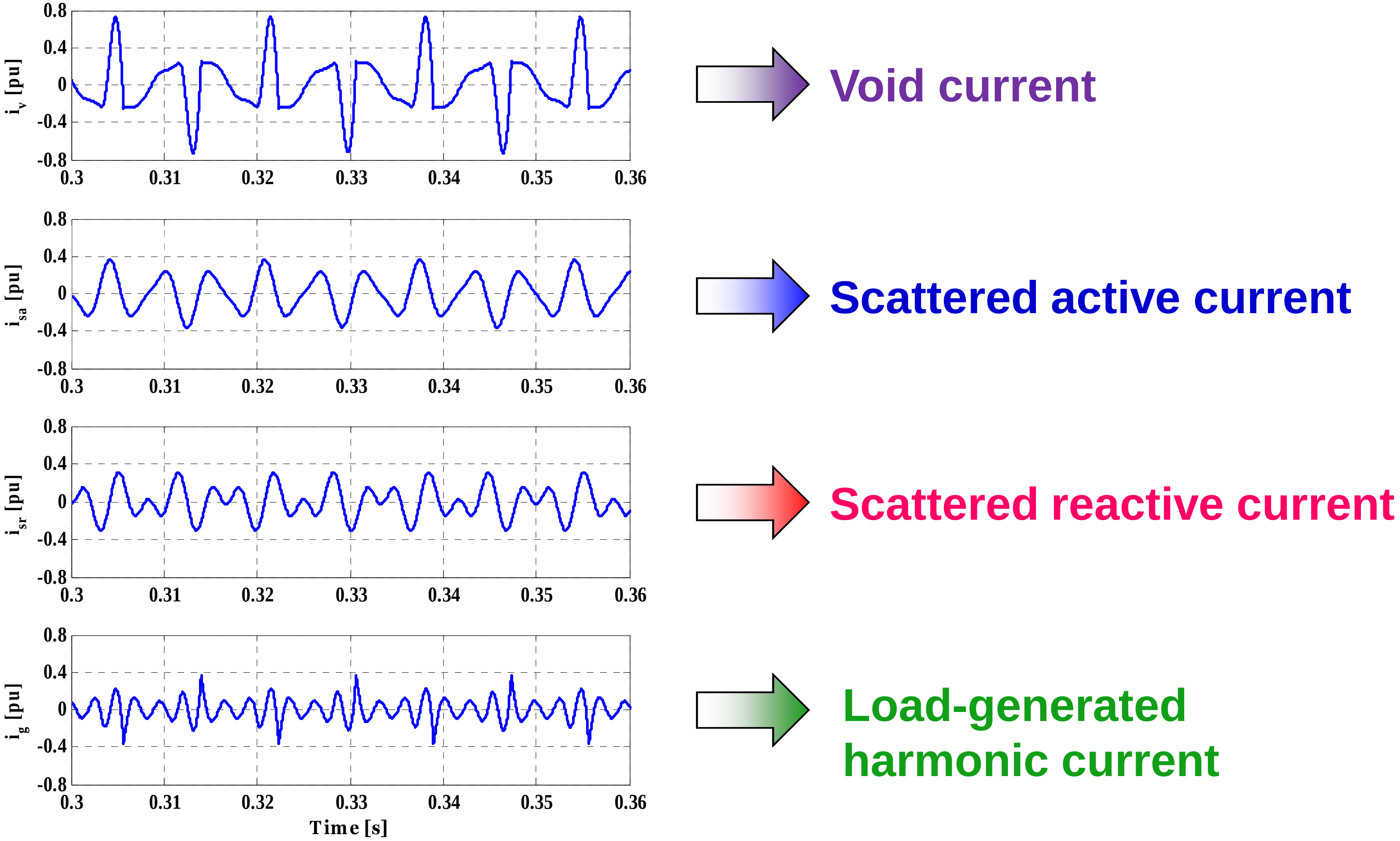
Reactive current

Void current

Non – sinusoidal voltage

Physical meaning of void current

Void Current Terms: Ohmic-inductive Load



Non – sinusoidal voltage

Seminar Outline

1. Motivation of work
2. Mathematical and physical foundations of the theory
   - Mathematical operators and their properties
   - Instantaneous and average power & energy terms in poly-phase networks
3. Definition of current and power terms in single-phase networks under non-sinusoidal conditions
4. **Extension to poly-phase domain: 3-wires / 4-wires**
5. **Sequence components under non-sinusoidal conditions**
6. **Measurement & accountability issues**

Conservative Power Theory

DEPARTMENT OF INFORMATION ENGINEERING
UNIVERSITY OF PADOVA

4. Extension to poly-phase domain: 3-wires / 4-wires

- Orthogonal current decomposition into active, reactive, unbalance and void terms
- Physical meaning of current terms
- Active power decomposition into active, reactive, unbalance and void terms
- Physical meaning of power terms

Orthogonal current decomposition
Extension to poly-phase: 3-wires / 4-wires

In poly-phase systems, the current components (active, reactive and void) can be defined for each phase:

✓ Active current

$$i_{an} = \frac{\langle u_n, i_n \rangle}{\|u_n\|^2} u_n = \frac{P_n}{U_n^2} u_n = G_n u_n, \quad n = 1 \div N$$

G_n= equivalent phase conductance

$$P_a = \langle \underline{u}, \underline{i}_a \rangle = P$$

$$W_a = \langle \underline{\hat{u}}, \underline{i}_a \rangle = 0$$

$$\boldsymbol{U}\,\boldsymbol{I}_a = \|\underline{u}\|\,\|\underline{i}_a\| \neq P$$

✓ Reactive current

$$i_{rn} = \frac{\langle \hat{u}_n, i_n \rangle}{\|\hat{u}_n\|^2} \hat{u}_n = \frac{W_n}{\hat{U}_n^2} \hat{u}_n = B_n \hat{u}_n, \quad n = 1 \div N$$

B_n = equivalent phase reactivity

$$P_r = \langle \underline{u}, \underline{i}_r \rangle = 0$$

$$W_r = \langle \underline{\hat{u}}, \underline{i}_r \rangle = W$$

$$\hat{\boldsymbol{U}}\,\boldsymbol{I}_r = \|\underline{\hat{u}}\|\,\|\underline{i}_r\| \neq W$$

✓ Void current

$$i_{vn} = i_n - i_{an} - i_{rn}, \quad n = 1 \div N$$

$$P_v = \langle \underline{u}, \underline{i}_v \rangle = 0, \quad W_r = \langle \underline{\hat{u}}, \underline{i}_v \rangle = 0$$

$$\boldsymbol{U}\,\boldsymbol{I}_v > 0$$

Orthogonal current decomposition
Extension to poly-phase: 3-wires / 4-wires

Active and reactive current terms can also be defined collectively, i.e., by making reference to an equivalent balanced load absorbing the same active power and reactive energy of actual load:

- ✓ **Balanced Active currents: minimum collective currents needed to convey active power *P***

$$\underline{i}_a^b = \frac{\langle \underline{u}, \underline{i} \rangle}{\|\underline{u}\|^2} \underline{u} = \frac{P}{U^2} \underline{u} = G^b \underline{u}$$

G^b= equivalent balanced conductance

$$U\, I_a^b = \|\underline{u}\| \, \|\underline{i}_a^b\| = P, \quad Q_a^b = 0$$

- ✓ **Balanced Reactive currents: minimum collective currents needed to convey reactive energy *W***

$$\underline{i}_r^b = \frac{\langle \underline{\hat{u}}, \underline{i} \rangle}{\|\underline{\hat{u}}\|^2} \underline{\hat{u}} = \frac{W}{\hat{U}^2} \underline{\hat{u}} = B^b \underline{\hat{u}}$$

B^b = equivalent balanced reactivity

$$\hat{U}\, I_r^b = \|\underline{\hat{u}}\| \, \|\underline{i}_r^b\| = W, \quad P_r^b = 0$$

Orthogonal current decomposition
Extension to poly-phase: 3-wires / 4-wires

Unbalanced currents account for the asymmetrical behavior of the various phases

- ✓ **Unbalanced Active currents**

$$\underline{i}_a^u = \underline{i}_a - \underline{i}_a^b \quad \Rightarrow \quad i_{an}^u = \left(G_n - G^b\right) u_n, \quad n = 1 \div N$$

$$P_a^u = P_a - P_a^b = 0$$
$$W_a^u = 0$$

- ✓ **Unbalanced Reactive currents**

$$\underline{i}_r^u = \underline{i}_r - \underline{i}_r^b \quad \Rightarrow \quad i_{rn}^u = \left(B_n - B^b\right) \hat{u}_n, \quad n = 1 \div N$$

$$P_r^u = 0$$
$$W_r^u = W_r - W_r^b = 0$$

Orthogonal current decomposition
Extension to poly-phase: 3-wires / 4-wires

✓ **Void currents:** **as for single-phase systems, they reflect the presence of scattered active, scattered reactive and generated terms.**

$$\underline{\dot{i}}_v = \underline{\dot{i}} - \underline{\dot{i}}_a - \underline{\dot{i}}_r = \underline{\dot{i}}_a^s + \underline{\dot{i}}_r^s + \underline{\dot{i}}_g$$

Scattered current terms:
Account for different values of equivalent admittance at different harmonics

Load-generated harmonic current:
Harmonic terms that exist in currents only, not in voltages

$$P_v = P - P_a - P_r = 0$$
$$W_v = W - W_a - W_r = 0$$

Orthogonal current decomposition
Extension to poly-phase: 3-wires / 4-wires

✓ Summary of current decomposition

$$\underline{i} = \underline{i}_a + \underline{i}_r + \underline{i}_v = \underline{i}_a^b + \underline{i}_a^u + \underline{i}_r^b + \underline{i}_r^u + \underline{i}_a^s + \underline{i}_r^s + \underline{i}_g$$

- ✓ $\underline{i}_a$ ***active currents***
  - $\underline{i}_a^b$ ***balanced active currents***
  - $\underline{i}_a^u$ ***unbalanced active currents***
- ✓ $\underline{i}_r$ ***reactive currents***
  - $\underline{i}_r^b$ ***balanced reactive currents***
  - $\underline{i}_r^u$ ***unbalanced reactive currents***
- ✓ $\underline{i}_v$ ***void currents***
  - $\underline{i}_a^s$ ***scattered active currents***
  - $\underline{i}_r^s$ ***scattered reactive currents***
  - $\underline{i}_g$ ***load-generated harmonic currents***

Orthogonal current decomposition
Extension to poly-phase: 3-wires / 4-wires

✓ **Summary of current decomposition**

$$\underline{i} = \underline{i}_a + \underline{i}_r + \underline{i}_v = \underbrace{\underline{i}_a^{\bullet b} + \underline{i}_a^{\bullet u}}_{\underline{i}_a} + \underbrace{\underline{i}_r^{\bullet b} + \underline{i}_r^{\bullet u}}_{\underline{i}_r} + \underbrace{\underline{i}_a^{\bullet s} + \underline{i}_r^{\bullet s} + \underline{i}_g}_{\underline{i}_v}$$

Each current component has a precise
PHISICAL MEANING and is computed in the time domain

Moreover, all current terms defined in the above equation are ORTHOGONAL, thus:

$$\left\|\underline{i}\right\|^2 = \left\|\underline{i}_a\right\|^2 + \left\|\underline{i}_r\right\|^2 + \left\|\underline{i}_v\right\|^2 = \left\|\underline{i}_a^{\bullet b}\right\|^2 + \left\|\underline{i}_a^{\bullet u}\right\|^2 + \left\|\underline{i}_r^{\bullet b}\right\|^2 + \left\|\underline{i}_r^{\bullet u}\right\|^2 + \left\|\underline{i}_a^{\bullet s}\right\|^2 + \left\|\underline{i}_r^{\bullet s}\right\|^2 + \left\|\underline{i}_g\right\|^2$$

Apparent power decomposition in poly-phase: 3-wires / 4-wires

$$A = \boldsymbol{U}\,\boldsymbol{I} = \|\underline{u}\|\,\|\underline{i}\| = \sqrt{P^2 + Q^2 + N^2 + V^2}$$

Active power:

$$P = \boldsymbol{U}\,\boldsymbol{I}_a^b = \|\underline{u}\|\,\left\|\underline{i}_a^b\right\|$$

Reactive power:

$$Q = \boldsymbol{U}\,\boldsymbol{I}_r^b = \|\underline{u}\|\,\left\|\underline{i}_r^b\right\|$$

Unbalance power:

$$N = \boldsymbol{U}\,\boldsymbol{I}^{\boldsymbol{u}} = \|\underline{u}\|\,\left\|\underline{i}^u\right\| = \sqrt{N_a^2 + N_r^2}$$

Void power:

$$V = \mathbf{U}\,\mathbf{I}_v = \|\underline{u}\|\,\|\underline{i}_v\| = \sqrt{S_a^2 + S_r^2 + V_g^2}$$

Unbalance Power Terms

Unbalance power:

$$N = \sqrt{N_a^2 + N_r^2}$$

✓ **Unbalance Active Power**

$$N_a = \boldsymbol{U}\,\boldsymbol{I}_a^u = \|\underline{u}\|\,\|\underline{i}_a^u\| = \boldsymbol{U}\sqrt{\sum_{n=1}^{N}\frac{P_n}{U_n^2} - \frac{P^2}{\boldsymbol{U}^2}}$$

✓ **Unbalance Reactive Power**

$$N_r = \boldsymbol{U}\,\boldsymbol{I}_r^u = \omega\boldsymbol{U}\,\frac{\sqrt{1+[THD(u)]^2}}{\sqrt{1+[THD(\hat{u})]^2}}\sqrt{\sum_{n=1}^{N}\frac{W_n}{\hat{U}_n^2} - \frac{W^2}{\hat{\boldsymbol{U}}^2}}$$

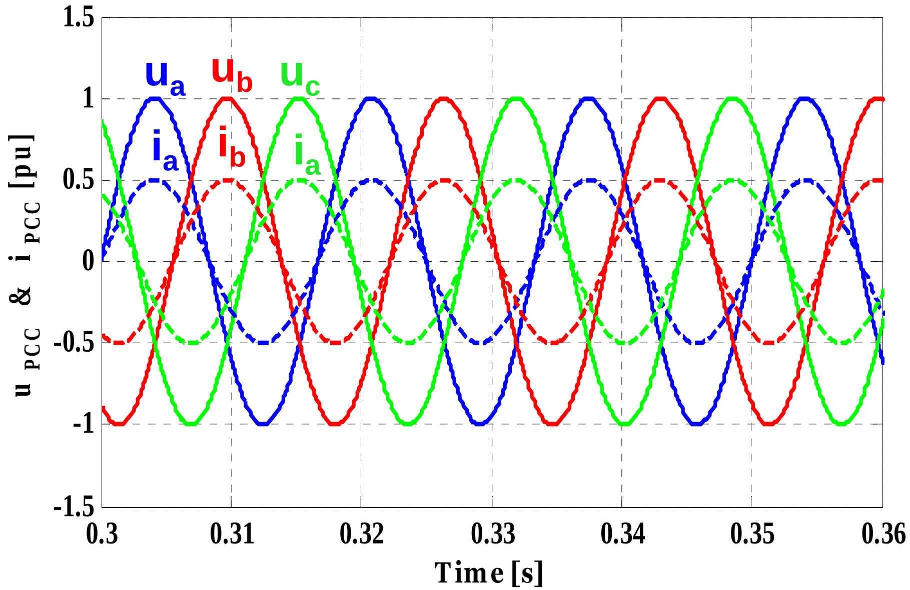
Unbalance active and reactive power vanish if the load is balanced

Skip examples

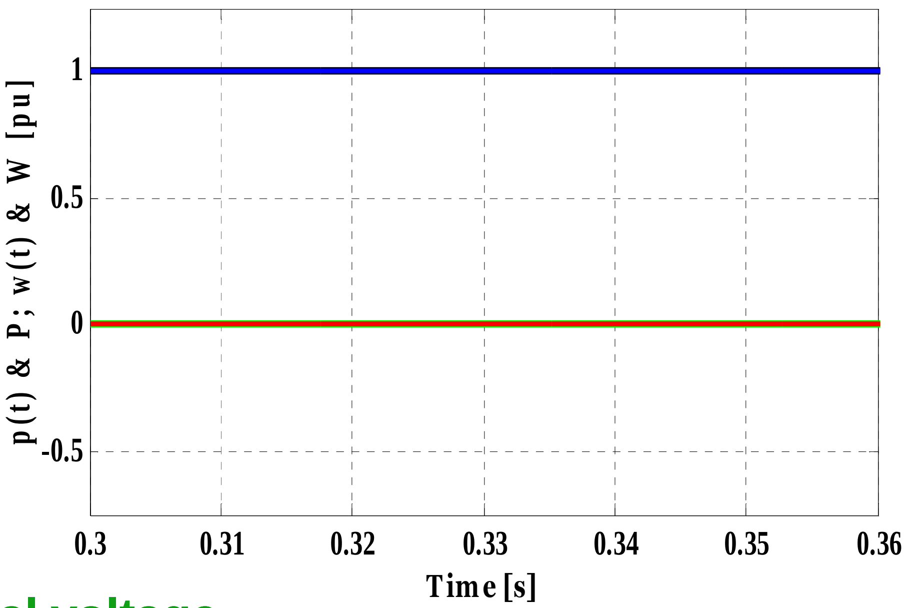
Application Examples

Example # 1 : 3-phase 3-wire – Balanced load
(Resistive)

Voltage and Current



Conservative Power Terms



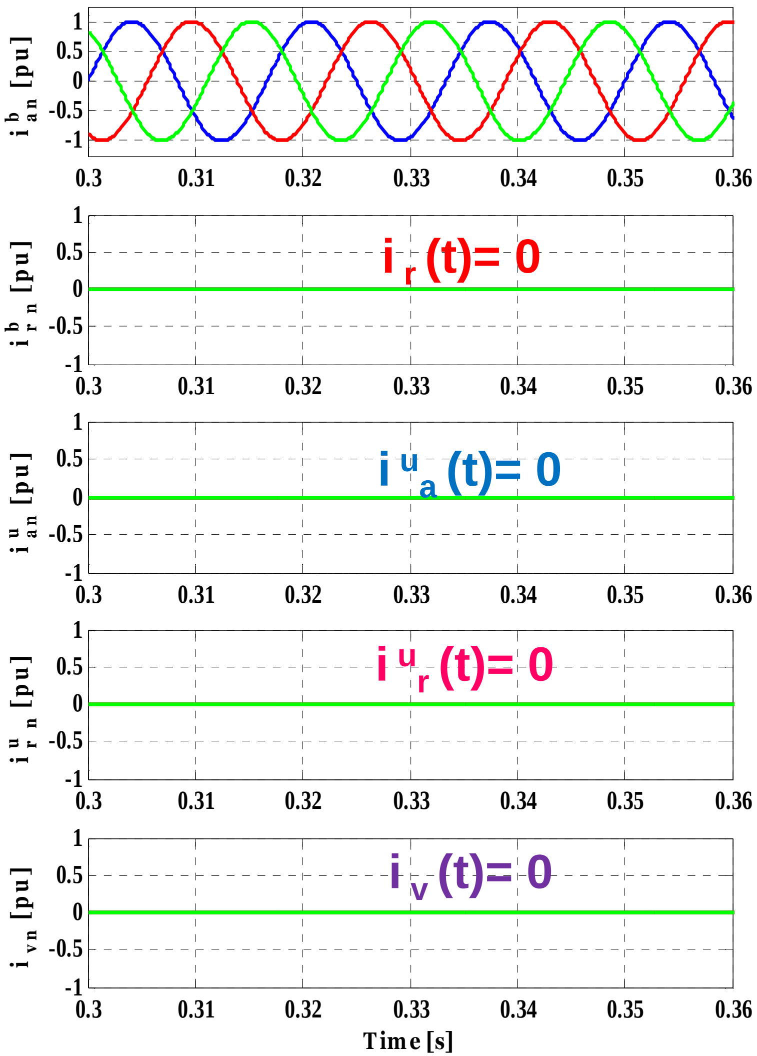
Sinusoidal voltage

Current = $i_{pu}(t)/2$

Application Examples

Example # 1 : 3-phase 3-wire – Balanced load

(Resistive)



Balanced active currents

Balanced reactive currents

Unbalanced active currents

Unbalanced reactive currents

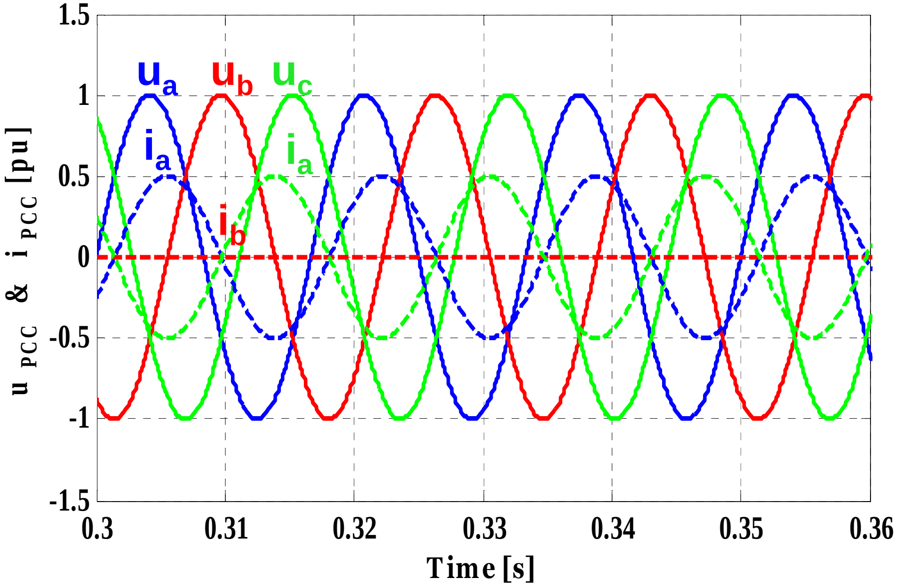
Void currents

Application Examples

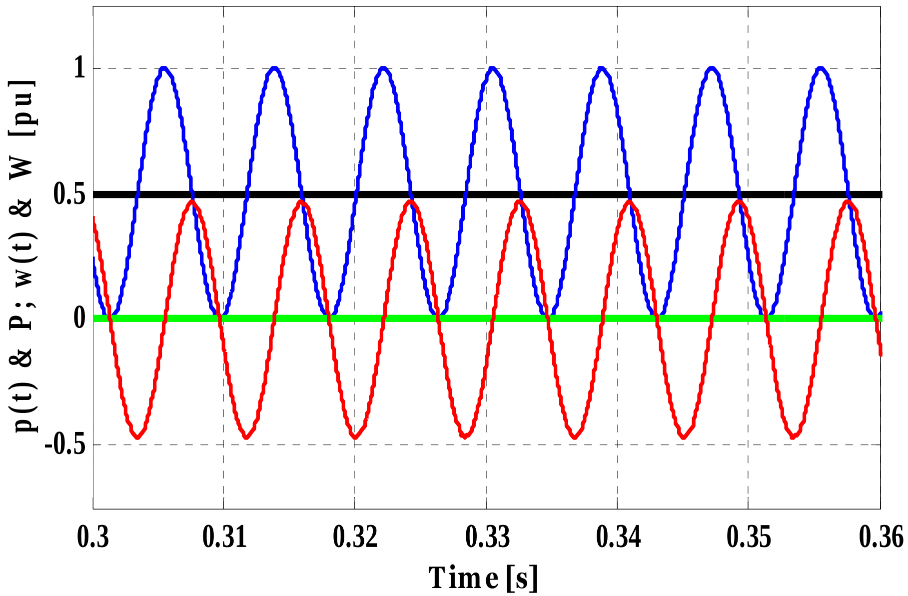
Example # 1 : 3-phase 3-wire – Unbalanced load

(resistor connected between two phases)

Voltage and Current



Conservative Power Terms



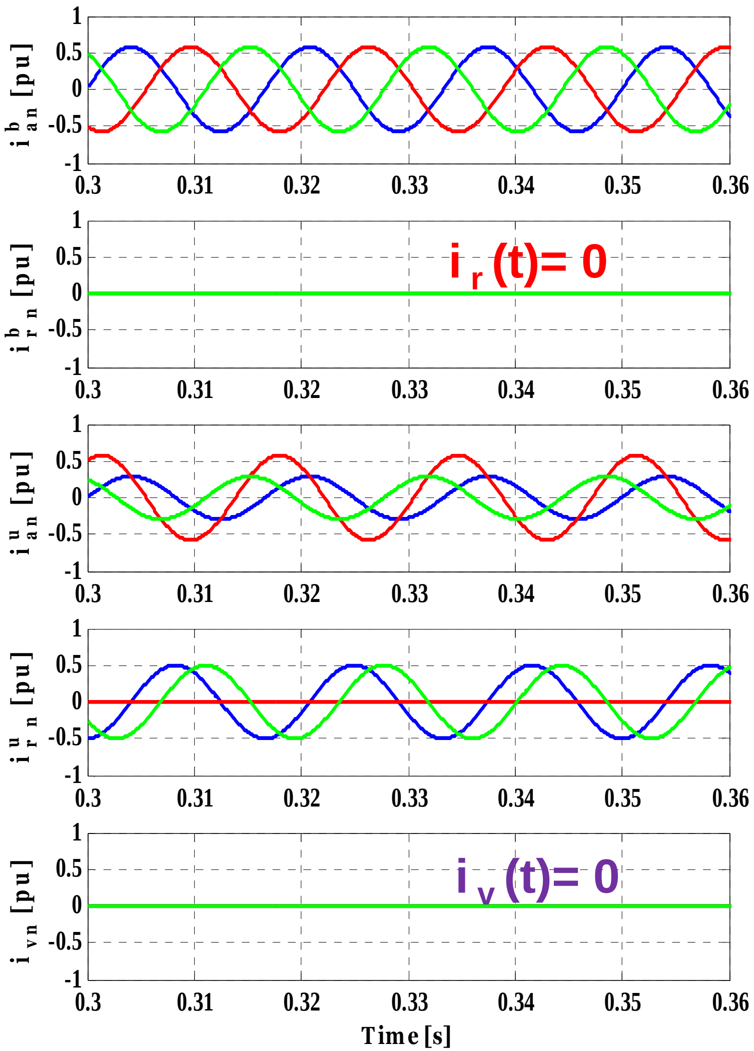
Sinusoidal voltage

Current = $i_{pu}(t)/2$

Application Examples

Example # 1 : 3-phase 3-wire – Unbalanced load

(resistor connected between two phases)



Balanced active currents

Balanced reactive currents

Unbalanced active currents

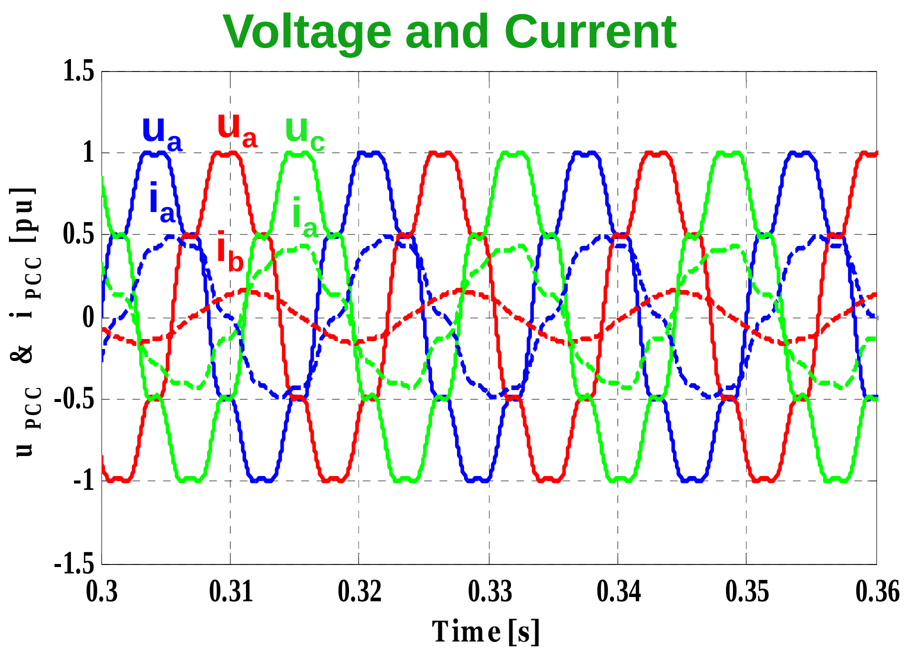
Unbalanced reactive currents

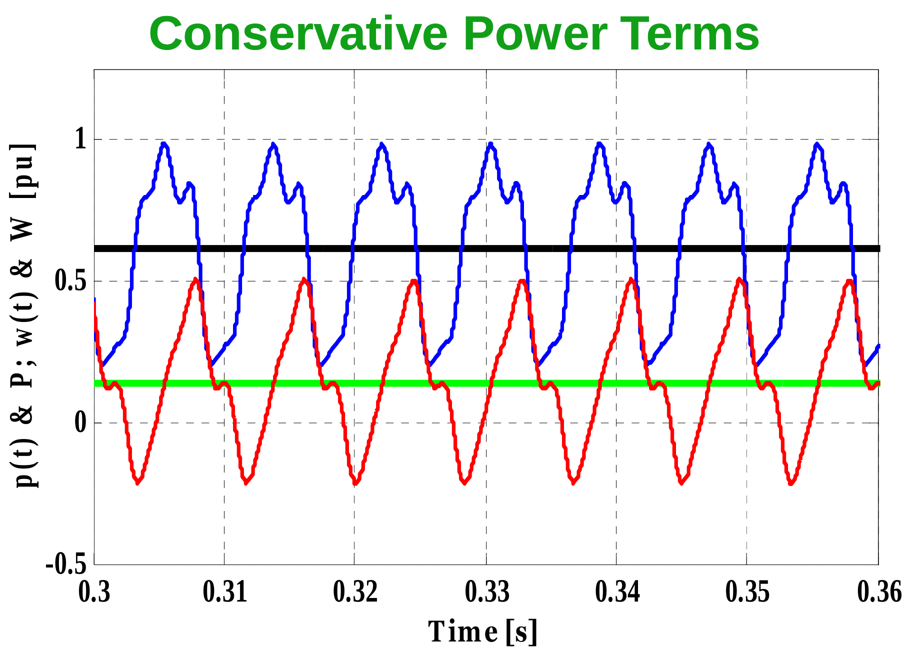
Void currents

Application Examples

Example # 3 : 3-phase 3-wire

Three-phase RL + Single-phase R load



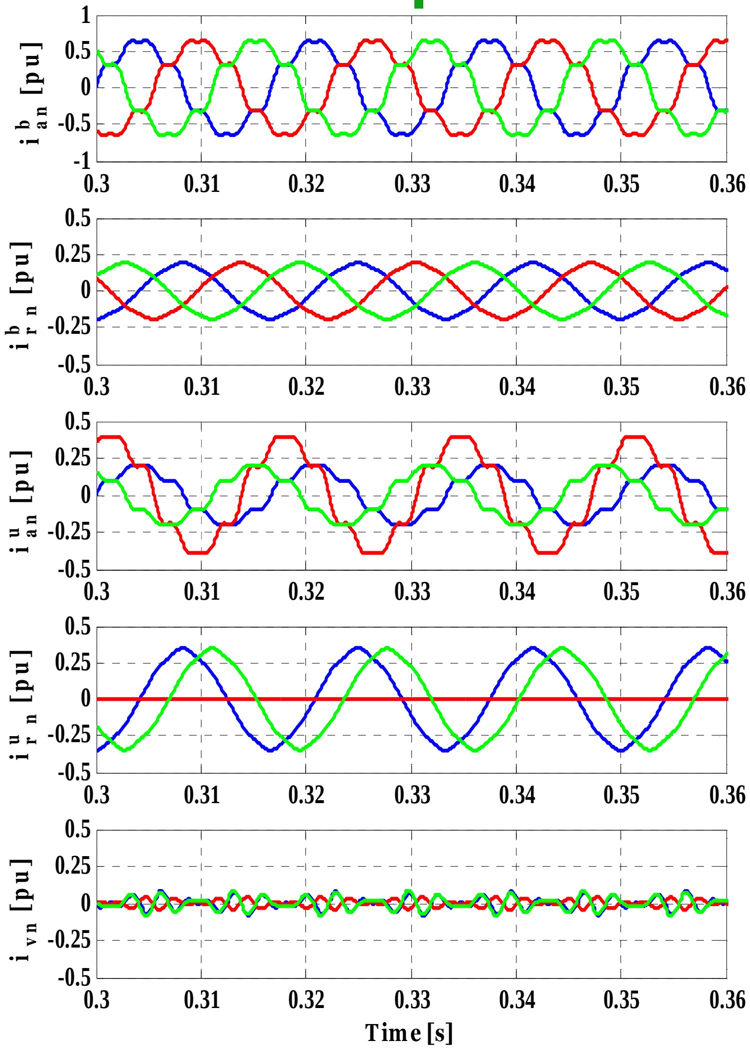


Symmetrical non-sinusoidal voltage

Application Examples

Example # 3 : 3-phase 3-wire

Three-phase RL + Single-phase R load



Balanced active currents

Balanced reactive currents

Unbalanced active currents

Unbalanced reactive currents

Void currents

Symmetrical non-sinusoidal voltage

Sharing of compensation duties

✓ **Orthogonal current terms:**

$$\underline{i} = \underline{i}_a + \underline{i}_r + \underline{i}_v = \underbrace{\underline{i}_a^b + \underline{i}_a^u}_{\underline{i}_a} + \underbrace{\underline{i}_r^b + \underline{i}_r^u}_{\underline{i}_r} + \underbrace{\underline{i}_{sa} + \underline{i}_{sr} + \underline{i}_g}_{\underline{i}_v}$$

Each current component has a precise PHYSICAL MEANING

- ✓ **Balanced Active currents convey active power P**
- ✓ **Balanced Reactive currents convey reactive power Q**
- ✓ **Unbalanced Active and Reactive currents account for asymmetrical behavior of the various phases**
- ✓ **Void currents reflect the presence of different behavior at different frequencies and/or generated current harmonics**

Sharing of compensation duties

$$\underline{i} = \underline{i}_a + \underline{i}_r + \underline{i}_v = \underline{i}_a^b + \boxed{\underline{i}_r^b} + \boxed{\underline{i}_a^u + \underline{i}_r^u} + \boxed{\underline{i}_v}$$

Reactive compensation

↓

Stationary Compensators (reactive impedances) & Quasi-Stationary Compensators (SVC, Static VAR Compensators)

Unbalance compensation requires controllable reactances (extended Steinmetz approach)

↓

Quasi-Stactionary Compensators (SVC)

Harmonic compensation requires high-frequency response

↓

Passive filters & Switching Power Compensators (SPC=APF+SPI)

Effect of compensation on power terms

Active power (balanced):	$P = \mathbf{U}\,\mathbf{I}_a^b$	**Compensation**
Reactive power (balanced):	$Q = \widehat{\mathbf{U}}\,\mathbf{I}_r^b$	$\xrightarrow{SVC} 0$
Unbalance power:	$N = \mathbf{U}\,\mathbf{I}^u = \mathbf{U}\sqrt{{\mathbf{I}_a^u}^2 + {\mathbf{I}_r^u}^2}$	$\xrightarrow{SVC} 0$
Void power:	$V = \mathbf{U}\,\mathbf{I}_v$	$\xrightarrow{SPC} 0$

APPARENT POWER

$$A = \mathbf{U}\,\mathbf{I} = \sqrt{P^2 + \cancel{Q}^2 + \cancel{N}^2 + \cancel{V}^2}$$

$$\xrightarrow{SVC,SPC} \quad A = \mathbf{U}\,\mathbf{I}_a^b = P$$

Seminar Outline

1. Motivation of work
2. Mathematical and physical foundations of the theory
   - Mathematical operators and their properties
   - Instantaneous and average power & energy terms in poly-phase networks
3. Definition of current and power terms in single-phase networks under non-sinusoidal conditions
4. Extension to poly-phase domain: 3-wires / 4-wires
5. **Sequence components under non-sinusoidal conditions**
6. **Measurement & accountability issues**

DEPARTMENT OF INFORMATION ENGINEERING
UNIVERSITY OF PADOVA

5. Sequence components under non-sinusoidal conditions

1. Problem statement
2. Goal of decomposition
3. Derivation of generalized symmetrical components in the time domain (extension of Fortescue's approach)
4. Analysis of generalized symmetrical components in the frequency domain
5. Orthogonality of sequence components
6. Application examples

Skip

1. Problem statement (1)

- ✓ Symmetrical components are very useful to simplify the analysis of three-phase networks under sinusoidal conditions

- ✓ It is important to extend the definition and application of symmetrical components to non-sinusoidal periodic operation

1. Problem statement (2)

✓ **Given periodic three-phase variables $f_a(t)$, $f_b(t)$, $f_c(t)$ of period T we define the following symmetry properties:**

- **Homopolarity (zero symmetry)**

$$f_a(t) = f_b(t) = f_c(t)$$

- **Positive (direct) symmetry**

$$f_a(t) = f_b\left(t + \frac{T}{3}\right) = f_c\left(t + \frac{2T}{3}\right)$$

- **Negative (inverse) symmetry**

$$f_a(t) = f_b\left(t - \frac{T}{3}\right) = f_c\left(t - \frac{2T}{3}\right)$$

2. Goal of decomposition

✓ **Given a set of generic three-phase variables:**

$$\underline{f} = \begin{vmatrix} f_a(t) \\ f_b(t) \\ f_c(t) \end{vmatrix}$$

we decompose them in the orthogonal form:

$$\underline{f} = \underline{f}^{\,z} + \underline{f}^{\,h} = \underline{f}^{\,z} + \underline{f}^{\,p} + \underline{f}^{\,n} + \underline{f}^{\,r}$$

where:

- f^{z} **are zero-sequence (homopolar) components**
- f^{h} **are non-zero sequence (heteropolar) components**
- f^{p} **are positive-sequence components**
- f^{n} **are negative-sequence components**
- f^{r} **are residual components**

3. Derivation of generalized symmetrical components

- **Zero sequence (homopolar) component**

$$\underline{f}^{z} = f^{z}(t)\begin{vmatrix}1\\1\\1\end{vmatrix} \qquad f^{z}(t) = \frac{f_a(t)+f_b(t)+f_c(t)}{3}$$

- **Heteropolar components**

$$\underline{f}^{h} = \underline{f} - \underline{f}^{z} = \begin{vmatrix}f_a^h(t)\\f_b^h(t)\\f_c^h(t)\end{vmatrix} = \begin{vmatrix}f_a(t)-f^{z}(t)\\f_b(t)-f^{z}(t)\\f_c(t)-f^{z}(t)\end{vmatrix}$$

3. Derivation of generalized symmetrical components

- **Positive sequence component**

$$f^{p}(t)=\frac{1}{3}\left[f_a^{h}(t)+f_b^{h}\left(t+\frac{T}{3}\right)+f_c^{h}\left(t+\frac{2T}{3}\right)\right]$$

$$f_a^{p}(t)=f^{p}(t),\quad f_b^{p}(t)=f^{p}\left(t-\frac{T}{3}\right),\quad f_c^{p}(t)=f^{p}\left(t-\frac{2T}{3}\right)$$

- **Negative sequence component**

$$f^{n}(t)=\frac{1}{3}\left[f_a^{h}(t)+f_b^{h}\left(t-\frac{T}{3}\right)+f_c^{h}\left(t-\frac{2T}{3}\right)\right]$$

$$f_a^{n}(t)=f^{n}(t),\quad f_b^{n}(t)=f^{n}\left(t+\frac{T}{3}\right),\quad f_c^{n}(t)=f^{n}\left(t+\frac{2T}{3}\right)$$

3. Derivation of generalized symmetrical components

- **Residual components**

$$\underline{f}^{r}=\begin{vmatrix} f_a^r(t) \\ f_b^r(t) \\ f_c^r(t) \end{vmatrix}=\begin{vmatrix} \dfrac{f_a^h(t)+f_a^h\left(t-\dfrac{T}{3}\right)+f_a^h\left(t-\dfrac{2T}{3}\right)}{3} \\ \dfrac{f_b^h(t)+f_b^h\left(t-\dfrac{T}{3}\right)+f_b^h\left(t-\dfrac{2T}{3}\right)}{3} \\ \dfrac{f_c^h(t)+f_c^h\left(t-\dfrac{T}{3}\right)+f_c^h\left(t-\dfrac{2T}{3}\right)}{3} \end{vmatrix}$$

- **Note:** **these components are computed independently for each phase and vanish in sinusoidal operation**

3. Derivation of generalized symmetrical components

- **Resulting decomposition**

$$\underline{f} = \begin{vmatrix} f_a(t) \\ f_b(t) \\ f_c(t) \end{vmatrix} = \begin{vmatrix} f^z(t) + f^p(t) + f^n(t) + f_a^r(t) \\ f^z(t) + f^p\left(t - \frac{T}{3}\right) + f^n\left(t + \frac{T}{3}\right) + f_b^r(t) \\ f^z(t) + f^p\left(t - \frac{2T}{3}\right) + f^n\left(t + \frac{2T}{3}\right) + f_c^r(t) \end{vmatrix}$$

4. Analysis in the frequency domain

Expressing variables $f_a(t)$, $f_b(t)$, $f_c(t)$ in Fourier series:

$$f_a(t) = \sum_{k=1}^{\infty} f_{ak}(t) = \sum_{k=1}^{\infty} \sqrt{2} F_{ak} \sin(k\omega t + \alpha_{ak})$$

$$f_b(t) = \sum_{k=1}^{\infty} f_{bk}(t) = \sum_{k=1}^{\infty} \sqrt{2} F_{bk} \sin(k\omega t + \alpha_{bk})$$

$$f_c(t) = \sum_{k=1}^{\infty} f_{ck}(t) = \sum_{k=1}^{\infty} \sqrt{2} F_{ck} \sin(k\omega t + \alpha_{ck})$$

we can determine, for each harmonic, the zero, positive, and negative components $f^{z}_{k}(t)$, $f^{p}_{k}(t)$, $f^{n}_{k}(t)$. Instead, residual harmonic components are zero because harmonic quantities are sinusoidal.

4. Analysis in the frequency domain

Contribution of harmonic sequence components to generalized sequence components

$$Harmonic\ order: k = 3m+1 \quad \forall m \in [0,\infty]$$

$$\boldsymbol{f}_k^p \Rightarrow \boldsymbol{f}_p, \quad \boldsymbol{f}_k^n \Rightarrow \boldsymbol{f}_n, \quad \boldsymbol{f}_k^z \Rightarrow \boldsymbol{f}^z$$

$$Harmonic\ order: k = 3m+2 \quad \forall m \in [0,\infty]$$

$$\boldsymbol{f}_k^p \Rightarrow \boldsymbol{f}^n, \quad \boldsymbol{f}_k^n \Rightarrow \boldsymbol{f}^p, \quad \boldsymbol{f}_k^z \Rightarrow \boldsymbol{f}_z$$

$$Harmonic\ order: k = 3m \quad \forall m \in [0,\infty]$$

$$\boldsymbol{f}_k^p \Rightarrow \boldsymbol{f}^r, \quad \boldsymbol{f}_k^n \Rightarrow \boldsymbol{f}^r, \quad \boldsymbol{f}_k^z \Rightarrow \boldsymbol{f}^z$$

5. Orthogonality of components

Given two sets of three-phase quantities $\underline{f}$ and $\underline{g}$, their sequence components obey the following general rules:

- ✓ **Scalar product**

$$\underline{f}^{z} \circ \underline{g}^{h} = \underline{f}^{z} \circ \underline{g}^{p} = \underline{f}^{z} \circ \underline{g}^{n} = \underline{f}^{z} \circ \underline{g}^{r} = 0$$

- ✓ **Internal product**

$$\left\langle \underline{f}^{p}, \underline{g}^{n} \right\rangle = \left\langle \underline{f}^{p}, \underline{g}^{r} \right\rangle = \left\langle \underline{f}^{n}, \underline{g}^{r} \right\rangle = 0$$

$$\left\langle \underline{f}, \underline{g} \right\rangle = \left\langle \underline{f}^{z}, \underline{g}^{z} \right\rangle + \left\langle \underline{f}^{h}, \underline{g}^{h} \right\rangle = \left\langle \underline{f}^{z}, \underline{g}^{z} \right\rangle + \left\langle \underline{f}^{p}, \underline{g}^{p} \right\rangle + \left\langle \underline{f}^{n}, \underline{g}^{n} \right\rangle + \left\langle \underline{f}^{r}, \underline{g}^{r} \right\rangle$$

- ✓ **Norm**

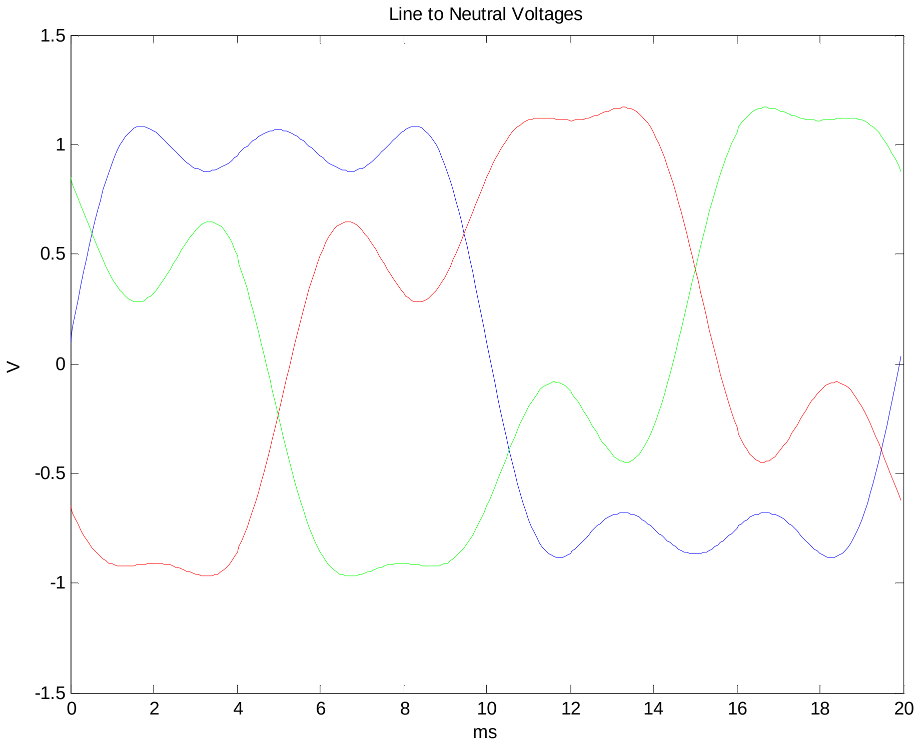
$$\left\| \underline{f} \right\|^{2} = \left\| \underline{f}^{z} \right\|^{2} + \left\| \underline{f}^{h} \right\|^{2} = \left\| \underline{f}^{z} \right\|^{2} + \left\| \underline{f}^{p} \right\|^{2} + \left\| \underline{f}^{n} \right\|^{2} + \left\| \underline{f}^{r} \right\|^{2}$$

6. Application Example

$$v_a(t) = sin(\omega t) + \frac{1}{3} sin(3\omega t) + \frac{1}{5} sin(5\omega t) + \frac{1}{10} sin(\omega t) + \frac{1}{10};$$

$$v_b(t) = sin(\omega t - \frac{2\pi}{3}) + \frac{1}{3} sin(3\omega t + \frac{2\pi}{3}) + \frac{1}{5} sin(5\omega t - \frac{2\pi}{3}) + \frac{1}{10} sin(\omega t) + \frac{1}{10};$$

$$v_c(t) = sin(\omega t + \frac{2\pi}{3}) + \frac{1}{3} sin(3\omega t - \frac{2\pi}{3}) + \frac{1}{5} sin(5\omega t + \frac{2\pi}{3}) + \frac{1}{10} sin(\omega t) + \frac{1}{10};$$

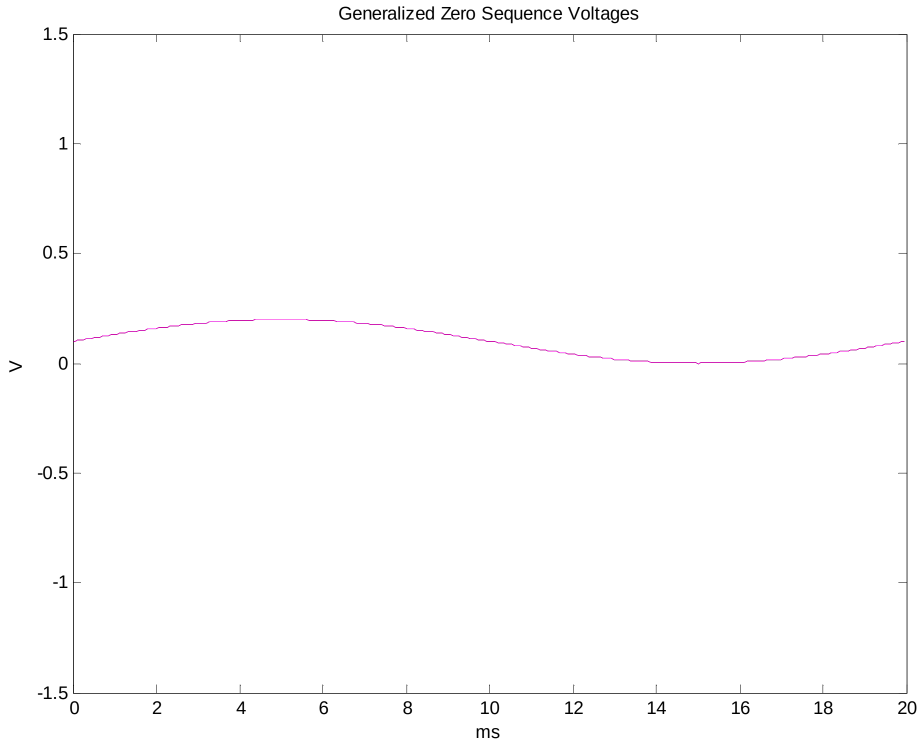


6. Application Example

$$v_a(t) = sin(\omega t) + \frac{1}{3}sin(3\omega t) + \frac{1}{5}sin(5\omega t) + \frac{1}{10}sin(\omega t) + \frac{1}{10};$$

$$v_b(t) = sin(\omega t - \frac{2\pi}{3}) + \frac{1}{3}sin(3\omega t + \frac{2\pi}{3}) + \frac{1}{5}sin(5\omega t - \frac{2\pi}{3}) + \frac{1}{10}sin(\omega t) + \frac{1}{10};$$

$$v_c(t) = sin(\omega t + \frac{2\pi}{3}) + \frac{1}{3}sin(3\omega t - \frac{2\pi}{3}) + \frac{1}{5}sin(5\omega t + \frac{2\pi}{3}) + \frac{1}{10}sin(\omega t) + \frac{1}{10};$$

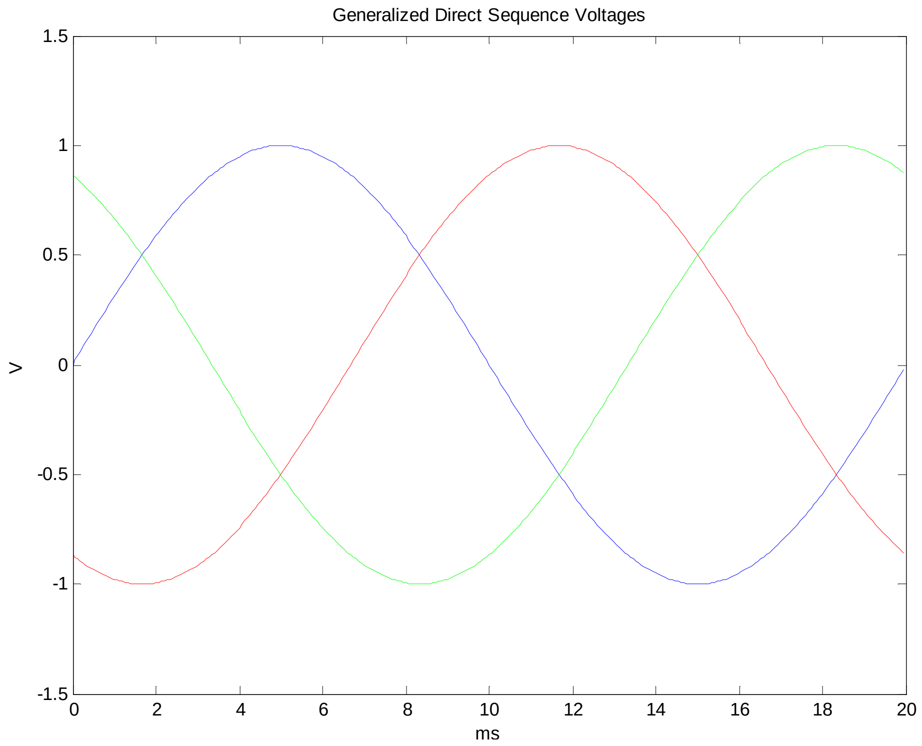


6. Application Example

$$v_a(t) = \sin(\omega t) + \frac{1}{3}\sin(3\omega t) + \frac{1}{5}\sin(5\omega t) + \frac{1}{10}\sin(\omega t) + \frac{1}{10};$$

$$v_b(t) = \sin(\omega t - \frac{2\pi}{3}) + \frac{1}{3}\sin(3\omega t + \frac{2\pi}{3}) + \frac{1}{5}\sin(5\omega t - \frac{2\pi}{3}) + \frac{1}{10}\sin(\omega t) + \frac{1}{10};$$

$$v_c(t) = \sin(\omega t + \frac{2\pi}{3}) + \frac{1}{3}\sin(3\omega t - \frac{2\pi}{3}) + \frac{1}{5}\sin(5\omega t + \frac{2\pi}{3}) + \frac{1}{10}\sin(\omega t) + \frac{1}{10};$$

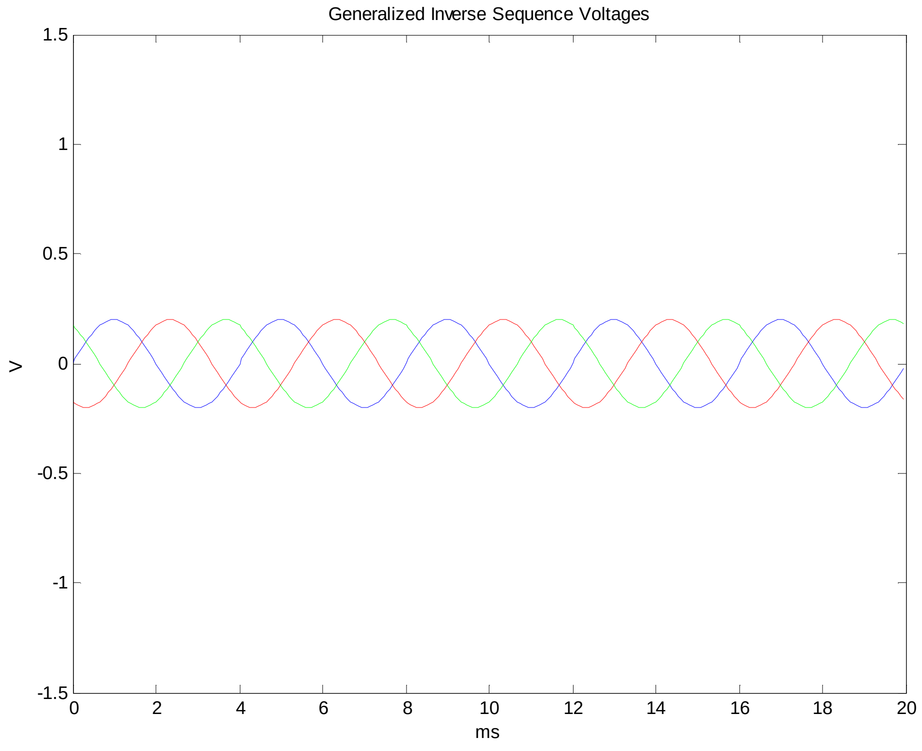


6. Application Example

$$v_a(t) = \sin(\omega t) + \frac{1}{3}\sin(3\omega t) + \frac{1}{5}\sin(5\omega t) + \frac{1}{10}\sin(\omega t) + \frac{1}{10};$$

$$v_b(t) = \sin(\omega t - \frac{2\pi}{3}) + \frac{1}{3}\sin(3\omega t + \frac{2\pi}{3}) + \frac{1}{5}\sin(5\omega t - \frac{2\pi}{3}) + \frac{1}{10}\sin(\omega t) + \frac{1}{10};$$

$$v_c(t) = \sin(\omega t + \frac{2\pi}{3}) + \frac{1}{3}\sin(3\omega t - \frac{2\pi}{3}) + \frac{1}{5}\sin(5\omega t + \frac{2\pi}{3}) + \frac{1}{10}\sin(\omega t) + \frac{1}{10};$$

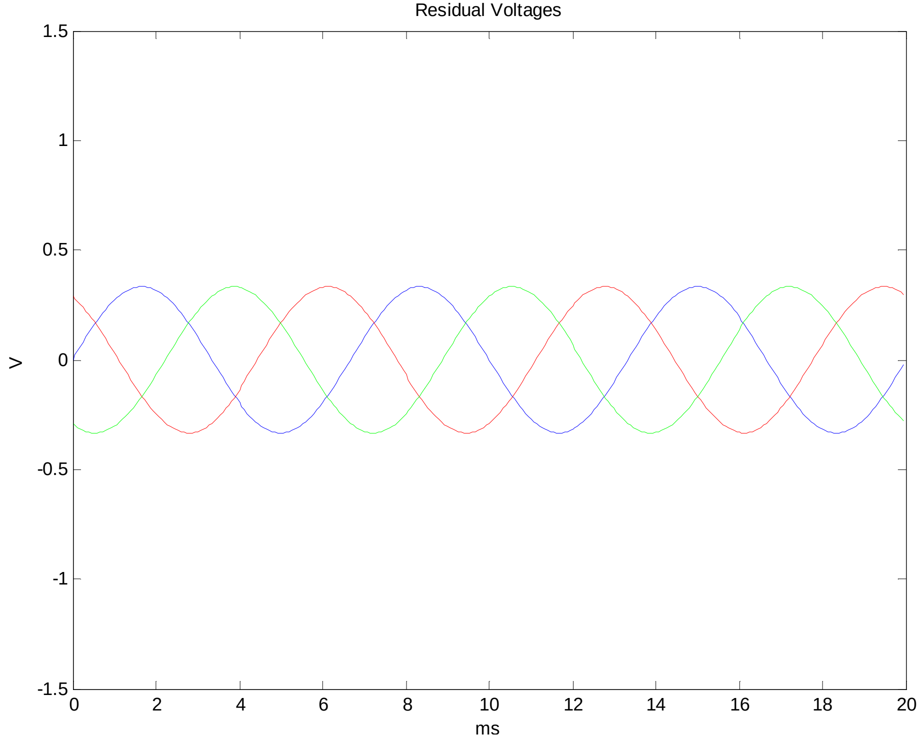


6. Application Example

$$v_a(t) = sin(\omega t) + \frac{1}{3}sin(3\omega t) + \frac{1}{5}sin(5\omega t) + \frac{1}{10}sin(\omega t) + \frac{1}{10};$$

$$v_b(t) = sin(\omega t - \frac{2\pi}{3}) + \frac{1}{3}sin(3\omega t + \frac{2\pi}{3}) + \frac{1}{5}sin(5\omega t - \frac{2\pi}{3}) + \frac{1}{10}sin(\omega t) + \frac{1}{10};$$

$$v_c(t) = sin(\omega t + \frac{2\pi}{3}) + \frac{1}{3}sin(3\omega t - \frac{2\pi}{3}) + \frac{1}{5}sin(5\omega t + \frac{2\pi}{3}) + \frac{1}{10}sin(\omega t) + \frac{1}{10};$$



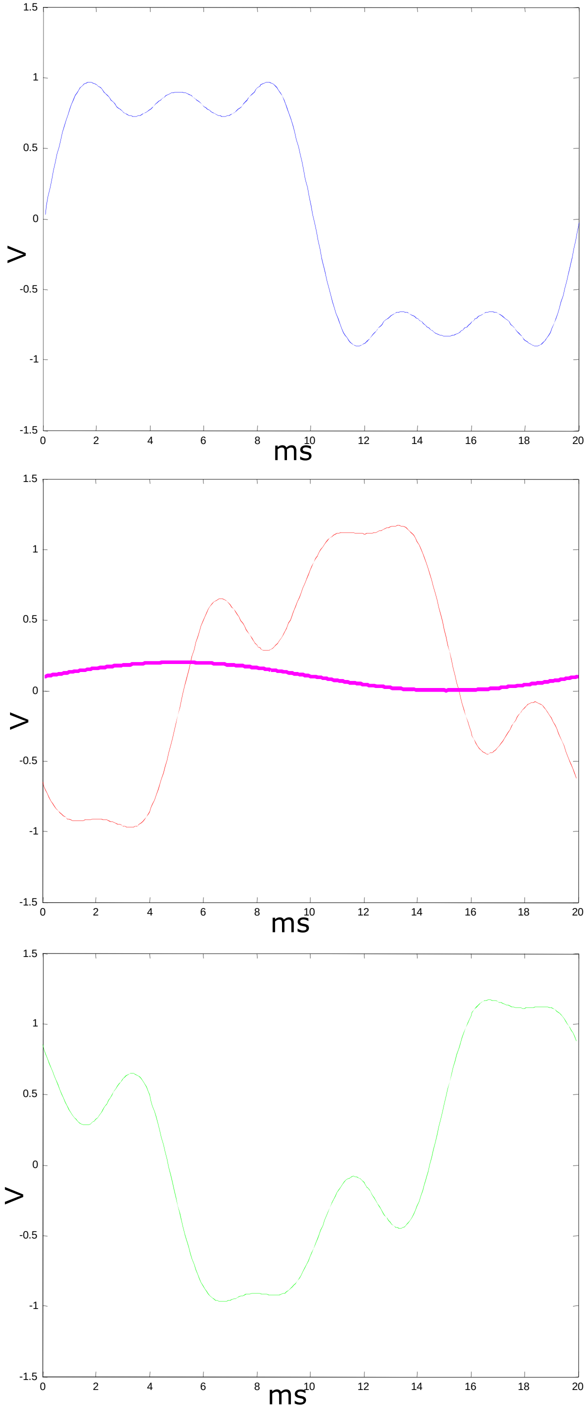
9. Summary - 1

- ✓ An extension of the sequence components in case of non-sinusoidal periodic operation has been proposed.
- ✓ It has been shown that three-phase currents (or voltages) cannot always be derived from generalized positive sequence, generalized negative sequence and generalized zero-sequence components. A residual component may be required.
- ✓ To compute the generalized positive and negative sequence components, the zero-sequence components should first be subtracted from the phase quantities, in contrast with the sinusoidal case where this is not necessary.
- ✓ In the sinusoidal case the residual component is absent and the other components reduce to the classical symmetrical components.

9. Summary - 2

- ✓ The generalized positive sequence, negative sequence, and zero-sequence components have complete phase symmetry. This implies that the three-phase analysis can be reduced to a single-phase analysis.
- ✓ The residual components do not have the same symmetry, and the corresponding three-phase analysis cannot be reduced to single-phase analysis. It corresponds to a periodic time function in each of the three phases with a period which is 1/3 of the line period; this simplifies the analysis because only 1/3 of the period must be studied.

Derivation of homopolar component



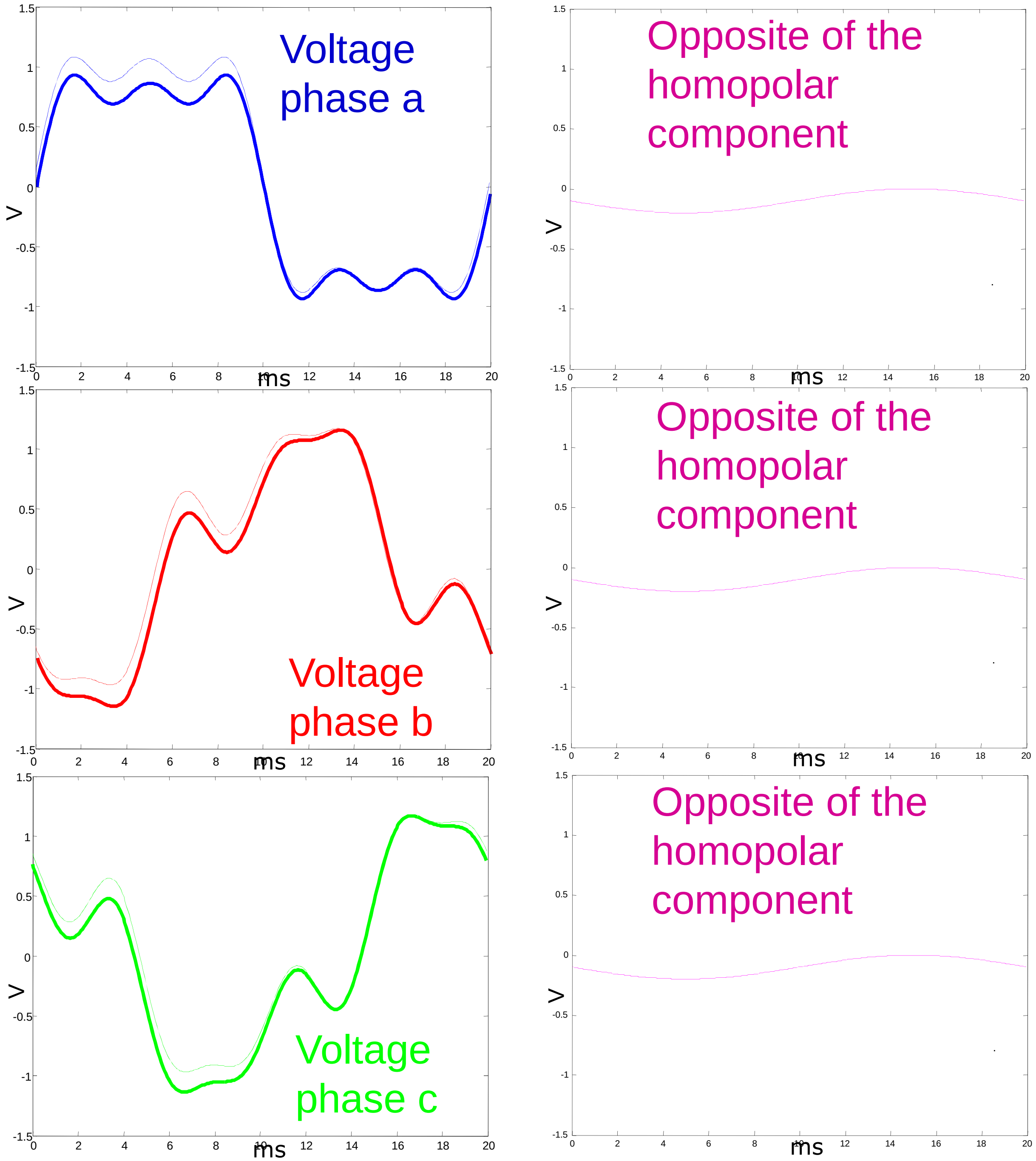
Voltage phase a

Voltage phase b

Homopolar component

Voltage phase c

Derivation of heteropolar component

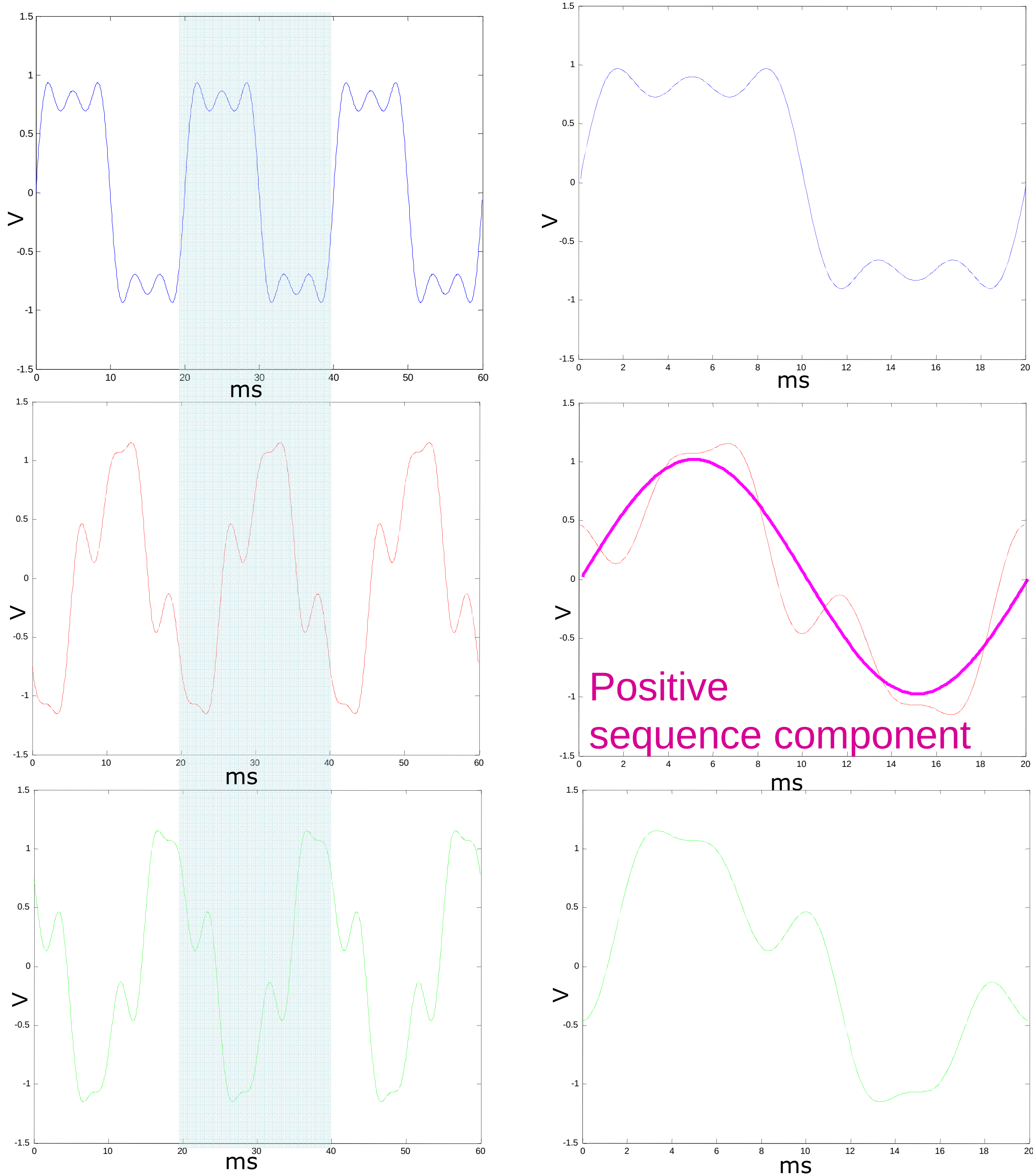


Derivation of positive sequence component (phase a)

Voltage phase a

Voltage phase b

Voltage phase c

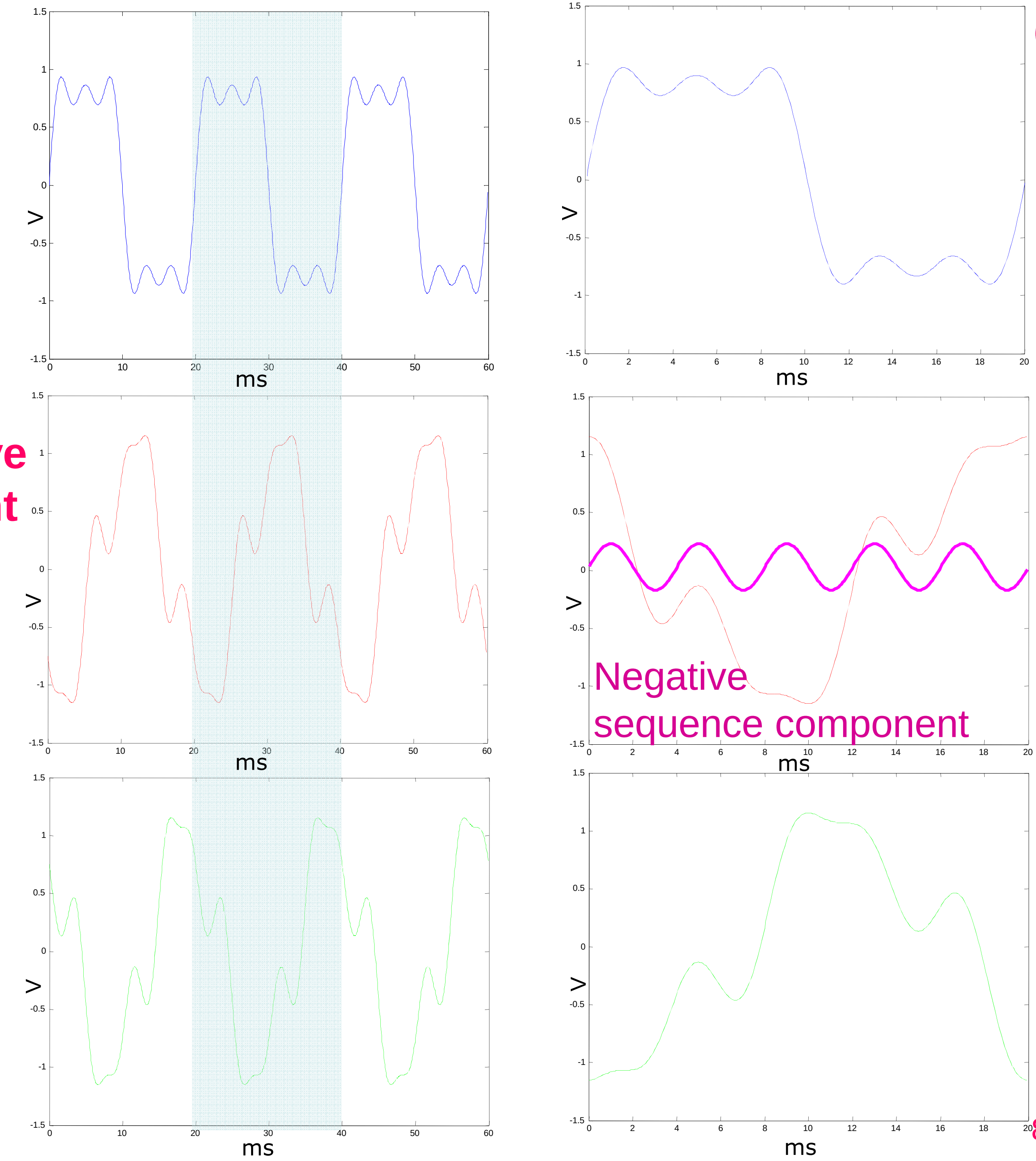


Derivation of negative sequence component (phase a)

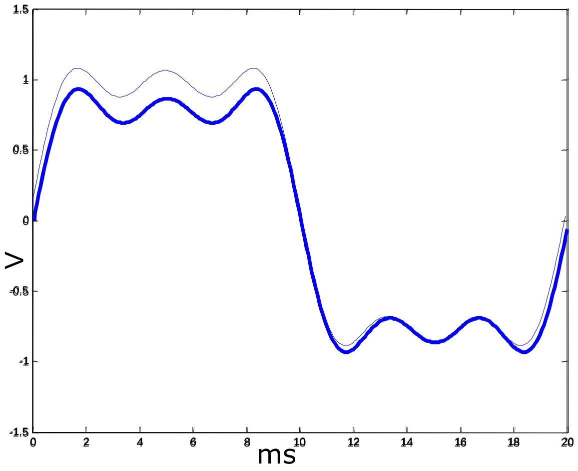
Voltage phase a

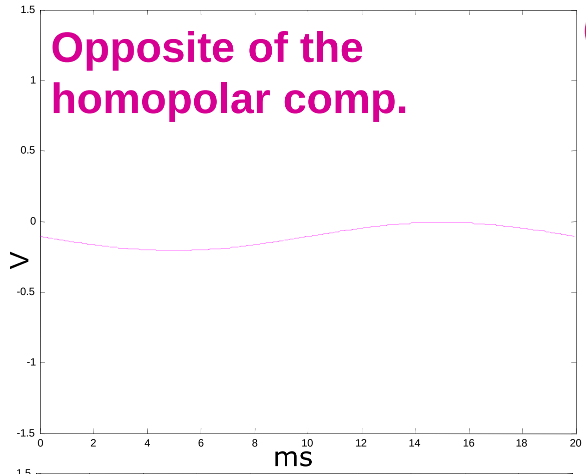
Voltage phase b

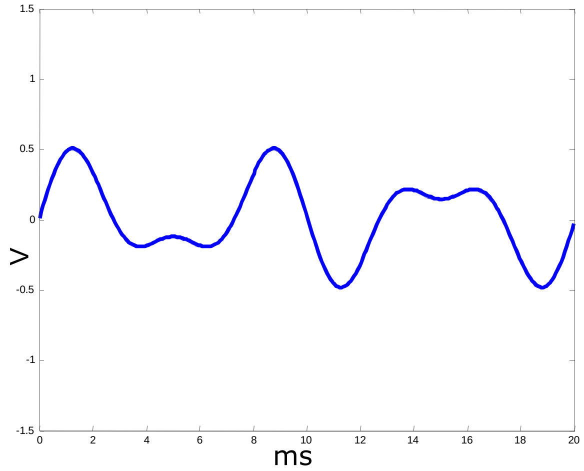
Voltage phase c

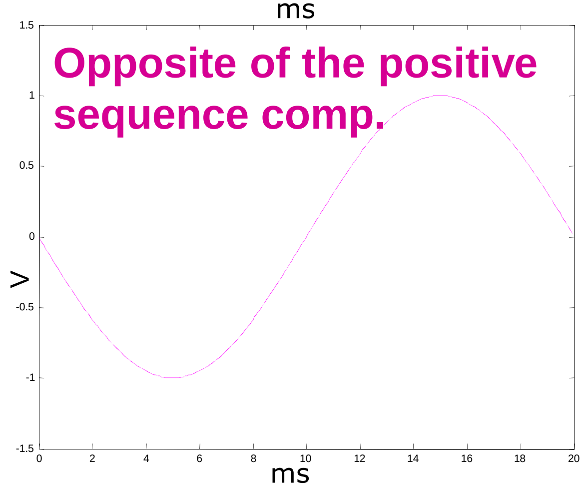


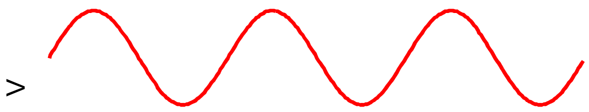
Derivation of residual component (phase a)





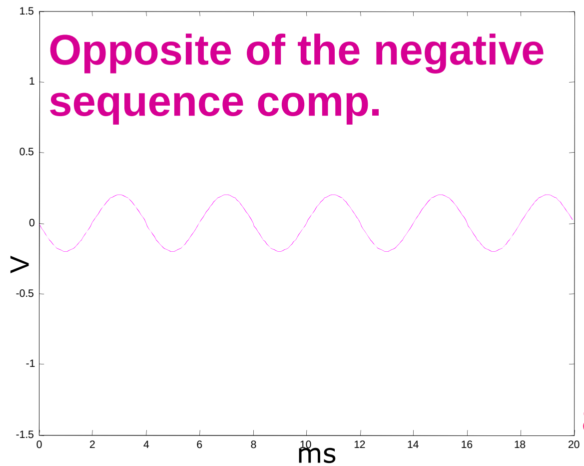






Residual component

ms



Seminar Outline

1. Motivation of work
2. Mathematical and physical foundations of the theory
3. Instantaneous and average power & energy terms in poly-phase networks
4. Definition of current and power terms in single-phase networks under non-sinusoidal conditions
5. Extension to poly-phase domain: 3-wires / 4-wires
6. Sequence components under non-sinusoidal conditions
7. **Measurement & accountability issues (basic approach)**

Measurement & accountability issues

- ✓ **Active and reactive current (and power) terms are affected by the presence of negative-sequence, zero-sequence and harmonic voltages**
- ✓ **A proper accountability approach must be adopted to depurate the power and current terms from the effects of voltage non-idealities, which are not under load responsibility**

If we assume that the supply voltages are sinusoidal and symmetrical with positive sequence, we have:

$$U_n = U_f^p, \quad n = 1 \div 3 \quad \Rightarrow \quad \boldsymbol{U} = \sqrt{3}\, U_f^p$$

$$\widehat{U}_n = \frac{U_f^p}{\omega}, \quad n = 1 \div 3 \quad \Rightarrow \quad \widehat{\boldsymbol{U}} = \frac{\sqrt{3}\, U_f^p}{\omega}$$

Phase current and power terms

- ✓ Assuming that the equivalent phase resistance remains the same irrespective of supply conditions, we can express the active current and power accountable to the load in each phase as:

$$i_{a\ell n} = G_n u_{fn}^p \Rightarrow P_{\ell n} = \langle u_{fn}^p, i_{a\ell n} \rangle = P_n \frac{U_f^{p\,2}}{U_n^2}$$

$$I_{a\ell} = \frac{1}{U_f^p} \sqrt{\sum_{n=1}^{3} P_{\ell n}^2}$$

- ✓ Similarly, if the equivalent phase reactivity remains the same irrespective of supply conditions, we can express the reactive current and power accountable to the load in each phase are:

$$i_{r\ell n} = B_n \widehat{u}_{fn}^p \Rightarrow W_{\ell n} = \langle \widehat{u}_{fn}^p, i_{r\ell n} \rangle = W_n \frac{\widehat{U}_f^{p\,2}}{\widehat{U}_n^2}$$

$$I_{r\ell} = \frac{1}{U_f^p} \sqrt{\sum_{n=1}^{3} Q_{\ell n}^2}$$

Balanced current and power terms

- ✓ **The total power terms accountable to the load are:**

$$P_\ell = \sum_{n=1}^{3} P_{\ell n} \Rightarrow G_\ell^b = \frac{P_\ell}{3U_f^{p\,2}}$$

$$W_\ell = \sum_{n=1}^{3} W_{\ell n} \Rightarrow B_\ell^b = \frac{W_\ell}{3\widehat{U}_f^{p\,2}} = \omega \frac{Q_\ell}{3U_f^{p\,2}}$$

- ✓ **The balanced current terms accountable to the load are:**

$$\underline{i}_{a\ell}^b = G_\ell^b \underline{u}_f^p \Rightarrow \boldsymbol{I}_{a\ell}^b = \frac{1}{\sqrt{3}} \frac{P_\ell}{U_f^p}$$

$$\underline{i}_{r\ell}^b = B_\ell^b \widehat{u}_{fn}^p \Rightarrow \boldsymbol{I}_{r\ell}^b = \frac{1}{\sqrt{3}} \frac{W_\ell}{\widehat{U}_f^p} = \frac{1}{\sqrt{3}} \frac{Q_\ell}{U_f^p}$$

Unbalanced current and power terms

- ✓ The unbalanced active current and power accountable to the load are :

$$i^u_{a\ell n} = i_{a\ell n} - i^b_{a\ell n} = \left(G_n - G^b_\ell\right) u^p_{fn}$$

$$I^u_{a\ell} = \sqrt{\sum_{n=1}^{3} \left(G_n - G^b_\ell\right)^2 U^{p\,2}_f} = \frac{1}{U^p_f} \sqrt{\sum_{n=1}^{3} P^2_{\ell n} - \frac{P^2_\ell}{3}}$$

- ✓ The unbalanced reactive current and power accountable to the load are :

$$i^u_{r\ell n} = i_{r\ell n} - i^b_{r\ell n} = \left(B_n - B^b_\ell\right) \widehat{u}^p_{fn}$$

$$I^u_{r\ell} = \sqrt{\sum_{n=1}^{3} \left(B_n - B^b_\ell\right)^2 \widehat{U}^{p\,2}_f} = \frac{1}{U^p_f} \sqrt{\sum_{n=1}^{3} Q^2_{\ell n} - \frac{Q^2_\ell}{3}}$$

Void current and power

- The void currents satisfy the condition:

$$\langle \underline{u}, \underline{i}_v \rangle = 0 \quad \Rightarrow \langle \underline{u}_f^p, \underline{i}_v \rangle + \langle \underline{u}_f^n + \underline{u}_f^z + \underline{u}_h, \underline{i}_v \rangle = 0$$

$$\langle \underline{\widehat{u}}, \underline{i}_v \rangle = 0 \quad \Rightarrow \langle \underline{\widehat{u}}_f^p, \underline{i}_v \rangle + \langle \underline{\widehat{u}}_f^n + \underline{\widehat{u}}_f^z + \underline{\widehat{u}}_h, \underline{i}_v \rangle = 0$$

- The void current terms which can be accounted to the load are therefore given by:

$$\underline{i}_{v\ell} = \underline{i}_v - \frac{\langle \underline{u}_f^p, \underline{i}_v \rangle}{3U_f^{p\,2}} \underline{u}_f^p - \frac{\langle \underline{\widehat{u}}_f^p, \underline{i}_v \rangle}{3\widehat{U}_f^{p\,2}} \underline{\widehat{u}}_f^p$$

- In fact, the fundamental component of the void current has been already accounted for in the active and reactive current terms.

Currents terms accountable to the load

✓ **Summary of current decomposition**

$$\underline{i}_{\ell} = \underline{i}_{a\ell} + \underline{i}_{r\ell} + \underline{i}_{v\ell} = \underline{i}_{a\ell}^{b} + \underline{i}_{r\ell}^{b} + \underline{i}_{a\ell}^{u} + \underline{i}_{r\ell}^{u} + \underline{i}_{v\ell}$$

All current terms are orthogonal, thus:

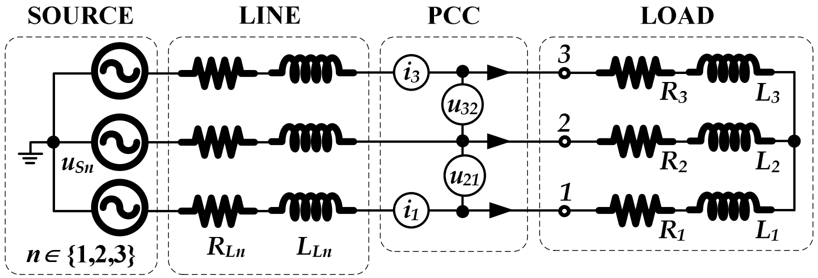
$$\mathbf{I}_{\ell} = \sqrt{\mathbf{I}_{a\ell}^{b\,2} + \mathbf{I}_{r\ell}^{b\,2} + \mathbf{I}_{a\ell}^{u\,2} + \mathbf{I}_{r\ell}^{u\,2} + \mathbf{I}_{v\ell}^{2}}$$

Apparent power accountable to the load

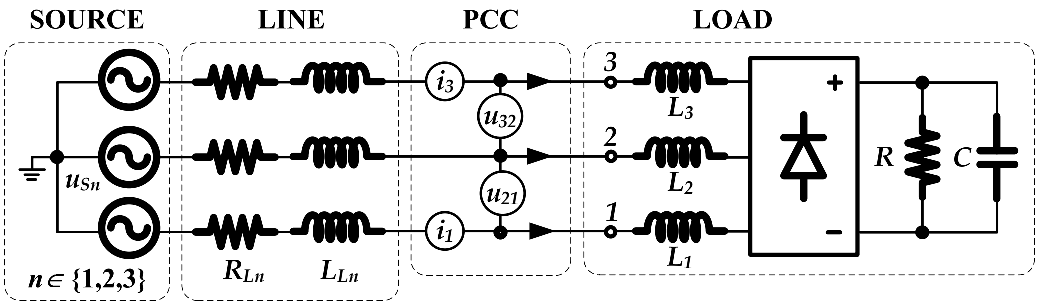
$$A_{\ell} = \boldsymbol{U}_{f}^{p}\,\boldsymbol{I}_{\ell} = \sqrt{P_{\ell}^{2} + Q_{\ell}^{2} + \underbrace{N_{a\ell}^{2} + N_{r\ell}^{2}}_{N_{\ell}^{2}} + V_{\ell}^{2}}$$

Application Examples: 3-phase 3-wire

Case I: Symmetrical sinusoidal voltages
Case II: Asymmetrical sinusoidal voltages
Case III: Symmetrical non-sinusoidal voltages
Case IV: Asymmetrical non-sinusoidal voltages



Balanced Load



Distorting Load

Case I	Case II
$U_1 = 127\angle 0^{o}$ Vrms	$U_1 = 127\angle 0^{o}$ Vrms
$U_2 = 127\angle -120^{o}$ Vrms	$U_2 = 113\angle -104{,}4^{o}$ Vrms
$U_3 = 127\angle 120^{o}$ Vrms	$U_3 = 147{,}49\angle 144^{o}$ Vrms

cases III and IV are the same of cases I and II with the addition of 10% of 5th and 7th harmonics

The line parameters are : $R_{L1} = R_{L2} = R_{L3} = 1 m\Omega$ and $L_{L0} = L_{L1} = L_{L2} = L_{L3} = 10\ \mu H$.

Application Examples: 3-phase 3-wire

Example # 1: Balanced Load

	CASE I		CASE II		CASE III		CASE IV	
	PCC	LOAD	PCC	LOAD	PCC	LOAD	PCC	LOAD
A	**1,0000**	**1,0000**	1,0000	**0,9634**	1,0000	**0,9840**	1,0000	**0,9538**
P	**0,7985**	**0,7985**	0,7985	**0,7693**	0,7913	**0,7758**	0,7945	**0,7556**
Q	**0,6020**	**0,6020**	0,6020	**0,5800**	0,6023	**0,5962**	0,6022	**0,5770**
N	0,0000	0,0000	0,0000	0,0000	0,0002	0,0002	0,0056	0,0061
V	0,0000	0,0000	0,0000	0,0000	0,1054	**0,1038**	0,0782	**0,0760**
λ	**0,7985**	**0,7985**	**0,7985**	**0,7985**	**0,7913**	**0,7885**	**0,7945**	**0,7922**

The load is accounted for less active, reactive and void power than the PCC

Application Examples: 3-phase 3-wire

Example # 2: Distorting load

	CASE I		CASE II		CASE III		CASE IV	
	PCC	LOAD	PCC	LOAD	PCC	LOAD	PCC	LOAD
A	1,0000	1,0009	1,0000	0,9050	1,0000	**0,9832**	1,0000	0,8973
P	0,8275	0,8280	0,8841	0,7668	0,8254	**0,8098**	0,8808	0,7570
Q	0,2432	0,2451	0,2135	**0,2245**	0,2422	**0,2417**	0,2135	**0,2232**
N	0,5060	0,5060	0,4156	**0,4250**	0,5055	**0,4980**	0,4185	**0,4231**
V	0,0000	0,0000	0,0000	0,0000	0,0674	**0,0666**	0,0581	0,0566
λ	0,8275	0,8273	0,8841	0,8473	0,8254	0,8237	0,8808	0,8437

Again the apparent and active power accounted to the load are always lower than those computed at PCC due to the depuration of the effects of voltage asymmetry and distortion

Defects of proposed accountability approach

- ✓ The equivalent phase conductance and reactivity are computed by considering the effect of fundamental and harmonic supply voltages as a whole. In practice, the load can have different response to fundamental and harmonic voltages.
- ✓ The void currents are not orthogonal to supply voltages if these latter become sinusoidal.
- ✓ Only the load impedance is modeled, while supply impedance is not estimated and enters the computation process indirectly, in a way that does not allow to fully analyze its effect.

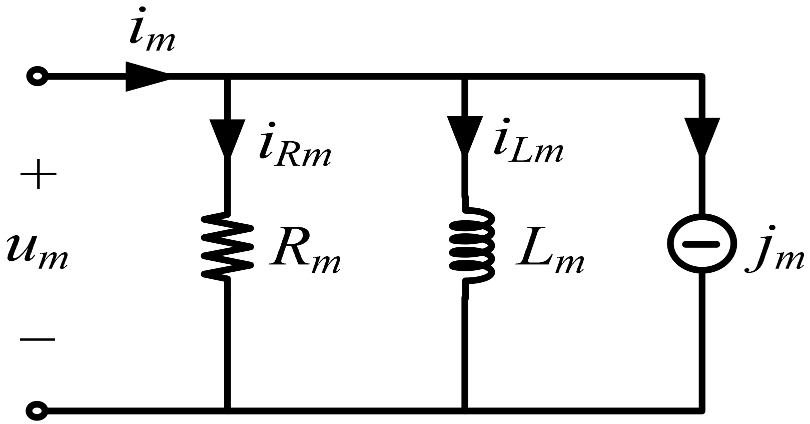
Seminar Outline

1. Motivation of work
2. Mathematical and physical foundations of the theory
3. Instantaneous and average power & energy terms in poly-phase networks
4. Definition of current and power terms in single-phase networks under non-sinusoidal conditions
5. Extension to poly-phase domain: 3-wires / 4-wires
6. Sequence components under non-sinusoidal conditions
7. **Measurement & accountability issues (extended approach)**

Extended approach to accountability

- ✓ Both load and supply are modeled based on measurements made at PCC
- ✓ Load modeling is done under sinusoidal conditions
  - This makes the load model more reliable, since harmonic effects are depurated
  - Moreover, the harmonic currents generated by the load are represented separately and their effect can directly be accounted for accountability purposes
- ✓ Supply modeling is made for three-phase symmetrical systems and allows estimation of no-load supply voltages and line impedances
- ✓ The extended accountability approach is more reliable than the basic one, and possibly avoids under- and over-penalization of the loads.
- ✓ Of course, better results can be achieved if a more accurate modeling of the load is available.

Load modeling (3-phase 4-wire) - 1



Single-phase equivalent circuit of 3-phase load seen from PCC

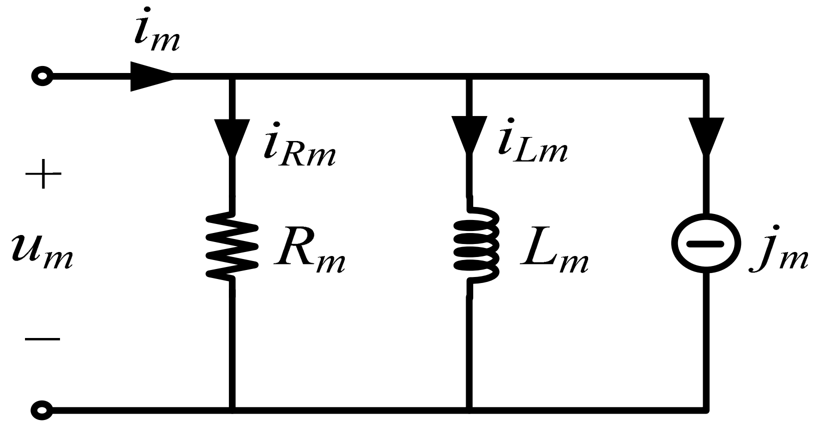
- ✓ **The passive parameters of the equivalent circuit are computed to suit the circuit performance at fundamental frequency, i.e.:**

$$i_m^f = \frac{u_m^f}{R_m} + \frac{\hat{u}_m^f}{L_m} \quad \Rightarrow \quad \begin{cases} P_m^f = \langle u_m^f, i_m^f \rangle = \dfrac{U_m^{f\,2}}{R_m} & \Rightarrow \quad R_m = \dfrac{U_m^{f\,2}}{P_m^f} \\ W_m^f = \langle \hat{u}_m^f, i_m^f \rangle = \dfrac{\hat{U}_m^{f\,2}}{L_m} & \Rightarrow \quad L_m = \dfrac{\hat{U}_m^{f\,2}}{W_m^f} \end{cases}$$

- ✓ **With this assumption current j_m is purely harmonic. In fact:**

$$j_m = i_m - \frac{u_m}{R_m^f} - \frac{\widehat{u}_m}{L_m^f} = i_m^f + i_m^h - \frac{u_m^f + u_m^h}{R_m^f} - \frac{\widehat{u}_m^f + \widehat{u}_m^h}{L_m^f} = i_m^h - \frac{u_m^h}{R_m^f} - \frac{\widehat{u}_m^h}{L_m^f}$$

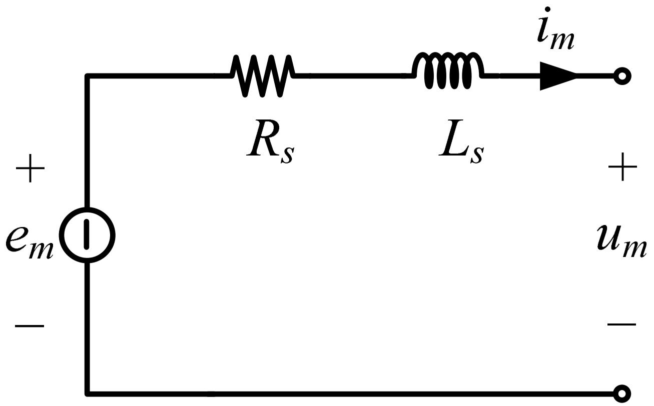
Load modeling (3-phase 4-wire) - 2



Single-phase equivalent circuit of 3-phase load seen from PCC

- ✓ **For the validity of the model we must assume that the equivalent circuit parameters remain the same within reasonable variations of the voltage supply, both in terms of asymmetry and distortion.**
- ✓ **This is only approximately true in real networks, but it makes possible an accountability approach based on measurement at the load terminals, without requiring a precise knowledge of the load itself.**

Supply modeling (3-phase 4-wire) - 1



Single-phase equivalent circuit of 3-phase supply seen from PCC

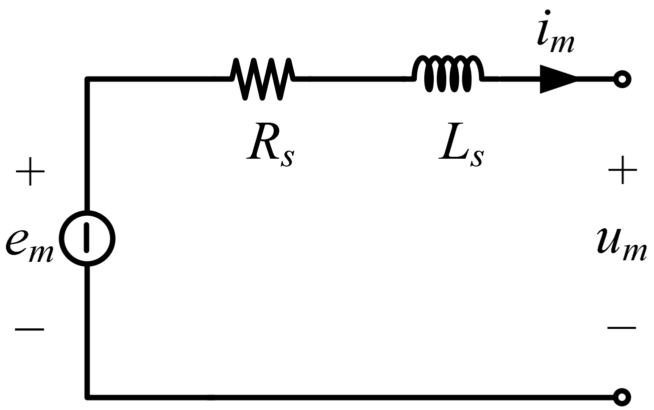
- ✓ **The passive parameters of the equivalent circuit are the same for all phases, due to supply lines symmetry. The circuit equations are:**

$$\mathbf{e} = \mathbf{u} + R_S\,\mathbf{i} + L_S\,\frac{d\,\mathbf{i}}{dt}$$

$$\Rightarrow \begin{cases} \mathbf{e}^p = \mathbf{u}^p + R_S\,\mathbf{i}^p + L_S\,\dfrac{d\,\mathbf{i}^p}{dt} \\ \mathbf{e}^n = \mathbf{u}^n + R_S\,\mathbf{i}^n + L_S\,\dfrac{d\,\mathbf{i}^n}{dt} \end{cases}$$

- ✓ **Due to its linearity, this equation can be applied separately to the fundamental positive-sequence voltage and current terms (index p) and the remaining terms (index n), which represent the unwanted current and voltage components.**

Supply modeling (3-phase 4-wire) - 2



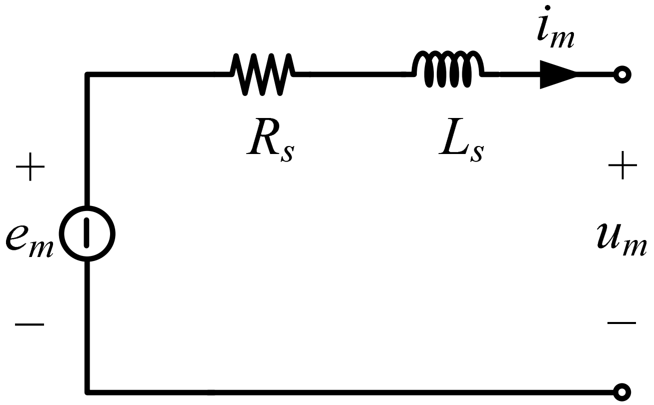
Single-phase equivalent circuit of 3-phase supply seen from PCC

- ✓ **The passive parameters of the equivalent circuit are selected so as to minimize the unwanted components of the supply voltages e^n, i.e., the function:**

$$\varphi = \left\|\mathbf{e}^n\right\|^2 = \sum_{m=1}^{M} \left\langle e_m^n, e_m^n \right\rangle =$$

$$= \left\|\mathbf{u}^n\right\|^2 + R_S^2 \left\|\mathbf{i}^n\right\|^2 + L_S^2 \left\|\frac{d\,\mathbf{i}^n}{dt}\right\|^2 + 2R_S \left\langle \mathbf{u}^n, \mathbf{i}^n \right\rangle + 2L_S \left\langle \mathbf{u}^n, L_S \frac{d\,\mathbf{i}^n}{dt} \right\rangle$$

Supply modeling (3-phase 4-wire) - 3



Single-phase equivalent circuit of 3-phase supply seen from PCC

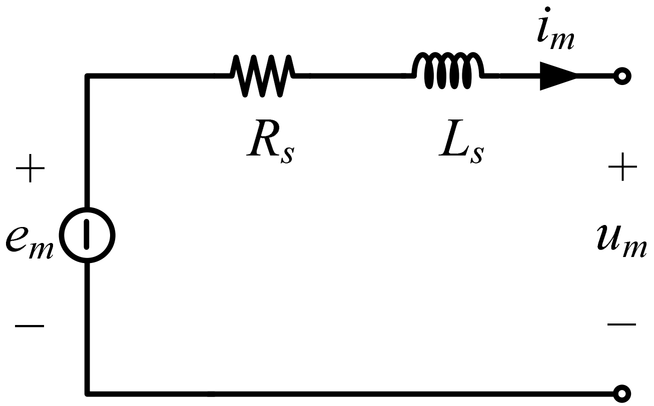
- ✓ **The result is expressed as a function of the quantities measured at PCC in the form:**

$$\frac{\partial \varphi}{\partial R_S} = 0 \quad \Rightarrow \quad R_S = -\left\langle \mathbf{u}^n, \mathbf{i}^n \right\rangle \Big/ \left\| \mathbf{i}^n \right\|^2$$

$$\frac{\partial \varphi}{\partial L_S} = 0 \quad \Rightarrow \quad L_S = -\left\langle \mathbf{u}^n, \frac{d\mathbf{i}^n}{dt} \right\rangle \Big/ \left\| \frac{d\mathbf{i}^n}{dt} \right\|^2$$

- ✓ **Obviously, only positive solutions are acceptable for R_S and L_S. In case of negative solution, the corresponding parameter is set to zero.**

Supply modeling (3-phase 4-wire) - 4



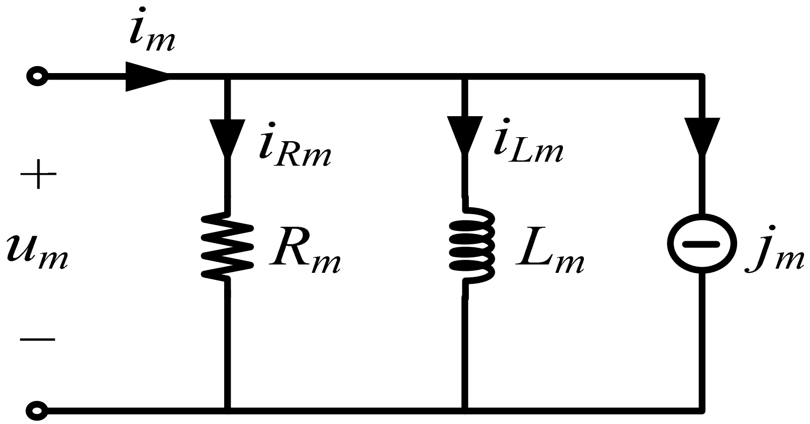
Single-phase equivalent circuit of 3-phase supply seen from PCC

- ✓ **Given R_S and L_S, we may compute the positive-sequence supply voltages to be included in the equivalent circuit :**

$$\mathbf{e}^p = \mathbf{u}^p + R_S\,\mathbf{i}^p + L_S \frac{d\mathbf{i}^p}{dt}$$

Note that e^p, u^p, i^p are the fundamental positive sequence components of the related voltages and currents, while e^n, u^n, i^n are calculated by difference from the original voltages and currents.

Accountability – Procedure (1)



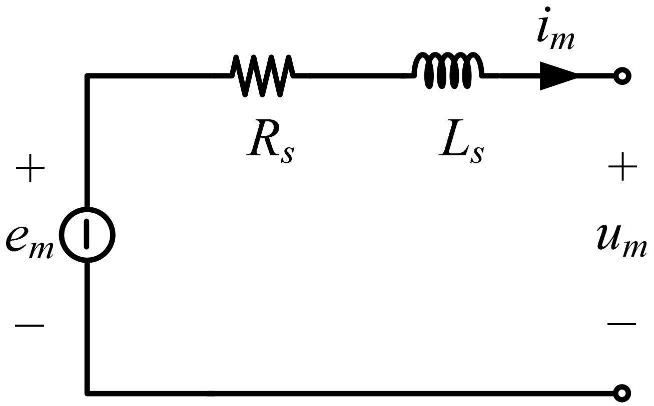
Single-phase equivalent circuit of 3-phase load seen from PCC

1. **From the voltages and currents measured at PCC we estimate the phase parameters R_m and L_m and the current source j_m of the equivalent circuit.**

$$R_m = \frac{U_m^{f\,2}}{P_m^f} \qquad L_m = \frac{\hat{U}_m^{f\,2}}{W_m^f}$$

$$j_m = i_m^h - \frac{u_m^h}{R_m^f} - \frac{\widehat{u}_m^h}{L_m^f}$$

Accountability – Procedure (2)



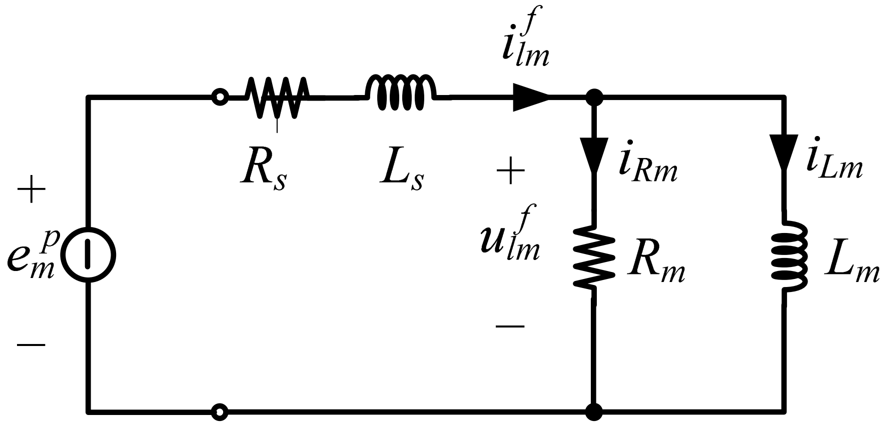
Single-phase equivalent circuit of 3-phase supply seen from PCC

2. **From the voltages and currents measured at PCC we estimate the supply line parameters R_S and L_S and the fundamental positive-sequence supply voltages e^p**

$$R_S = -\left\langle \mathbf{u}^n, \mathbf{i}^n \right\rangle \Big/ \left\| \mathbf{i}^n \right\|^2 \qquad L_S = -\left\langle \mathbf{u}^n, \frac{d\,\mathbf{i}^n}{dt} \right\rangle \Big/ \left\| \frac{d\,\mathbf{i}^n}{dt} \right\|^2$$

$$\mathbf{e}^p = \mathbf{u}^p + R_S\, \mathbf{i}^p + L_S \frac{d\mathbf{i}^p}{dt}$$

Accountability – Procedure (3)



Equivalent circuit for the computation of fundamental voltages at PCC

4. **Applying now the positive-sequence supply voltages at the input terminals of the equivalent circuit, we may determine the fundamental phase currents $\mathbf{i}_\ell^f$ absorbed by the load under these supply conditions and the corresponding fundamental phase voltages $\mathbf{u}_\ell^f$ appearing at the PCC terminals.**

✓ **Note that the currents and voltages at PCC may result asymmetrical due to load unbalance. This non-ideality must obviously be ascribed to the load, since the voltage supply and distribution lines are symmetrical**

Accountability – Procedure (5)

5. **Finally, the load voltages and currents at PCC, which are accountable to the load are given by:**

$$\mathbf{u}_\ell = \mathbf{u}_\ell^f + \mathbf{u}_\ell^h$$
$$\mathbf{i}_\ell = \mathbf{i}_\ell^f + \mathbf{i}_\ell^h$$

✓ **We can now compute all power terms accountable to the load and the corresponding performance factors.**

$$P_{\ell m} = \langle u_{\ell m}, i_{\ell m} \rangle \qquad W_{\ell m} = \langle \hat{u}_{\ell m}, i_{\ell m} \rangle$$
$$P_\ell = \sum_{m=1}^{M} P_{\ell m} \qquad W_\ell = \sum_{m=1}^{M} W_{\ell m}$$
$$N_{\ell a} = \dots \qquad N_{\ell r} = \dots$$
$$V_\ell$$

$$\Rightarrow \quad \begin{array}{l} G_{\ell m}, B_{\ell m}, \mathbf{I}_{\ell a}, \mathbf{I}_{\ell r} \\ G_\ell^b, B_\ell^b, \mathbf{I}_{\ell a}^b, \mathbf{I}_{\ell r}^b \\ \mathbf{I}_{\ell a}^u, \mathbf{I}_{\ell r}^u \\ \mathbf{I}_{\ell v} \end{array}$$

Performance factors

- ✓ **Distortion factor:** $$\lambda_D = \sqrt{1 - \frac{V^2}{A^2}} = \sqrt{\frac{P^2 + Q^2 + N^2}{A^2}}$$

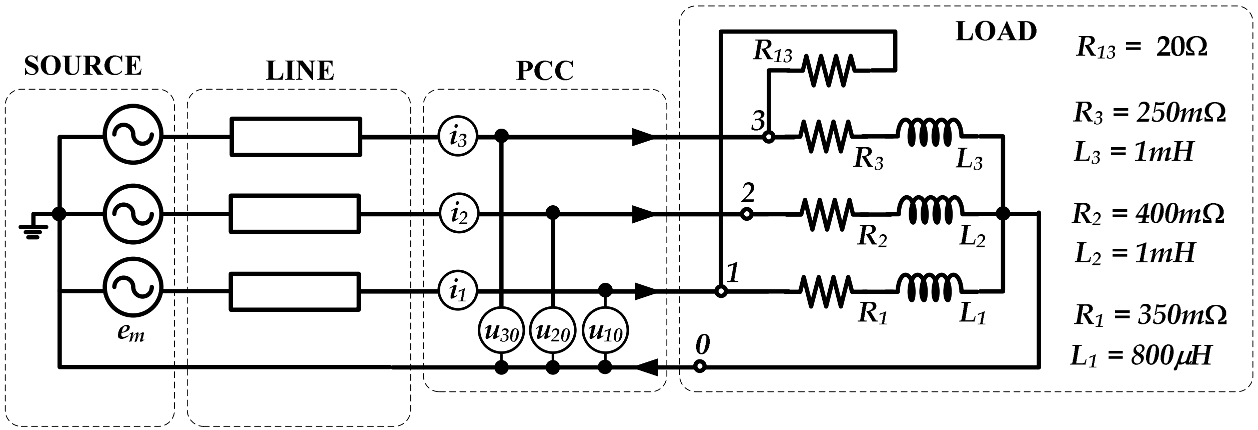
- ✓ **Unbalance factor:** $$\lambda_N = \sqrt{1 - \frac{N^2}{P^2 + Q^2 + N^2}} = \sqrt{\frac{P^2 + Q^2}{P^2 + Q^2 + N^2}}$$

- ✓ **Reactivity factor:** $$\lambda_Q = \sqrt{1 - \frac{Q^2}{P^2 + Q^2}}$$

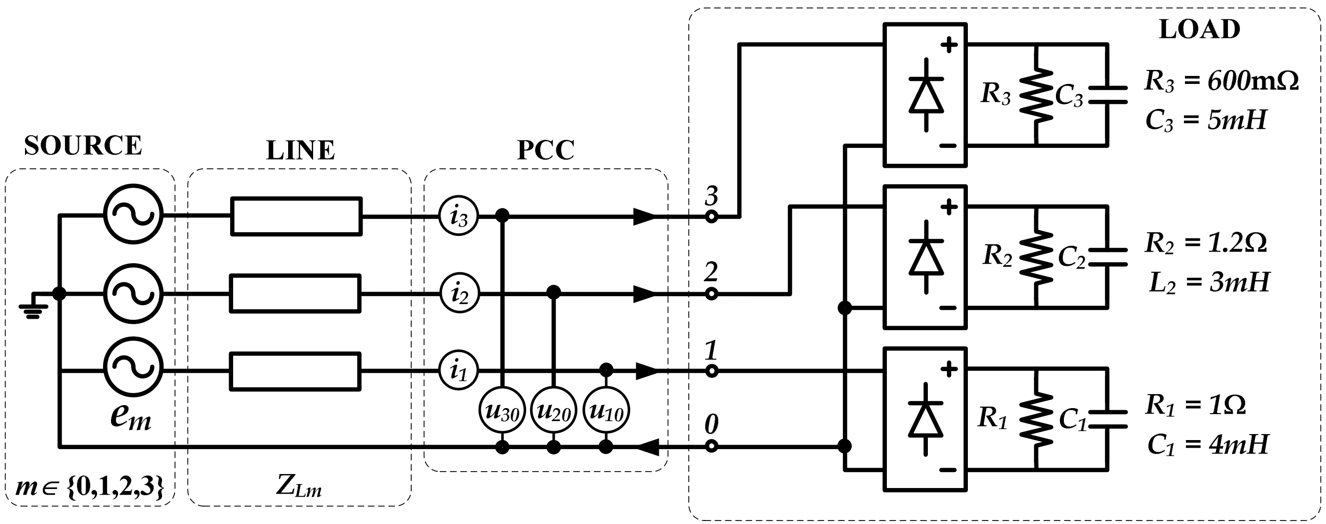
- ✓ **Power factor:** $$\lambda = \frac{P}{A} = \frac{P}{\sqrt{P^2 + Q^2 + N_a^2 + N_r^2 + D^2}} = \lambda_Q \lambda_N \lambda_D$$

Application Examples: 3-phase 4-wire

Load circuits



A. Unbalanced linear load



B. Unbalanced nonlinear load

Line parameters: R_{L1}= R_{L2}= R_{L3}= 10.9 mΩ - L_{L1}= L_{L2}= L_{L3}= 38.5 μH

Application Examples: 3-phase 4-wire

Supply conditions

Case 1: Symmetrical sinusoidal voltages

Case 2: Symmetrical non-sinusoidal voltages

Case 1	Case 2
$e_1 = 127\angle 0$ Vrms	$e_1 = Case\ (I) + \sum H_1$ Vrms
$e_2 = 127\angle -120$ Vrms	$e_2 = Case\ (I) + \sum H_2$ Vrms
$e_3 = 127\angle 120$ Vrms	$e_3 = Case\ (I) + \sum H_3$ Vrms

- ✓ **In case 2 the terms called ΣH represent the harmonic contents of the phase voltages.**
- ✓ **Each phase voltage includes 2% of 3rd harmonic, 2% of 5th harmonic.**
- ✓ **The phase angle of each harmonic term is the phase angle of the fundamental voltage (as in Case 1) multiplied by the harmonic order.**

Application Examples: 3-phase 3-wire

Case A: Unbalanced linear load

	Case A.1		Case A.2	
	PCC	Load	PCC	Load
***A* [KVA]**	95.522	98.722	95.263	96.571
***P* [KW]**	65.224	67.862	65.231	65.089
***Q* [KVA]**	67.698	69.813	67.729	69.597
***N* [KVA]**	15.158	16.334	15.163	15.582
***D* [KVA]**	0.017	0.074	1.627	1.649
λ	0.6828	0.6874	0.6847	0.6740
λ_Q	0.6938	0.6970	0.6937	0.6831
λ_N	0.9872	0.9862	0.9872	0.9869
λ_D	1.0000	0.9999	0.9999	0.9999

The load is penalized for its unbalance, especially in case A.1 (sinusoidal and symmetrical supply voltages)

Application Examples: 3-phase 3-wire

Case B: Unbalanced nonlinear load

	Case B.1		Case B.2	
	PCC	Load	PCC	Load
A **[KVA]**	93.267	94.007	89.494	91.207
P **[KW]**	63.909	63.334	62.738	62.763
Q **[KVA]**	33.274	35.376	33.599	36.159
N **[KVA]**	20.873	21.143	20.619	21.296
D **[KVA]**	55.421	55.916	50.190	51.171
λ	0.6852	0.6737	0.7010	0.6881
λ_Q	0.8870	0.8730	0.8815	0.8665
λ_N	0.9605	0.9601	0.9605	0.9594
λ_D	0.8043	0.8039	0.8279	0.8278

The apparent, reactive and unbalance power accounted to the load are higher than those computed at PCC